NATURAL LANGUAGE, HISTORY AND THE INTERPRETATION OF PROCESS OF LEARNING/TEACHING[1]

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Abstract: *In cultural integration the relationship between mathematics languages and natural languages has a very important role. In mathematics education there is an important problem about the relationship between epistemology, history, and the communication of mathematics.*

If the interpretation of phenomena during learning or understanding utilises the semiotic approach of mathematics, then we have the instruments to analyse verbal messages and non-verbal messages. A classification of semiotic interpretations of the history of mathematics as 1) history of syntax of mathematical languages, 2) history of semantics of mathematical languages, 3) history of pragmatic languages, give the means for the a priori interpretation of understanding/learning phenomena.

Historical epistemological representations are possible routes to pupils′ knowledge (a priori and a posteriori). In this perspective the history of mathematics is of service in research in mathematics education: the researcher has special requests for the historian of mathematics.

In this paper we present an experience of the preparation of a Chinese man for his final college examination. We compare the structure of Chinese language with algebra. The history of mathematics is a basic key in the interpretation of the didactic problem.

Riassunto: *Nell'integrazione culturale un ruolo molto importante é giocato dalle relazioni tra lingua naturale e linguaggi matematici.*

Nella Didattica delle Matematiche esiste un problema importante è quello riguardo la relazione tra Epistemologia, Storia e comunicazione delle Matematiche.

Se si utilizza l'approccio semiotico delle Matematiche abbiamo strumenti utili per poter interpretare i messaggi verbali e non verbali.

Una classificazione semiotica della storia delle Matematiche come 1) Storia della sintassi dei linguaggi matematici, 2) Storia della semantica dei linguaggi M., 3)

[1] (Editors' Note: This paper is a work in progress, and is not yet fully developed in English. However we regard some of the ideas to be both new and interesting, and therefore have included it in its present form as a way to facilitate their further exploration.)

Storia della Pragmatica dei linguaggi M, fornisce degli strumenti interpretativi dei fenomeni di insegnamento/apprendimento a priori.

Le rappresentazioni storico-epistemologiche sono delle possibili stravedi conoscenza degli allievi (a priori e a posteriori).

In questa prospettiva la storia delle matematiche è al servizio della ricerca in didattica. Il ricercatore ha delle particolari richieste per lo storico delle Matematiche.

In questo lavoro si presenta lo studio di un caso: la preparazione di un giovane Cinese per l'esame di licenza media. La storia delle Matematiche è la chiave per interpretare il problema didattico.

Introduction

In the study of the process of learning and teaching, the history of mathematics has an important role (Spagnolo, 2000).

In this paper we present a theoretic point of reference for the didactic view, and examples of situations of teaching in different cultures.

In particular the theoretic study has as a reference the theory of situations of Guy Brousseau (Brousseau, 1997, 1998) with a semiotic revision in this connection with interpretations of phenomena of learning/teaching (Spagnolo, 1998). The semiotic point of view is very important in all three components: syntactic, semantic and pragmatic (Spagnolo, 2001).

A paradigmatic example is pertinent with a particular approach of ethnomathematics in the Chinese culture.

1.0 HISTORY COMES TO MEET THE INTERPRETATIONS OF EXPERIENCES OF ETHNOMATHEMATICS

In the ethnomathematical approach there is the importance of language in relation with the context.

This paper analyses an experience with 3 interpretative dimensions: languages with syntactical and semantic relations, the history of mathematics, and the style of understanding.

1.1. The experimental context: The experience of Tong.

Tong, a Chinese man aged 32, attended a Chinese school up to the age of 16. Subsequently he joined his mother in Italy in order to work there near their Chinese restaurant. In Italy it is necessary to have done middle school examinations to be able to open a business.

Preparation of Mathematics is usually examined in the last class of college (age: 13). Tong was preparing for these examinations.

A classic problem in the Italian classroom is:

"The lateral area of a right prism is $2016m^2$ and the base is a right-angled triangle with the hypotenuse of 20m and a cathetus 4/5 of the hypotenuse. Compute the volume of right prism."

Tong´s solution was:

$$V = \left(\frac{\sqrt{(20^2) - (20 \cdot \frac{4}{5})^2} \cdot (20 \cdot \frac{4}{5})}{2} \right) \cdot \left(\frac{m^2 2016}{\left(20 + (20 \cdot \frac{4}{5}) + \sqrt{(20)^2 - (20 \cdot \frac{4}{5})^2} \right)} \right)$$

$$V = \left(\frac{\sqrt{400 - 256} \cdot 16}{2} \right) \cdot \left(\frac{m^2 2016}{\left[20 + 16 + \sqrt{400 - 256} \right]} \right)$$

$$V = \left(\frac{12 \cdot 16}{2} \right) \cdot \left(\frac{m^2 2016}{48} \right)$$

$$V = m^2 96 \cdot m42 = m^3 4032$$

Another problem problem was:

"The bases of a right trapezium measure 32cm and 20cm and the oblique side 13cm. Compute the area of surface."

