ASPECTS AFFECTING PUPIL`S THINKING IN MATHEMATICS DURING INTERACTION RESEARCHER-PUPIL [1]

Jana Kratochvílová, Charles University, Faculty of Education, Prague, Czech R.

Ewa Swoboda, Rzeszów University, Rzeszów, Poland

Abstract

This paper will describe the research tool that helps the researcher to reflect on the experimental work being carried out by focussing on the interactions between a researcher and a student communicating about mathematics. This tool enables the researcher to analyse thinking processes of both the researcher and the pupil, during their communication. The problem of language and communication is analysed from the point of view of different spheres – cognitive, social and emotional. These analyses give an opportunity to see how those spheres influence each other and how two-way interaction affects a pupil`s thinking. The paper gives an example of the analysis using this research tool.

1. Rationale

The increase of interest concerning everything that happens in a class during mathematics lessons has influenced both the following perspectives: inner perspective (researches undertaken according to constructivist paradigms) and external perspective concerning the conditions of the learning processes in mathematics (research inspired by the socio-cultural theory). Cobb [p. 13, 1994] states: “The socio-cultural perspective informs theories of the conditions for the possibility of learning, whereas theories developed from the constructivists perspective focus on what students learn and the process by which they do so.”

In this approach there is a problem of language and communication. It is supposed that on one hand a pupil constructs his/her own mathematical knowledge in the frame of language s/he owns [Brown, 1997] but on the other hand, individual perspective is gained to a great extent by cultural apparatus. Language creates space in which thinking can form [Bussi, 1998]. Language is perceived as a bridge connecting two research perspectives: socio-cultural and constructivistic. Therefore communication in mathematics is one of main areas of research nowadays in mathematics education [Boero et al., 1998; Brown, 1997; Bussi, 1998; Van Dormolen, 1986; Jirotková, Littler, 2002; Pirie, 1998; Sierpinska, 1998; Steinbring, 1998].

Our interest in communication has not appeared randomly. Analysing our own experiments we have found many various phenomena characterizing interactions in communicating. We have focused on different mathematical and social ways of understanding, eg. the ways of giving a task, the ways of formulating an answer, the non-verbal ways of transmitting information. We have analysed an ability to use

[1] The contribution was partially supported by GACR no. 406/01/P090 and GAUK no. 309/2002 APP.

received information and the influence of the received information on the researcher's own thinking processes (or the thinking processes of a pupil). We have been interested in the influence of an emotional sphere on mathematical thinking.

2. Research tool

During our analyses of experimental work we have used the method of analysis which was elaborated in a Prague seminar (led by M. Hejný) called Atomic Analysis [Stehlíková, 1999]. In this method a pupil's performance is divided into the smallest and meaningful parts called atoms which are then analysed.

However we have analysed our experiments from more points of view than just the pupil's. They are analysed mainly from the point of view of the researcher and from the point of view of different spheres – cognitive, social, emotional. Therefore for the analysis we have constructed a row-column table. Rows in the table divide the protocol of the experiment into the smallest parts that are analysed. Columns from the left side analyse the researcher and in an analogue way columns from the right side analyse the pupil. We have distinguished four main spheres of analysis (placed in columns): cognitive (C), language (L), social (S) and emotional (E).

• Cognitive sphere: - Mathematics is carried out in this sphere and all actions in a given situation are determined by the existing net of connections (cognitive structure). Here we find: aims, expectations, knowledge associated with concept in some way, schemes of action, concept-in-action [Skemp, 1979; Vergnaud, 1998]. In this sphere decisions about action and language are made.

• Language sphere: - Language is a tool for expressing one's own ideas (concerning the cognitive sphere) and at the same time it is a tool for communication (concerning with social sphere). Not all ideas are verbalized, for example there need not be an equivalence between an idea and words expressing the idea.

Many research papers are aimed at language analysis and there are various classifications of verbal statements and various functions of language [Dormolen, 1986; Pirie, 1998; Skemp, 1979]. For our purposes we have accepted and extended the language classification of Pirie [1998]. In the paper Pirie has distinguished: 1. everyday language, 2. verbal language, 3. symbolic language, 4. visual language, 5. non-verbal language, 6. quasi-language. We have also looked at these categories from the point of view of two domains: mathematical (statement connected with mathematics) and social (statement connected with something outside of mathematics). In the table we have used an abbreviation involving a number, one of Pirie's categories and the letter 'm' or 's' for one of the domains, mathematical or social (eg. 4m means visual language relating to mathematics).

