METAPHOR IN YOUNG CHILDREN'S MENTAL CALCULATION

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Abstract: In this study 7-9 year old children performed mental calculations and were then asked 'What was in your head when you were thinking of that?'. The language they used in response appeared to be related to their previous classroom activities. It will be argued that the metaphoric nature of this language is an indication of the influences on the children's conceptual and procedural knowledge of mathematics. These metaphors may be 'theory-constitutive' and could be restrictive rather than 'generative' for the children's future learning

INTRODUCTION

As a working definition **'**metaphor**'** can be taken to be the use of the language of one context to communicate thoughts about another. Paivio (1979,p150) suggests that:

> For the student of language and thought, metaphor is a solar eclipse.

Paivio makes the point that during a solar eclipse aspects of the sun's appearance may be viewed that are usually not apparent. In the context of the study of thinking the suggestion is that metaphor gives us a view of the thought process that would otherwise not be available to us. In this paper examples of children's language, used to describe a mental calculation, will be given to illustrate the ways in which their thinking may have been influenced by their classroom activities.

The implication for the classroom is that children's use of words may indicate the ways in which they think and thus attention to children's language can provide teachers with information about differences and commonalities in these ways of thinking.

It can be argued that children's language use might merely indicate the norms of their speech community but I will follow Lakoff and Nunez's (1997, p32) suggestion that:

> Metaphor does not reside in words; it is a matter of thought. Metaphorical linguistic expressions are surface manifestations of metaphorical thought.

In 'Metaphors We Live By' Lakoff and Johnson (1980) have suggested that metaphor both indicates *and shapes* our conceptualisations. The language derived from classroom activities might thus both indicate and shape children's conceptualisation of number and number operations. In this case the pedagogic representations used by teachers (words, drawings, physical materials, real life contexts) may provide the 'metaphors we calculate by'. This is not an argument for linguistic determinism but rather that language and thought are interdependent. We talk the way we do because we think the way we do and we think the way we do because of our previous experiences. Our language is a result of individual and collaborative sense-making. Metaphor is thus an indication of what has been *met afore* (before).

COGNITIVE METAPHOR

The use of the language of one domain to communicate thoughts about another may indicate a ‘cognitive metaphor’. This implies that our conceptualisation of the target domain has the same structure as the source domain. Reddy (1979) gave examples of the ‘conduit metaphor’ of communication to illustrate that everyday behaviour reflects our metaphorical understanding of experience. In his view the English language suggests a preferred framework for conceptualising ‘communication’ that can bias the thought process. For example use of “give me an idea” or “put it into words” appears to assume that language transfers thoughts. Words are seen as containers of thought and language functions like a conduit to transfer thought between people. Reddy argued that the language of containers and conduits is so all-pervasive that it requires great conscious effort to communicate about communication in any other way. One of its pernicious influences is that teachers and learners can feel cheated when the supposed transfer of knowledge from one to the other is not achieved.

Schon (1979) used the term ‘generative metaphor’ to imply that the perspective of one domain of experience is applied to another. He suggested that metaphor may account for the way in which we think about things, make sense of reality, solve problems and subsequently frame questions about reality. In particular, when problem posing derives from the generative metaphor the range of solutions is constrained. This is because attention is focused by the perspective determined by the generative metaphor. His examples were taken from the field of social policy. For example if the problem is described as ‘fragmentation’ then the obvious solution is ‘amalgamation’. Framing the problem by use of the word “fragmented” in relation to social services generates the solution of joining them up whilst if they are called “autonomous” then those services may be left alone. The restrictive nature of cognitive metaphor was noted also by Pylyshyn (1993) in respect to the use of visual terms for mental images. He suggested that these metaphoric terms do not explain the phenomenon but they give us a way of describing it that may inhibit further thinking, because we feel no need to explain processes in the source.

