INVESTIGATING THE RELATIONSHIP BETWEEN ENGLISH LANGUAGE AND MATHEMATICAL LEARNING

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This paper reports on a preliminary investigation into the mathematics learning of undergraduate students for whom English is a second language. What are the English-language competencies required to learn mathematics at undergraduate level? Is a general English-language competency sufficient, or is a specific competency required, such as a knowledge of mathematical discourse? Do L1 and L2 students process mathematics in different ways? A test that assesses mathematical understanding in five ways was given to first year mathematics students in a New Zealand university. The results indicate that L2 learners have larger than expected difficulties with text, and that they mistakenly rely more on symbolic modes of working. This study provides directions for further research.

Este artículo informa sobre una investigación preliminar de los conocimientos de los estudiantes universitarios para quien el ingles es un segundo idioma. ¿Que capacidades de la lengua inglesa son necesarias para aprender matemáticas en la universidad? ¿Es suficiente para tener una capacidad general, o es necesario para tener una capacidad específica, tal como el conocimiento del discurso matemático? ¿Los estudiantes L1 y L2 tratan matemáticas en formas distintas? Un test que valore el entendimiento matemático en cinco formas se daba a estudiantes del primer año en una universidad de Nueva Zelanda. Los resultados indican que L2 principiantes tienen dificultades con los textos mas grandes que como era de esperar, y se fian erróneamente mas en modos simbólicos para trabajar. Esta investigación da las direcciones para otras investigaciones.

Educational institutions in developed countries are increasingly accepting immigrant, refugee or minority students whose language background is not the same as the language of instruction. There is thus increasing interest in language requirements for tertiary study, and in the provision of support programmes. This investigation was prompted by a review of the English language requirements for tertiary institutions in New Zealand. The focus is mathematics because this subject is often taken by students with low English language abilities who are under the impression that they will not be so disadvantaged. This subject is probably perceived by them to be relatively language-free.

Thus the motivation for the study is to investigate the level of English language proficiency required to successfully undertake undergraduate mathematics study at a university. Are students who have English as an additional language (L2) performing differently in undergraduate mathematics than students who have English as a first language (L1)? Is there a minimum English proficiency threshold below which the student will be significantly disadvantaged, or is mathematics at this level language independent? Are the answers to these questions the same for students learning mathematics in French or Spanish or other world languages? If there is a language-based disadvantage for L2 students, is this related to general English ability, or is it related to mathematical discourse in particular?

Within mathematics, it is relevant to ask whether L1 and L2 students learn mathematics in similar ways. Several modes of communication are possible: textual, symbolic, diagrammatic, graphical. Do L1 and L2 students use these modes with similar frequency and success? Do they, or are they able to, swap between modes with similar facility? Does the mode of presentation make a difference to their ability to interact with the mathematical question?

The study reported here was a first step towards addressing these questions. First year mathematics students were asked to answer mathematics questions using each of five modes of communication. Profiles of L1 and L2 students, and profiles of L2 students with different English proficiency levels, were constructed and compared. The intention of this preliminary study was to get some indication of where the differences between L1 and L2 learners might lie, to estimate the extent of these differences, and to generate more specific research questions that would give rise to significant, as opposed to indicative, results.

English for Specific Purposes

Estimates of the variability in academic performance due to English language ability are that this is up to 10% where the International English Language Testing Service (IELTS) band is over 6.0 (as is the case for university students), and that it is higher for humanities and social science subjects in comparison with mathematics or science subjects (Elder, 1993; Graham, 1987). However many L2 students attend English for Academic Purposes (EAP) courses in preparation for university study. The content of these courses vary, but commonly include: study skills such as those needed for library use, research and autonomous learning; strategies and skills for vocabulary learning, reading, listening and speaking in academic contexts; academic writing; and specific study of general academic vocabulary. In addition, there are specific language courses dealing with mathematical terminology, syntax, understanding symbols, and listening to lectures (Perkins, Barton, & Brown, 1994).

Dudley-Evans and St John (1998) describe English for Specific Purposes (ESP) using both absolute characteristics (such as "meets specific needs of the learner") and variable characteristics (such as "assumes a basic knowledge of the language system"), but they emphasise ESP reflecting the discipline it serves. It thus includes both EAP and the mathematics-oriented course described above.

ESP has developed from the 1960s and 70s when it was regarded as register analysis, for example, the grammar and vocabulary of technical English was compared with general English. The focus shifted in the 1980s towards discourse analysis where the form of the language was related to its use (Dudley-Evans & St John, 1998, p22). Another development was the skills-centred approach that investigated the thinking processes underlying language use, and that led into a learning-centred approach focussed on the learning needs of the student (Hutchinson & Waters, 1987). Such developments lead to the conclusion that, should this study indicate a need for ESP courses (either EAP or ESP-Mathematics), then considerable work is necessary to understand the language needs of students with respect to mathematics and mathematics learning.