• Social sphere: - Learning and teaching mathematics (also experimental work in mathematics education) takes place in a social context. Development of mathematical concepts is placed in the net of mental and social connections [Presmeg, 1998]. It has influence on the epistemological status of mathematics as a discipline (school mathematics as knowledge cannot be logically deduced from from mathematics as a scientific knowledge) [Steinbring, 1998][2]. Mathematics is a certain type of "game" in which *knowledge* is not as important as *how to reach the knowledge* [Arzarello et al., 1999] and this is a problem for pupils and teachers [Koutsoulis, Makrides, 2000]. In addition all interpretations of words and behaviour are loaded socially and culturally.

• Emotional sphere: - Human actions could emerge or be inhibited by emotions (not only by conscious decisions – the cognitive sphere) and also all statements could have emotional colour which has its importance in communication.

3. Experiment

The aim of experiment was to investigate an ability to use mathematical knowledge of similar figures in atypical situations. The prepared situation gave the possibility of understanding similarity in everyday sense or in the sense of mathematical definition of similarity. The research tool was a set of objects that were not mathematically similar but all of them were bracelets of different thickness and diameter and made from different materials.

The student Kuba (fifteen-years-old) learned only one definition of similar polygons and solids in school. Half a year later he took a part in the experiment described below. One of the authors of this paper was a researcher who run the experiment and later on analysed the whole experiment with the second author.

Res.01: *(showing the set of bracelets)* Consider if these objects have any relation with mathematical concept of similar figures.

Kuba 01: *(showing no doubt)* These are similar.

Res.02: Mathematically?

Kuba 02: Yes. From the mathematical point of view.

Res.03: *(with surprise in her voice)* From the mathematical point of view?

Kuba 03: Yes.

Res.04: ……………….?

Kuba 04: Because in everyday language they are not similar at all.

4. Analysis of the experiment

Why did the researcher not react (Res. 04) to Kuba (03)? Did the researcher not understand Kuba? Did Kuba`s reaction (Kuba 04) help the researcher to understand what had happened?

[2] Importance of social sphere was discussed in one of working group (WG4) "Social interactions in mathematical learning situations" in the conferences CERME1 (1999) in Osnabruck and CERME2 (2001) in Marianske Lazne.

Tab.1

Researcher					Situation	Student			
Sphere						Sphere			
No	C	L	S	E		C	L	S	E
1	**Two understandings of the word *similar*** 1. everyday: *nearly the same, a little bit different.* 2. mathematical: *the relation of similarity among the 3-dimensional objects*		**Performed the experiment in the relationship: researcher - participant of experiment**		*Just before the experiment.*	**One understanding of the word *similar*** *as the mathematical relation among geometrical objects*		**Performed the experiment in the relationship: teacher – student**	
2	Two models of similarity: *1. the mathematical relation among 3-dimensional objects* *2. everyday meaning* **Dominance of**	4m 4s			Res. 01*: Showing the set of bracelets.* Res.: Consider if these objects have any relation with mathematical	**The word *mathematical* is dominance in the formulation.**			

	formulation is word *similar*	2m			concept of similar figures.	Mathematical= to mathematise, to project into the world of mathematics			
3	Misunderstanding Which model of similarity (from mentioned above) is chosen by Kuba			Suddenness	K 01: *Showing no doubt.* These are similar.	Model of similarity: the mathematical relation among 2-dimensional objects	4m 2m		Self-confidence
4	**Mathematical = existing in the domain of mathematics. There the concept *similar* is defined according to formal criteria.**	2m	**Signal of unrealised expectation versus belief in mathematical abilities of the student**.	Exasperation from misunderstandin g	Res. 02: Mathematically?	Mathematically = to mathematise, to project into the world of mathematics		**Considering researcher as teacher in " school game" – the teacher is checking me**	Self-confidence
5					K 02: Yes. From the mathematical point of view.	Bracelets = physical models representing circles	2m	Going towards the the researcher`s expectations	Self-confidence