This seems to suggest that conceptual metaphor may constrain thought. Lakoff and Johnson (1980), however, argue that communication is based on the same conceptual system that we use in thinking and acting so that the language a person uses is a source of evidence for what their conceptual system is like. In an elegant study, which attests to this view, that language and thought are interdependent, Pederson (1995) attempted to test the relations that might exist between linguistic and non-linguistic thought. He questioned whether linguistic parameters determine the non-linguistic cognitive operations, whether the reverse might be true or whether there is a general cognitive structure for both. He recognised that, even when a difference in cognitive performance is demonstrated by different linguistic populations, there might be other cultural or environmental factors which determine the difference. His comparative study thus used two populations sharing the same cultural features but differing linguistically. He chose two Tamil sub-communities in Southern India. His experiments involved spatial relationships, because spatial reasoning is believed to be based on a common human perceptual system and on universal elements of human environments, such as gravity and permanence of objects. Thus effects which are associated with different language in this field ought to be apparent, if they exist.

The two groups chosen differed in their use of terms for position. One group habitually used relative position terms (left, right, front, back) the other habitually used absolute position terms (north, south, east, west). He found that performance in his experiments *did* correlate with language use.

Subjects were asked to look at objects on one table then go to another table in a different room and stand at 180° to their original position. They were tested on: memory of position of objects, simplified return journey for a complicated outward journey and relative positions of three objects. Pederson found that those who used relative position terms gave answers which were opposite in orientation to those who used absolute position terms. He thus claimed this as evidence of linguistic relativity. His study suggested that whether the internal mental coding was linguistic or not, language and thought seemed to be based on the same conception.

METAPHOR IN MATHEMATICS

Concrete-material representations used for place value such as Dienes blocks, hundred squares and number tracks, which are intended to be 'structure-oriented', are sometimes referred to as 'physical metaphors' (Resnick and Ford, 1981). It can thus be argued that these representations are intended to provide the source for metaphors. Sfard (1994) suggested that in mathematics the meaning of abstract concepts is often created through the construction of an appropriate metaphor and that metaphors are projections from the tangible world onto the universe of ideas. In her view 'reification' (when mental objects replace processes) is the birth of a metaphor. Thus, in mathematics, metaphor can bring the target concept into being rather than simply make comparisons between existing concepts (Sfard, 1997).

Whilst literary metaphor may work at the microscopic word- or sentence- level, a macroscopic view is also needed for the systems of metaphoric models used in teaching and learning mathematics. Pimm (1995) has drawn attention to 'manipulation' as the core metaphor for 'doing' mathematics.

The manipulation of concrete referents of numbers, for instance adding more counters or taking some away, provides the physical and linguistic metaphors for mathematical operations. Addition, putting together or counting more, then becomes synonymous with increasing. Subtraction becomes synonymous with taking away, thus decreasing. When the metaphor 'Subtraction is Take Away' is the theory-constitutive model for a child then subtraction of negative numbers becomes problematic. Similarly the metaphors 'Multiplication is Lots Of' and 'Division is Sharing' leave children ill equipped for calculations with anything other than natural numbers. In the same way manipulation of symbols can provide restrictive metaphors, for example 'Multiplication By Ten is Adding a Nought'.

It has been suggested (Lakoff and Nunez, 1997) that there are three basic 'grounding' metaphors for arithmetic:

'Arithmetic is Object Collection'	numbers are collections of objects and operations are acts of forming collections.
'Arithmetic is Object Construction'	numbers are physical or mental objects and operations are acts of object construction.
'Arithmetic is Motion'	numbers are locations on a path and operations are acts of moving along the path.

The first two may be seen as instances of the more general 'Arithmetic is Object Manipulation'.

Previous studies of childrens' mental calculation have analysed the strategies used rather than the language. See for instance Carpenter, Hiebert and Moser, 1981; Beishuizen, 1993; Thompson, 1997a

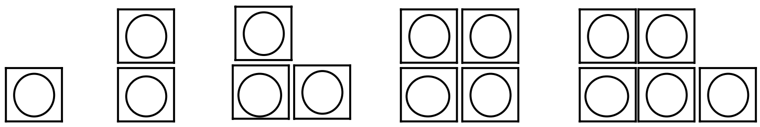
METHOD

This study attempted to answer the question: 'What evidence is given in the language used by children to describe their mental calculations that previous classroom activities have influenced their thinking about number and number operations?'