Can we use models from applied linguistics to understand the interaction between language and mathematics performance? Douglas (2000) proposes a model in which strategic competence mediates between background knowledge and linguistic competence. Strategic competence has two parts: metacognitive strategies and communication strategies. Communication strategies are those that bring relevant content knowledge to the linguistic task at the right time in the right way. This can easily be re-interpreted as bringing relevant linguistic knowledge to the mathematical task. Metacognitive strategies are non-language competencies that contribute to the linguistic task. Re-interpretion characterises metacognitive knowledge as non-mathematical competencies that contribute to a mathematical task, for example, the strategy of using trial and error. These types of strategies might be addressed in an ESP-Mathematics course.

What does the literature in applied linguistics lead us to expect in a study investigating mathematical performance across groups that differ in English language abililty? If we are to investigate the interaction between language knowledge and content knowledge, then we must first be able to distinguish between them. This is difficult because content knowledge is communicated through language. Potentially, mathematics offers the opportunity to separate these because some mathematical concepts can be communicated diagrammatically or symbolically.

A major study of reading performance (Clapham, 1996) showed that, for postgraduate level tasks, test takers performed better at reading tests when they were in their subject area. At undergraduate level no significant difference was found. Furthermore, poorer English speakers did not benefit from their background knowledge (this was a threshold effect, not a steady increase). On the other hand, there was another threshold, above which background knowledge mattered little. That is, proficient English speakers used their language skills to compensate for lack of background knowledge. Clapham's study is the reverse of what is being asked in this paper. We wish to know what is the effect of English language ability on mathematics tests, not what is the effect of mathematics ability on language tests. Nevertheless, we can interpret Clapham's results for our current study. They lead us to expect that, at undergraduate level or below, mathematical performance is affected by general English proficiency rather than specific mathematical English proficiency. It also indicates that L1 students will not be affected by whether mathematical tasks are presented in English text or symbolically or diagrammatically.

Mathematical Discourse

Research in multilingual settings has identified language as a vehicle for mathematics learning as important areas of investigation (Gorgorió & Planas, 2001). Research in schools where students have very poor ability in the language of instruction have shown the depth and complexity of the learning disadvantage, including, for example, a flow-on effect to the use of visual imagery (Gorgorió & Planas, 2001) and exclusion from significant mathematical discussion (Setati & Adler, 2000). In undergraduate mathematics learning, some competence in the language of instruction can be assumed, and significant learning may take place in the first language where there is a large group of learners with that language. However, texts, lectures and assessments take place in the language of instruction, so to what extent do language factors impinge on mathematics learning?

Research on bilingualism and mathematics learning (see Secada, 1992, for a review) shows a significant relationship between language and mathematics learning, although the situation is admitted to be complex. Cummins (1986) long ago postulated threshold levels at which advantages may apply for speakers of more than one language. Are these effects apparent in undergraduate mathematics? What are the language characteristics that create advantage or disadvantage? Investigating these questions requires a close examination of mathematical discourse.

Before considering mathematical discourse itself, it is important to acknowledge the other components of mathematical communication: Douglas (2000, p. 60) sets out a framework including: setting, participants, purpose, form and content, tone, language, norms of interaction, and genre. Although not considered in this study, these may be as important as the discourse itself, and need to be borne in mind for more detailed future studies.

There is a considerable body of literature that examines the nature of mathematical discourse. Halliday (1975) wrote:

> We can refer to a 'mathematics register', in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is, not mathematics itself), and that a language must express if it is used for mathematical purposes. ... It is the meanings, including the styles of meaning and modes of argument, that constitute a register, rather than the words and structures as such. We should not think of a mathematical register as solely consisting of terminology. (p. 65)

The mathematics register in English is the distinct way in which mathematical meaning is expressed in that language. Dale & Cuevas (1987) describe it in terms of unique vocabulary and syntax (sentence structure), and discourse (whole text features). For example, vocabulary characteristics include the use of common words with specialized meanings; syntax characteristics include increased use of logical connectives, and discourse characteristics include increased density of meaning, increased use of passive voice, and the need for multi-directional reading (for detailed examples see Dale & Cuevas, 1987; MacGregor & Moore, 1991). Some of the characteristics of natural language carry over into mathematical discourse, for example, sentence word order, logical structures, and grammatical conventions. Thus mathematical discourse is a mixture of characteristics that are peculiar to mathematics, and characteristics that derive from natural language. There are studies that examine languages other than English for characteristics that might affect mathematics learning in that language, for example Galligan's review of features of Chinese (Galligan, 2001). Han & Ginsburg (2001) showed that Chinese terminology is "clearer" than English, and this affects performance. However no studies of mathematical discourse in other languages have been found, so we do not know how different they might be from English mathematical discourse.