6	**Putting the formulation from *the mathematical point of view* into the same scheme as the formulation *mathematically***	5s 2m	Gradation of aggression, implication that the student`s response is not right	**Exasperation from misunderstanding and unrealised expectation**	Res. 03: *With surprise in her voice.* From the mathematical point of view?			Receiving the signal of unrealised expectation	
7					K 03: Yes.	Bracelets = physical models representing circles	1m	Creating space for the researcher	Need to explain the situation
8	Insufficiency of adequacy of Kuba`s response with researcher`s models of similarity	5m 5s	Expectation of explanation	Helplessness	Res. 04: ....?			Receiving the signal of expectation and explanation	
9					K 04: Because in everyday language they are not similar at all.	**Mathematical similarity of bracelets if they are projected into circles, in everyday language the objects with no projection**	6m	The trial of explanation of own idea	

In the Table 1 we present our analysis in which we try to answer the questions above. The places in the table having influence on understanding between the participants including also the places in which the aims and expectations of both, the researcher and the student differs, are in bold.

5. Several examples of influences on the misunderstanding

In general behaviour if one participant of the experiment influences the behaviour of another participant (the student ⇔ the researcher) and moreover these influences coming from the all spheres that we have distinguished, they might cause misunderstanding. From our table we show some examples of the influences on the misunderstanding developed in the experiment.

I. Influence in the cognitive sphere

Tab. 2

No	Researcher	Situation	Student
1	Two concepts connected with the word *similar* 1. everyday: *nearly the same, a little bit different.* 2. mathematical: *the relation of similarity among the 3-dimensional objects*	*Just before the experiment*	One concept connected with the word *similar* *as the mathematical relation among geometrical objects*

The researcher expected two types of responses from the student to the challenge: “no, these objects are not similar” with mathematical understanding similarity or “the objects are a little bit similar” in everyday understanding the word ‘similar’. However for the student the word ‘similar’ – in combination with the researcher being taken by the student as a teacher – might connect only with the mathematical concept of similarity that the student had learned in the school where emphasis was put on the similarity of two-dimensional figures, mainly polygons.

II. Influence in the language sphere (two different interpretations of one word)

Tab. 3

No	Researcher	Situation	Student
4	Mathematical = existing in the domain of mathematics. There the concept *similar* is defined according to formal criteria.	Res. 02: Mathematically?	Mathematically = to mathematise, to project into the world of mathematics

For the researcher the word 'mathematically' concerned the all concepts existing in the domain of mathematics. In the researcher's understanding the word was connected with the mathematical characteristics of similar solids in the definition. The solids are physical objects. For the student the word 'mathematically' concerned the process of mathematisation which is a process of abstracting particular characteristics from various representations of concepts and relations. He projected the physical objects into geometrical figures – the circles of different diameters or into the concept of circle. One of the possibilities was associated with the concept of similarity.

III. Influence in the social sphere

Tab. 4

No	Researcher	Situation	Student
1	Performed the experiment in the relationship: researcher –participant	*Just before the experiment.*	Performed the experiment in the relationship: teacher – student

Different social interpretations of the roles of both participants in the experiment caused different understanding of the given problem. The student got to know the researcher as a mathematics teacher. The researcher got to know the student as a very clever boy, thinking independently. Therefore the expectations of both were different: The researcher expected the student to understand the problem as a situation on the boundary between mathematics and real life. The student thought that the researcher's interest concerned only mathematics. Misunderstanding starting in social sphere had a main impact for the whole meeting.

IV. Influence in the emotional sphere

Tab. 5

No	Researcher	Situation	Student
6	Exasperation from misunderstanding and unrealised expectation	Res. 03: *With surprise in her voice.* From the mathematical point of view?	

The dissonance between the researcher's expectations and the student's reactions caused the researcher to react emotionally. The student's reaction (K 02) was so surprising for the researcher that she had put the student's words in the question (Res. 03) with wonderment in her voice. The researcher's reaction strongly suggested that she did not expect such response. In addition the researcher's own scheme of situation together with her emotional reaction made an obstacle in reading the student's intention in the dialogue. It blocked the possibility of opening the researcher's mind to other (not expected) responses. The question: "Mathematically?" (Res.02) was answered by the student: "Yes. From the mathematical point of view" (K.02). The restatement, where student has changed the structure of the sentence, might have prompted the researcher to interpret it in different way.