Lesson observations and pupil interviews were conducted with two classes from Year 3 (pupils aged 7 and 8 years) in Bright Cross, a school for children aged 5 to 11 years in a large middle-income village near Birmingham U.K., from September 1998 to July 1999. The same pupils were also observed and interviewed in the following year. The 80 children in the whole year had been placed in one of three groups for mathematics lessons based on their previous attainments. Lessons with the high attainment and the middle attainment groups were observed and a sample of 14 pupils from the first and a sample of 12 pupils from the second were interviewed in December, March and July in each year. The samples were chosen to represent the spread of attainment levels in each group.

Over the six interviews 45 calculation questions were used, no more than eight in an interview. Each was presented verbally and followed by the question "What was in your head when you were thinking of that?" The children were not told whether the answer was correct or incorrect. All children received the same questions. They did not use pencil and paper. They did not have any physical manipulatives to use during the interview. Two questions (17 add 9 and 48 add 23) were repeated in each interview. Other questions reflected recent classroom work (597 add ten, what is the difference between 27 and 65? 7 multiplied by five? what is a third of 48? etc).

In order to discover whether different language might develop from different classroom activities a second school was visited. The school chosen for comparison was in Peacehaven on the south coast of England. This school has adopted a very different approach to the teaching of number and number operations to the approach used in many UK schools. The teachers at Peacehaven use Stern plates (see Fig. 1) and other visual representation of number, as an alternative to the use of counting to introduce number and number operations (Wing 1996). Children here are encouraged to discover and learn number bonds by 'fitting' these number pattern plates together rather than by counting-on.



…etc

Fig 1
Stern plates

Only one interview was conducted at Peacehaven with a sample of six Y2 pupils. They were selected by the teacher to be representative of the Y2 cohort. The children in the sample for each school all had English as a first language. None was from an ethnic minority.

RESULTS

Dienes blocks and small cards with digits printed on them were commonly used to represent numbers in the lessons I observed. Pupils were encouraged to group the tens and units separately to perform addition and subtraction. They "put" the tens "together" and the units "together". The separate digit cards were also manipulated. To demonstrate addition of ten, for instance, the tens digit was "changed" whilst the rest were "left alone".

The children's language reflected the classroom activities when they described their mental calculations. For instance:

48 + 23 Well I **got the tens** and then added them up then that - made, 60 and then 8 and the 3.

97 + 10 I **move the 7** and then I knew which one, I know I got to add a ten on, I can't add a ten so I **put the one in front** of it and then a **nought in the middle.**

Round 246 6 is nearer to 10 than nought and if you just add them on you can just do it without the - like **push** the hundreds - away, and the tens away and then you just **do it** without it then you **put them back together** again.

30 + * = 80 I know 5 add 3 is 8 so you just add, **change** the 30, the 3 **into** the 30, and add 5, that should be 50.

200 more I was trying to **get away** all the units um and tens and hundreds, and, add another one on to the thousands.

The three grounding metaphors of Lakoff and Nunez can provide a coarse categorisation of the language that children use in describing their mental calculations. The categories may be termed ‘collection’, ‘motion’ and ‘construction’. An example of responses to “17 add 9” and “65 subtract 29” is given to illustrate each of these categories:

‘collection’

Children used the language of manipulation of concrete objects and counting. This included counting on their fingers and counting in tens. Use of the words “add”, “take”, “more”, “gives”, “with” was characteristic of these responses.

17+9 ... I was **counting up in ones**

65-29 **took away** twenty off the number then took away **the rest that was left over**.

‘motion’

Children used language related to position and directed movement such as the words “go”, “up”, “down”, “back”.

