THE EVOLUTION OF GRAPHIC REPRESENTATIONS IN A VYGOTSKIJAN PERSPECTIVE

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Abstract: In this paper we propose a constructivistic approach to the Cartesian plane. We look at it as a linguistic-graphic structure grounded in a cognitive function recognizable in the natural drawings of children. Then we hypothize for it a development process following the same path as in Vygotskjian model for natural vs. scientific language. Finally, we present a teaching experiment whose intent is to provide some empirical evidence for our hypotheses.

1. Introduction and theoretical framework

Graphic representations, because of their perceptual character, constitute a powerful support to conceptualize abstract entities, as long as they are recognized as cultural tools. They favour the construction of concepts, in the dynamic process understanding/development, exactly in the same way as language does. According to this, the number line is not simply one of the many possible ways of representing numbers, it is a structure that supports the formation of the concept of order relation, among numbers and in a general sense. In the same way the Cartesian plane, far from being just a tool for geometrical visualization of algebraic formulas, can be viewed as the visual-operative structure that allows and promotes the construction of the concept of relation between two variables.

It is well known that students encounter several difficulties in managing graphs, not only as tools for transferring properties from one mathematical domain to another (geometrical and algebraic), but especially as tools for representing and interpreting natural phenomena. Many authors have reported about teaching experiments on this subject, but, as far as we know, all of them refer to more or less teen-ager students. See for example (Bazzini, 2001) and (Robutti & Ferrara, 2001), also for an extensive bibliography.

We have tried a very early approach to the problem. In *Mind in Society* (Vygotskij, 1978), Vygotskij asserts that language allows children to "cognitively perceive", namely to substitute the direct perception that human beings share with animals with a mediated one. Analogously, in the case of motion of a material point, we claim that the Cartesian plane is the "linguistic" structure that allows to cognitively perceive the space-time relation, while what is under our direct perception is the trajectory of motion. (see for this what (Lakoff & Núñez, 2000) say about the source-path-goal general schema).

We quote again Vygotskij: "The general law of development states that awareness and control are peculiar properties of the higher stage of development of any function.

They come late into being and are necessarily preceded by a stage of unconscious and unintentional functioning of any form of mental activity. In order to be aware of something, we must already possess that thing. In order to control something, we must have at our disposal the thing on which to exert our will." (Vygotskij, 1934, page 236 of the Italian edition, our translation). In our opinion, the Vygotskijan model of development of higher psychical functions can be applied to the cognitive function of graphic language: in order to do this, a) a genuinely cognitive function must be recognized in the spontaneous drawings of children, together with their more widely known expressive and communicative functions; b) this cognitive function must follow a cultural process of development which should be promoted in the educational process.

According to this framework, we believe that the above mentioned difficulties depend on an incomplete development of the cognitive function of graphic representation. This cultural function ("higher", in the sense of Vygotskij), does not emerge suddenly or passively, but grows slowly and differently for different pupils in a long and complex evolutionary process supported by learning. Therefore, the answers to students' failures need to be found very early, by designing activities in primary schools, which promote awareness and development of the cognitive function of graphic language.

Usually, in the first years of school, children are requested to utilize iconic language only to represent their experiences in the form of spontaneous drawings. These activities essentially fulfil an expressive or, less often, a communicative function, while children's cognitive structures, incorporated in their drawings, are quite neglected. A few years later, 8-to-12-years-old children are introduced to the concept of Cartesian representation in a mathematical context. This reinforces the prejudice of an outstanding dichotomy between "natural" drawing and mathematical representation, which closely resembles the Piagetian dichotomy between natural and scientific thought.