It is accepted that L2 students need to learn specific mathematical technical vocabulary. However, there is also an unanswered question as to whether L2 students learning mathematics need to re-learn mathematical discourse in the new language. To what extent would L1 learners also benefit from instruction in mathematical discourse? Further, does this discourse have different characteristics at different levels of mathematics?

Another aspect of this study is the difference between textual, diagrammatic, symbolic and graphical representations of mathematical concepts. However, because the main focus is language, it is not intended to elaborate representation theory in this paper. Suffice to say that a considerable literature exists (e.g., English, 1997; Goldin & Janvier, 1998). There is also, in embodied cognition, the idea that all representations are metaphorical and language based (Lakoff & Nunez, 2000). Thus it is acknowledged that the assumed possibility of presenting questions about mathematical concepts in five independent ways needs to be questioned before substantive conclusions can be drawn.

This Study

This initial investigation of the English language basis of mathematical understanding of undergraduate mathematics students used a test that presents mathematical questions using five different modes: general English text; mathematical technical text; symbols; diagrams; and graphs. Four mathematical concepts were chosen, each of which should be understandable by first year undergraduate mathematics students. A short question testing understanding of this concept was written in each of the five modes. The general English text mode uses words predominantly from the 2000 most frequently occurring words (Nation, 1996), while the technical text mode uses specialised language that students at this level should understand. Both these modes use a minimum of numbers or symbols. No text at all is used for the symbolic, diagrammatic and graphical modes.

During October, 2002, and January, 2003, two first year undergraduate mathematics classes at The University of Auckland were invited to attend an extra tutorial at which the 30-minute test was administered. More than half the population of about 1000 students were L2 students. A total of 54 L2 and 29 L1 students volunteered to take part..

A methodological problem for this study was to prevent students carrying their understanding of a mathematical concept from one of the presentation modes to another. It was decided that each student should only be asked each mathematical question using two modes, and that the order of these presentations should be randomised. For this study it was assumed that the L1 and L2 groups were each homogeneous with respect to mathematical ability, and that the number of students was sufficiently large for the results of the tests to be amalgamated in these groups. It was further assumed that the amalgamation would give a profile in which success in each question, and in each mode of each question, could be taken to be independent.

The test papers therefore consisted of eight questions. In each test, each of the four mathematical questions was presented in two ways: one of the textual modes (general English text or technical text), and one of the other modes (symbolic, diagrammatic, or graphical). The combination of questions and modes of presentation were randomised, and randomly distributed to the students. Each question was on a separate page, with blank spaces in which students were encouraged to note their working. Students were asked to attempt questions in the order in which they appeared in the test, and not to return to earlier questions. Each test also asked students their first language, and, where that language was not English, what prior mathematics learning in English they had had, whether they had done an EAP course, and their IELTS or TOEFL score if they had one.

The results were processed by making a subjective judgement of whether the question was understood, marking its correctness, and recording the mode of any working. Understanding was judged by examining the working and answer, and deciding whether they were in line with the question posed. If no working and no answer were shown, then it was assumed that the question was not understood. The results were grouped for L1 and L2 students, and for different English language backgrounds of L2 students. No significance testing was performed because the questions and evaluation criteria were not considered robust. This was a first study and question design and analysis all need more work. All conclusions are therefore to be regarded as interesting hypotheses that need rigorous confirmation.

Results

What Disadvantages are L2 Students Experiencing?

To investigate the problems created by text, all text questions were amalgamated. The results show a 20% difference between L1 and L2 students in understanding of text questions, and no difference in understanding non-text questions (symbolic, diagrammatic or graphical). The percentage of correct answers, given that the question was understood, was about 70% for both groups for all types of questions, indicating that their mathematical abilities were about the same.

This 20% difference in understanding may not translate into such a high difference in mathematical undergraduate achievement for two reasons. First, in most mathematics examinations, questions are written in a combination of modes, thus the lack of textual understanding can be overcome using another mode. Second, in an examination only a proportion of questions are answered correctly. Thus the 20% difference is reduced by those questions that would have been answered wrongly anyway. We estimate that this research indicates a difference in achievement of at least 10%, that is, at the highest levels indicated in the literature for undergraduate university students. This is unexpectedly high for mathematics.