17+9 ... I thought that if it was a ten it would be twenty-seven then I **went back** a number

65-29 I **started at** twenty and counted **along to** sixty

‘construction’

Children used the language of manipulation of symbols. This included place value language, derived facts and known facts. Use of the words “is”, “equals”, “make”, “sums”, “the”, “it” was characteristic of these responses.

17+9 ... I just add **one from seven** to **the nine** so it **becomes ten** then add **the ten** onto **the sixteen**

65-29 I **rounded twenty-nine to thirty** then I took thirty from sixty-six

Over all interviews 51% of responses were categorised as ‘object construction’, 34% as ‘object collection’ and 15% as ‘motion’.

COMPARING COMMUNITIES

The language used by individual pupils at Bright Cross was sufficiently different to suggest that they had formed their own mental constructions yet sufficiently similar to suggest it was rooted in the common classroom activities. An alternative reason for the commonality of the language could be that all young children talk like this about mental calculation.

However when the Peacehaven children were interviewed their language *and* methods were different from those used at Bright Cross. Y2 children at Peacehaven were asked to calculate 17 add 9 in their heads and then were asked "What was in your head when you were thinking of that?" They had no materials or paper available to use.

Barbara	That was where I had the 17 on the **plates**, and then I had the 9 - I **chopped the 9** - took 3, chopped it off and then there was, I added the 3 onto the seven, teen and then I had the 6 left and then that was 26.
Lawrence	I like see like 17 cars and 9 people and **I put them in the pattern** then I add them up I took the three off - the 9 and I **put it onto** the 17, that made 20 then there were 6 left from the 9.
David	**I got numbers up to a hundred in my head** and I got 9 and 17 and I added the 3 onto the 17 and added the 6 onto the 20.
Clive	I got a picture of the 7 and I got a picture of the 9 that looked like **them joined together and making the number** I just said.

An important point here is that none of the pupils interviewed at Peacehaven counted. In contrast at Bright Cross 11 of the 26 pupils interviewed in the main study still counted for "17 add 9" at the end of Y3 (a year older than the Peacehaven children). The pupils were of similar achievement levels. This might be seen as evidence that the children's early classroom activities have influenced their methods of calculation *and* the corresponding language that is used.

DISCUSSION

The use of metaphor has been presented as an indication of understanding of one situation in terms of previous experience. The notion that conceptual metaphors provide a basis for both language and behaviour might imply that language and behaviour are both indicators of the underlying conceptualisation. The examples of metaphoric language given for the pupils of Bright Cross serve to illustrate that language of manipulation of materials and symbols and the language of position on a number line was used when children made mental calculations. The theoretical perspective presented suggests that these children may possibly have used different conceptualisations for number operations based on their classroom experience of manipulating materials or symbols and using the number line. The fact that metaphors of object creation were three times more common than metaphors of motion may be because the majority of the pupils' time in the observed classrooms was given to written algorithms and very little time given to number line activities. Pupils in the comparison school used quite different language and methods which reflected their different classroom activities.

If the language use apparent in these primary school classrooms is indicative of generative cognitive metaphors then they may be 'restrictive'. For instance children, in using the language of counting and grouping of objects, may be prevented from developing deeper understandings of arithmetic. The metaphoric language that children use may indicate that their thinking is rooted in one particular pedagogic representation. If this is the case then future teachers of these pupils may have difficulty communicating with them if they use a different metaphor for calculation.

We may, however, use a variety of different metaphors which indicates that we have different ways of thinking about things. We talk of 'ideas', for instance, as: plants (ideas come to fruition), products (ideas need refining), commodities (ideas are worthless), resources (we use an idea) etc. The language we use indicates how we are conceptualising 'ideas' at the time of the utterance. Furthermore successful functioning in daily life can require shifting of metaphors when we need to use a variety of metaphors to understand some concepts.

These children may prove not to have a restrictive cognitive metaphor but teachers need to be aware of the difference in conceptualisation that the different language may imply and help pupils develop their range of metaphors.

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