On the contrary, when the role of graphic representations as a "scientific language" is acknowledged, and therefore the Vygotskijan "general law of development" is assumed for them, some noteworthy consequences derive in the educational practice: it becomes necessary to take into account the roots of this language in the first years of school, and therefore to favour children's ability to produce and utilize graphic representations "to understand" and to share their achievements. In much the same way as the verbal language gradually develops, and its syntactical and semantical structures become more and more rich and complex, the graphic language can (and, we say, must) develop, starting from a prevalent narrative-pictorial function and integrating more and more deeply its cognitive and communicative functions. In a Vygotskijan perspective, the crucial role of the teacher is that of acting as an (adult) mediator of this cultural process: so he/she will create suitable contexts, stimulate motivation and favour social interaction.

In the following sections we will report about a teaching experiment designed and performed on the basis of the above research hypotheses. Some particular transition episodes will also be carefully analyzed, and some conclusive remarks will follow.

2. The teaching experiment

The activity we are going to describe is a part of a wider experimental project that started in 2000-2001 in the first class of a primary school and continues in 2001-2002 in the second class (Iannece, Nazzaro & Tortora, 2001). This project is in turn designed and carried out in the spirit of "Capire si può" ("It is possible to understand"), a long term research project directed by Paolo Guidoni (for theoretical references see (Guidoni, 1985)). The whole subproject aims at leading children to undertake the long cultural path connecting their various experiences of movement to the scientific concept of motion. A lot of bodily knowledges (related to perception, self-acting motory schemata, etc.), often unconscious, are tied to those early experiences. Our approach starts from there in order to reach a satisfactory intellectual control of the concept of space-time relation, the purely "scientific" core of every motion.

In a Vygotskijan perspective, the process by which concepts are constructed necessarily includes the development of a suitable language. Therefore, the scientific concept of motion cannot be grasped if the Cartesian plane is not available, and in turn this "linguistic" tool can emerge and be organized only when an intellectual need based on experience requests it. In the sequel, in order to stress the singularity of bidimensional representation of motion, (in contrast with any other informal way of representing it), we will reserve to the former the term "scientific" as opposed to "natural". This choice is made in accordance with the distinction natural/scientific, occurring in the verbal language development.

The segment of the project under our attention refers to 25 seven-years-old students. It was designed and realized:

a) In order to verify our research hypothesis about the development of graphic language;

b) In order to investigate how the didactic mediation can guide and support this development.

Before and also throughout the activity, children underwent many motion experiences, in which their "cognitive unconscious", in the sense of (Lakoff & Núñez, 2000), was evoked, actualized and made explicit.

The experiment starts with the reading of a tale, freely adapted from the novel *The seven Messengers* (Buzzati, 1958). In essence, the story is the following:

"The prince of an imaginary kingdom leaves his castle for an endless trip toward the borders of the kingdom. Every day he travels for 50 km and every

night he rests together with his troops. At the end of the first day he sends his faithful knight back to the castle to bring a message. The knight covers 100 km in a day and comes back to rejoin the prince. Then again and again, while the prince goes on, the knight goes back and forth between the castle and the prince, bringing messages".

In the first stage, children go to gymnasium, where they are invited to stage the story into a play. This play provides throughout the activity a powerful cognitive support, linking action with thought. During the successive stages, children are allowed to re-enter into the play whenever they need to construct or verify by action the meanings of graphic representations. Sometimes the teacher decides about this opportunity, but often children directly ask for it (as when Daniele says: "*Mistress, I don't understand anything, let's go back to gym!*").

The remaining part of the teaching experiment is constituted by an alternation of individual drawings of the story and classroom discussions where their meanings are collectively analyzed and negotiated. Discussions are essentially among peers while the teacher's main concern is that of keeping children's attention on the shared goal of "understanding". In this way pupils become more and more aware of the link between action and reasoning provided by a "good" representation.