Tong’s solution was:

$$\overline{AB} = cm20$$
$$\overline{DC} = cm32$$
$$\overline{BC} = cm13$$
$$A_s = \frac{(cm32 + cm20) \cdot \sqrt{(cm13)^2 - (cm32 - cm20)^2}}{2}$$
$$A_s = \frac{cm52\sqrt{(cm13)^2 - (cm12)^2}}{2}$$
$$A_s = \frac{cm52 \cdot cm5}{2} = cm^2 130.$$

(50 problems solved by Tong have been collected.)

In synthesis the procedure was :

- What are the things that I know (AB, BD, BC)?
- What are the things that I do not know (EC, BE)?
- What formula must I to utilise?

In Italian schools analogous problems are solved with what could be called "Euclid's Method". This way consists of step by step obtaining the elements for the resolution in the final formula.

For example the above-mentioned problem could be solved in the following way:

$$
\begin{aligned}
&\overline{AB} = \overline{DE},\\
&\overline{EC} = \overline{DC} - \overline{DE}\\
&\overline{EC} = cm32 - cm20 = cm12\\
&\overline{BE} = \sqrt{\overline{BC}^2 - \overline{EC}^2}\\
&\overline{BE} = \sqrt{(cm13)^2 - (cm12)^2} = cm5\\
&A_S = \frac{(B+b)\langle\ h}{2}\\
&A_S = \frac{\left(\overline{DC} + \overline{AB}\right)\langle\ \overline{BE}}{2}\\
&A_S = \frac{cm52\langle\ cm5}{2} = cm^2 130.
\end{aligned}
$$

The method of Tong is an "algebraical" way.

Why did he use this synthetic solution? Why was the solution was not in many steps as done by the (Western) Italian students?

1.2 The context of structure of Chinese language

Needham (1959) says that the mental mechanism for the building and the recognition of ideograms by association is a "mental equation".

" More primitive elements of Chinese language were generally pictograms, that is pictures reduced to the essential, made conventional, at the end very stylised. Naturally, concrete objects as the heavenly bodies, animals, plants, implements and instruments could be easier pictured. We reported some in the first part of the list two, having them origins in a Haluon's short popular leader. You will note, the most part of them, in the course of the time, have been included in radicals, (you can see later); but, it isn't always like that: hsiang (elephant) isn't a radical, but it has been classified under the radical number 152 (shih, pig) on the other hand, hu (wine's recipe) has been classified under the radical 33 (shid, studious). This depended on decisions made by lexicographers of successive ages.

In this way, the writing's fan extended and includes indirect symbols by different types of metaphoric substitution, like the part for the whole, the attribute for thing, the effect for the cause, the instrument for the activity, the gesture for the action, and so on. The list shows as the word chin, go up, derives from the picture of two footprints turning up; and as the word fù, that means "summit" derives from the ancient pot's picture.

A third characters class has composed of semantic combinations of two or more than two pictograms, making those called compound by association. In this way fu, wife, has composed by women's signs, hand and broom, fu, father, by the ancient signs of hand and stick; hao, to love, or good, combines signs of women and child.

A particularly interesting example, is the word that means male or man, nan, that includes the radicals of plough and field, indicates " who uses his force in the fields". Obviously, the sounds of the different elements lose themselves in the sound " that *result", because this sign existed before that, to represent it, the scribes associated signs having other sounds,*

So, we have a sort of equation: li + thine = nan.

These equations make up a mental half-conscious foundation for people acquiring familiarity with the language" (Needham, 1981).

1.3 The point of view of history

The point of view needed for communication in mathematics is that different mathematics' are languages with semiotic interpretation (Spagnolo, 1995, 1998).

What is a semiotic interpretation of mathematics languages? It includes the following.

- A syntactic point of view: syntax of mathematics languages, Bourbaki and structuralism, Formalism.
- A semantic point of view.
    - In mathematics languages "set theory" is a base structure. For example: The group is defined using a set: the set of Integers, the set of symmetries, etc.
    - In algebraic language, for example: 4x+2 and 2(2x+1) have different senses but they denote the same function (the same set of ordinate couples). $(x+5)^2=x$ and $x^2+x+1=0$ denote the same object (the Empty set) but have a different sense (Arzarello et al., 1993).
- A pragmatic point of view: the point of view of communication, or the didactic point of view

1.4 In the history of mathematics

- The history of syntax of mathematics languages: see Bourbaki (1963) for the evolution of algebra:
- The law of composition: Egyptian and Babylonian have a complex system of norms of calculation on Natural numbers >0 and Rational numbers >0, commutativity of the product of rational numbers (Euclid, theory of magnitude), Diofantus, - 2 pages - , XVII century law of composition in algebra (Gauss), theory of substitutions (Lagrange), Galois (groups of substitutions, XIX century (2 pages). (9 pages)
- The history of the semantics of mathematical languages: several books with titles "History Mathematical Thought"[2]. The history of thought scours mathematics languages analysing the "senses" attributed to mathematical concepts, before they could play a role in organised mathematical language.