	Total Number Questions	% Understood	% Correct	% Correct if Understood
All Text Questions				
L1 Students (n=29)	116	91	63	69
L2 Students (n=54)	216	72	50	70
All Non-Text Questions				
L1 Students (n=29)	116	86	63	73
L2 Students (n=54)	216	87	60	70

Table 1 Understanding of Text and Non-Text Questions

When the text question differences are broken down for the L2 students, it is not surprising that the level of understanding is greater for those who have been learning mathematics in an English-speaking environment for 6 years or more. It is surprising that these more experienced students still have more than 10% understanding disadvantage compared with L1 students. Similar results occur for those who have higher and lower IELTS scores (although the number of such students was small).

All Text Questions	Total Number Questions	% Understood	% Correct	% Correct if Understood
L1 Students (29)	116	91	63	69
L2 Students (54)	216	72	50	70
Less than 6 Years (34)	136	69	50	72
6 or More Years (20)	80	78	51	66
IELTS <= 6.0 (17)	68	71	54	77
IELTS >= 6.5 (11)	44	80	59	74

Table 2 Analysis of L2 Students

Profile Differences Between L1 and L2 Students

The data can also be analysed for each type of question separately. The comparison between the general text and the technical text indicates that L2 students experience more difficulties with the technical text. This result is the opposite of what was expected from the literature. The implication is that courses that deal specifically with mathematical discourse may well be useful for these students.

	Everyday Text			Technical Text		
	% U/std.	% Corr.	% Corr. if U/std.	% U/std.	% Corr.	% Corr. if U/std.
L1 Stds.	93	64	69	90	62	69
L2 Stds.	76	53	70	68	48	70

	Symbols			Diagrams			Graphs		
	% U/std.	% Corr.	% Corr. if U/std.	% U/std.	% Corr.	% Corr. if U/std.	% U/std.	% Corr.	% Corr. if U/std.
L1 Stds.	84	65	77	84	63	75	90	61	68
L2 Stds.	89	59	66	82	53	65	89	70	79

Table 3 Question Modes and Understanding

There was no guarantee in the question design that the five types of questions were of equal difficulty, thus the above data is only useful when L1 and L2 comparisons are made. It is notable that L2 students have a profile where they understand symbolic and graphical questions best, then diagrammatic questions, and text questions least. L1 students, on the other hand, understand text questions best, then graphical ones, and symbolic and diagrammatic least. What is most interesting—and needs further investigation—is the ability of L2 students with symbolic, diagrammatic and graphical questions (given that they are understood). It appears that they have great success with graphical questions but very poor success with symbolic and diagrammatic ones. L1 students exhibit a slight reverse tendency. The relatively poor success of L2 students with symbolic questions becomes important after considering the final set of data.

Differences in Preferred Mode of Working

The analysis of the working shown by students exposes another area of difference between L1 and L2 students: the L1 students use working in 20% more of the questions. When proportions of the modes of working are considered, L2 students use symbolic methods more than L1 students, and graphical methods less. When the working mode with the different types of questions is analysed (the data is not given here), then it transpires that it is in the text questions that L2 students use the symbolic mode with greater frequency.

This result and the previous one (poor success with symbolic modes) lead to the conclusion that L2 students revert to (or rely on) symbolic mode when having textual difficulty, but that this is a false security, as they do not perform well in this mode.

Table 4 Preferred Mode of Working

	L1 Students			L2 Students		
Mode of Working	Number of Questions with Working	% of All Questions [232]	% of Questions Showing Working	Number of Questions with Working	% of All Questions [432]	% of Questions Showing Working
Symbolic	112	48%	62%	170	39%	67%
Diagrammatic	27	12%	15%	48	11%	19%
Graphical	34	15%	19%	34	8%	13%
Text	7	3%	4%	1	0%	0%
TOTAL	180	78%	100%	253	58%	100%

Final Comments

The study is undertaken at a particular level of mathematical facility (first year university) and linguistic ability (most students are at a language proficiency of IELTS band 6.0 or above). Research (Clapham, 1996) indicates that any results may not be consistent with other levels of ability in either mathematics or language.

This study provides an initial insight into how language affects mathematical understanding. It indicates that L2 learners at university level suffer a greater disadvantage in mathematics than is expected from the literature, about 10%. There is evidence that the technical language is particularly important, not just everyday English. Finally it suggests that L2 students rely, unjustifiably, on symbolic modes when they are unsure. These three conclusions need to be substantiated by larger and more rigorous studies, and further research into the role of symbolic modes of working for L2 students is needed. Mathematics is not “language-free” and the particular vocabulary, syntax and discourse it presents challenges for L2 learners.

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