Special attention is devoted to the different strategies necessary to face continuities and discontinuities of the process. Let us try to exemplify this assertion. From the beginning, children have three tools at their disposal (action, natural language, drawings) in order to attain a satisfactory mental control of the situation described in the story. The combined action of these equally useful tools potentially enhances comprehension; but, unfortunately, the graphic language appears inadequate to represent the space-time relation experimented in action and verbally describable. So, our initial purpose is to make children aware of this inadequacy – a breaking point in the understanding process. Following Vygotskij and Fischbein, we believe that the awareness of such a conflict is essential to reach the scientific representation of motion. Catching the opportunity from an intuition of Daniele ("*On one side we can put the space, on the other side the time*"), the teacher suggests the use of a very rough version of Cartesian plane. A few months later Danilo, looking at the display of a computer in a CBR (Computer Based Ranger) laboratory session, will cry: "*We too did utilize two straight lines in this way, to represent space and time!*".

After the introduction of bidimensional representation, the process continues with progressive syntactical and semantical refinements, following the same path suggested by Vygotskij for the development of scientific language (Vygotskij, 1934).

3. Methodology and analysis of crucial episodes

In order to verify the validity of our proposal, namely a Vygotskijan model for the development of graphic language and its cognitive function, we need to observe not

only achievements, but mainly processes. We do this by analyzing the sequence of children's representations compared with their verbal descriptions/explanations in collective discussions (Bartolini Bussi, 1998a and 1998b).

In this report we do not summarize the whole evolutive process, but we focus our attention on two particular moments:

A) an excerpt of the classroom discussion, relative to the drawings of three pupils, Marco, Giulia and Giuseppe, that are good prototypes of different initial represensions of the story;

B) the passage where children, discussing about a shared representation (the "space-roll"), feel the necessity of a "scientific" tool.

(We apologize for our poor translation into English of the children's words, where some particular slang and dialectal expressions are inevitably missed).

A) 1 Teacher: *Let's listen to Marco.*

   2 Marco: (see Fig. 1) *This is the tree where they put the tents for the night, then the prince send the knight for the message.*

   3 Teacher: *But, where does the knight go, while the prince and his troops go forward?*

   4 Marco: *Here, look, I've put him far from the troops. Then he has arrived, and afterwards he has gone away, and again he's come back.*

   5 Sara: *The map: I don't see the map!*

   6 Titti: *Here's the map, look: the castle, the wood, the fence, the first ravine, the second ravine, the small castle, the lake, the meadow, the second small castle and the farm.*

   7 Sara: *Well, he drew the map, but he didn't draw the children and the road.*

   8 Daniele: *Time is missing: things change, when time passes.*

   9 Teacher: *What is it that changes?*

  10 Daniele: *Where people stay.*

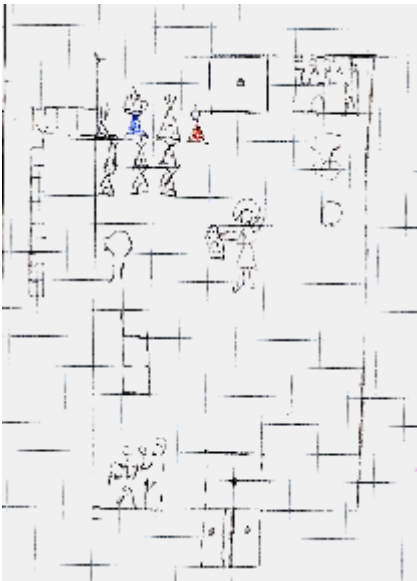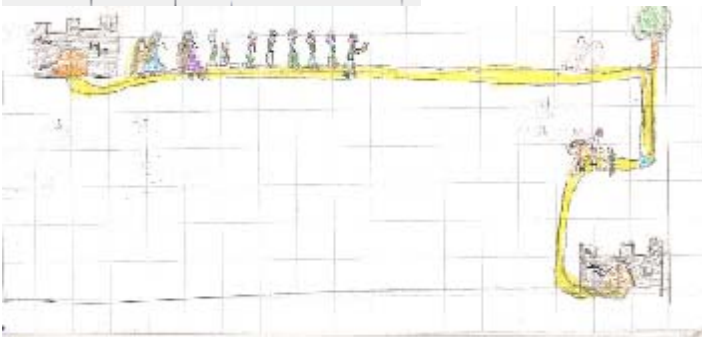Fig. 1   Fig. 2