[2] Examples: M. Kline, Mathematical thought from Ancient to Modern Times.

- For example the book *History of mathematics, history of problems* (The inter-irem commission, Ellipse, Paris)
- The history of the function concept (also in Piaget, *Epistémologie et psycologie de la fonction, Etudes d'épistémologie génétique*, 1968, Presses Universitaries de Frances). The point of view of psychology is privileged. Every study of mathematical concepts is done using a historical study.
- Morris Kline, *Mathematical Thought from Ancient to Modern times*, 1972. The history of semantics and syntax are not completely separated in this example.
- The history of the Eudoxus-Archimedes Postulate (see Spagnolo, Les obstacles epistemologiques: Le postulate d'Eudoxe-Archimede, 1995);

- The history of the pragmatics of mathematics languages: There is the history of communication of mathematics.
  - What was there to know (Savoir, Sapere) in a determinate historical period?
  - What was the real knowledge of students?

In this perspective there are many important historical sources: books, official curricula, registers of teachers, reviews of mathematics, and reviews of mathematics education. (In Italy there are reviews of mathematics education since 1870. In Palermo there is the review "Il Pitagora" (1874-1919)).

1.5 The comparative schema: history, languages, Mathematics

History of Mathematics in China: Algebraic Thought	**Chinese Language**	**History of Mathematics in the West: Geometrical Thought**	**Natural Languages in the West**
• Chu Chang Suan Shu: Roots, false position, solution of particular systems. 300 before C.-200 B.C. • Chao Chun Chin, Commentary to Chou Pei: 200-300 After C.	≈1000 after C. Chinese language as it is in present. In the Chinese hieroglyphic language 80% of hieroglyphics are for association.	Euclid: Axiomatic geometry 300 before C. • Geometry as first interpretation of physical world; • Geometry as science of to	Aristotle's Logic is like a model of Western languages. In particular Euclidean Geometry is like a

• Wang Hsiao Tung, Equations of 3rd degree. 625 a.C. • Chhin Hiu-Shao, Numerical solutions of equations of degree>2. 1247 b.C. • Ricci and Hsu Kuang-Chhi, Translation of Euclid's Geometry. 1607.	 This hieroglyphic MAN is composed in two parts STRENGTH and FIELD (a mental equation: says Needham)	argue. Fibonacci 1200 introduced algebra in the West with his "Liber abaci". • Geometry as a hypothetical -deductive system (Hilbert, Grunlagen der Geometrie, 1899)	model of Aristotle's Logic.

The hieroglyphic language is also the bearer of the concept of variable. For example in the following two hieroglyphics

to pick (cai) and to protect (bao) have the same part, referring to "person":

This part is "variable". Sometimes the variable part is in a semantic context, sometimes in a syntactical context.

B. Kosko (1993) says that Fuzzy Logic (Fuzzy Thinking) is a base of Eastern thinking. There is the relationship between the development of Fuzzy Logic in Japan, for example, and technology that applies Fuzzy Logic. Kosko says that Fuzzy Thinking is the way of Eastern people approach the solution of problems.

What is the relationship between Fuzzy Thinking, hieroglyphic languages, and algebraic language?

Enrica Lemut (2000) suggests that technology and mathematics are the mediators of Systemic Thinking. The action of Systemic Thinking is global, but acts locally: "A system's components are not only the individual elements composing it but also the network of interactions among them and their purpose" (Lemut, 2000, p. 183).

Is algebraical thinking part of systemic thinking?

E. Lemut agreed.

History allows the semiotic interpretation of phenomena of learning/teaching:

- Meditation about the structure of natural language (syntax);
- Meditation about Logic;
- History shows how languages are constructed (semantics);
- Pragmatic: study of process of learning/teaching.

Conclusions

We can certainly affirm that in the case of Tong the algebraic structure of his natural language (Chinese) has allowed him to make explicit entire processes of the resolution of problems in a different way from those generally developed in Western culture. His final examination in mathematics was excellent.

The study of the case of "Tong" has provided evidence for the need to use all the possible cultural tools to our disposition: the internal and external history of mathematics, the knowledge of the structure of natural language, the different logical referents, and the different inferential systems.

All of this indicates a series of open theoretical problems that can be investigated from the experimental point of view:

1. To what extent does natural language impinge on schemes of reasoning?
2. What are the underlying contributions of natural language and socio-cultural characteristics (beliefs, etc…)?
3. Would the same results be obtained in multicultural situations of teaching and learning of the mathematics?

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