11 Teacher: *So Sara, is she right?*

12 Daniele: *Yes, she is. We don't see day by day where the prince and the knight are.*

13 Giulia: *Here* (pointing at her drawing, see Fig. 2) *there are the troops that follow the prince and here* (idem) *is the knight, which goes back to the castle.*

14 Leo: *The sun, I don't see the sun.*

15 Giuseppe: *Nor the night.*

16 Teacher: *Giuseppe, now explain your drawing.* (See Fig. 3)

17 Giuseppe: *Here* (pointing at his drawing) *you see the castle, here the sky, here the clouds… that the night is coming, and here again the sun. Then this is the map and the knight that goes on and back, and this* (pointing at the prince) *is running too.*

18 Federica: *But in your drawing it is not shown that they go to sleep. You had to use another sheet to show that the sun was not up anymore and that they went to sleep.*

19 Giuseppe: *No, no, here there is the sun, but afterwards there is the moon.*

20 Federica: *O.k., here there are the sun and the moon, again the sun and the moon, but I don't see what they are doing with the sun or when it is night and they sleep. You had to use another sheet and make the night.*

21 Rachele: *But it is useless to represent the nights, because in the night nothing happens, they sleep and when they get up they go their way again starting from where they stopped…If I draw a mark on a sheet, and put the sheets one under the other, one understands that the prince stops now in a place, afterwards in another place…*
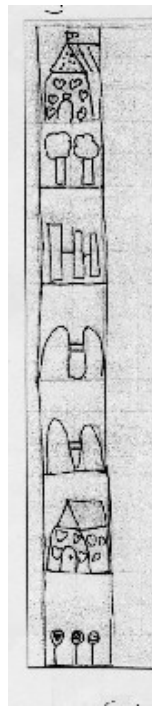

Fig. 3   Fig. 4

     The drawings made by Marco, Giulia and Giuseppe exhibit different solutions to the problem of adequately describe the story. Giuseppe (see Fig. 3) has a rudimentary intuition of bidimensionality (note that, in his drawing, the castle is big, while the signs that allude to variations of time and space are much smaller), but, as Federica says (#20), "*I don't see what* (people) *are doing*". In Marco's drawing (see Fig. 1) time is hidden in the circularly ordered mark points; and Federica's previous remark (#20) could be applied to it as well. Giulia's solution (see Fig. 2) is in a sense the opposite: the focus is on people while the "map" is neglected; the two castles drawn might be interpreted as a sign of time passing. So, different initial graphic solutions correspond to different "ways of looking and thinking" (models): we get empirical evidence of the fact that children's drawings contain what they see and feel, but also what they think and know. The above reported discussion shows how the children's attempts to explain the content of their drawings, together with the objections made

by their fellows, constitute the starting point of a collective negotiation which allows to converge toward a shared way of looking. The proposal made by Rachele (#21) marks the moment in which, from the analysis of drawings, the attention shifts to selecting the truly meaningful elements with respect to the goal of representing motion (she says: *It is useless to represent the nights, because in the night nothing happens*").

B) Starting from Rachele's proposal, children decide to realizy "space-rolls". Every roll is built by putting together the sheets in which all the halts of the prince are represented. Then, as the following discussion reports, children try to verify whether the goal is reached and the rolls express all the things that a good representation (as previously negotiated) must contain: space, time, and what people do.

22	**Danilo:**	(The roll – see Fig. 4 - represents) *time and space because making five steps requires time…*
23	**Andrea:**	*For me, it represents only the space.*
24	**Marco:**	*No, both.*
25	**Sara:**	*When we were playing, you made the sun and the moon: so you made the time…*
26	**Giuseppe:**	*But in the drawings I don't see the sun and the moon.*
27	**Ilaria:**	*The sun doesn't say to you: first day, second day…*
28	**Andrea:**	*But we are speaking about five steps and mark points, not about the teacher when she was making the sun.*
29	**Alessio:**	*No. If space and time are put together…they do not fit.*
30	**Daniele:**	*Why?*
31	**Alessio:**	*And we don't see them and…*
32	**Andrea:**	*I think that Alessio was meaning that space and time don't…for example, if you make the sun and the moon, there is the time but not the space.*
33	**Alessio:**	*Yes.*
34	**Daniele:**	*Yes, but he himself has said that this thing doesn't matter!*
35	**Andrea:**	*Yes, I've said that it doesn't matter that the teacher makes the sun and the moon, because we were speaking about our drawings. Now we are speaking about space and time and therefore this does matter.*
36	**Roberta:**	*But when you make five steps, you spend time and also space…*
37	**Andrea:**	*Well, I know that I spend time in making steps, but…*
38	**Daniele:**	*He's saying what to do in order to draw both things, how to put together space and time.*

39 Andrea: *Oh, yes, but how can we draw it?*

40 Daniele: *We must find a different way to make it clear that making steps requires time.
...We are looking for a kind of drawing where everybody understands that time
passes while they go on.*

In all the items of this discussion and in particular in the words of Danilo and Roberta (#22 and #36) the awareness of space-time relation is evident, as well as the effort to find a way for representing motion. But, while Danilo, Marco and Roberta (#22, #24 e #36) refer to their bodily "direct perception" of the relation to decide about the validity of its representation by means of the roll, Sara and Ilaria (#25 and #27) refer to the first symbolic representation of motion realized in the play, to "see" the time. What Andrea says is particularly interesting, since a progress can be recognized in his words. In fact, initially (#23 and #28) he speaks about rolls, whereas afterwards (#32, #35, #37 and #39) he comes to a generalization: both in the play and in the drawings either space or time can be seen but never in conjunction. The distinction between perceiving, understanding and representing the space-time relation confusely emerges in Alessio's words (#29), when he clearly recognizes the difficulty of putting together the two things in any drawing. This is a turning-point: from now on the inadequacy of iconic tools is acknowledged by almost every child. So Daniele (#38 and especially #40) can finally affirm that new and more powerful tools are needed.

While the discussion goes on, Daniele is drawing: he puts the marking points (i. e. the prince's halts) one above the other on a vertical line, and many small circles on a horizontal one; in every circle there is a child who traces an arising or a setting sun. Meanwhile he is saying: "*On one side we can put the space, on the other side the time*". The teacher catches the opportunity, and leads children again to gymnasium, to favour a general condivision of Daniele's intuition.

4. Concluding remarks

The problem of verifying a research hypothesis concerning development dynamics is a difficult one and requires a very long time. In particular, to be sure that an early didactic approach as ours is effective, it would be necessary to wait many years: indeed, only when pupils go to secondary school is it possible to verify whether they are able to adequately employ the Cartesian plane. However, our whole teaching experiment, not entirely examined in this report, provides some empirical evidence for our research hypothesis. Namely, we have observed occurrences of the same tokens that Vygotskij indicates as characteristic of the language development:

a)   the bidimensional representation undergoes successive syntactical and semantical rearrangements;

b) the scientific representation is interwoven with individual cognitive structures;

c) in the development process there are continuity and discontinuity stages.

In our approach real action is crucial and not simply a link between school learning and everyday experience. Indeed, we believe, in accordance with the recent theory of embodied mathematics (Lakoff & Núñez, 2000) that mathematical concepts, structures and tools can and should be grounded on bodily experience. We hope that our experimental research, in which we try to let the Cartesian plane emerge as a structure that organizes bodily experiences, could be a small contribution to the advancement of these studies.

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