6. Conclusion

From this analysis it is possible to see how one of the spheres of student influences one of the spheres of researcher and vice versa. But also the four spheres of student or researcher influence each other. Eg. The researcher`s misunderstanding in the cognitive sphere could influence the social sphere (eg. expectation of explanation) and the emotional sphere (eg. exasperation, helplessness) and then the emotional sphere could influence language sphere (eg. the researcher is so surprised that she cannot find any words for reaction). In addition the social perception of the meeting from the beginning influenced both the interpretation of the given problem and the behaviour of the researcher and student.

In the light of the analisis, it is possible to see that the researcher was not aware of the fact how complex the situation is. She was concentrated on the mathematical coating. Neither did not she take into account various functions of language, control of her gestures nor non-verbal ways of communication, as well as, she did not consider the social coating and social customs.

Aquisition of the mathematical knowledge by the child takes place mainly during math lessons. Teacher, conducting activities, has no chance to make such detailed analysis of his/her action as presented by us. Critical assesment of our own behaviours and believes during research session can be helpful in this matter. For teacher it is very important to know what kind of components of interactions have impact for pupil's mathematical thinking. It is also very important to show some examples. The knowledge about aspects affecting pupil`s thinking in mathematics could be a part of theoretical teacher's preparation.

References:

Arzarello F., Dorier, J.L., Hefendehl-Hebeker, L. & Turnau, S. (1999). Mathematics as a Cultural Product. In I. Schwank (Eds.), *Proceedings of the First Conference of the European Society for Research in Mathematics Education (CERME I),* Forschungsinstitut für Mathematikdidaktik, Osnabrück, vol. 1, pp. 73 – 80.

Boero, P., Pedemonte, B., Robotti, E. & Chiappini, G. (1998). The "Voices and Echoes Game" - the interiorization of crucial aspects of theoretical knowledge in a Vygotskian perspective: ongoing research. In: *Proceedings of PME XXII (International Group for the Psychology of Mathematics Education),* Stellenbosch, vol.2, pp. 120 – 127.

Brown, T. (1997). Mathematics Education and Language, Interpreting Hermaneutics and Post-Structuralism. *Kluwer Academic Publishers*, Dordrecht, Boston, London.

Bussi, M.B. (1998). Verbal Interaction in the Mathematics Classroom: a Vygotskian Analysis. In H. Steinbring et al. (Eds.), *Language and Communication in the Mathematics Classroom*, The National Council of Teachers of Mathematics, Inc. Reston, Virginia.

Cobb, P. (1994). Where is the mind? Constructivists and socio-cultural perspectives on mathematical development. In: *Educational Researcher*, 23 (7), pp. 13-19.

Van Dormolen, J. (1986). Textual Analysis. In B. Christiansen, A. G. Howson, M. Otte & D. Reidel (Eds.), *Perspectives on Mathematics Education,* Publishing Company, pp. 141-171.

Jirotková, D. & Littler, G.H. (2002). Investigating Cognitive and Communicative Processes through Children`s Handling of Solids. In: *Proceedings of PME XXVI (International Group for the Psychology of Mathematics Education)*, Norwich, vol. 3, pp. 145-152.

Koutsoulis, M. & Makrides, G. (2000). Classroom Culture and Math Achievement. In A, Gagatsis & G. Makrides, *Proceedings of the Second Mediterranean Conference on Mathematics Education, 7-9 January 2000, Nicosia-Cyprus,* pp. 79 – 92.

Pirie, S. (1998). Crossing the Gulf between Thought and Symbol: Language as (slippery) Stepping-Stones. In H. Steinbring et al. (Eds.), *Language and Communication in the Mathematics Classroom,* National Council of Teachers of Mathematics, Reston, Virginia, pp. 7-29.

Presmeg, N. (1998). Balancing Complex Human Worlds: Mathematics Education as an Emergent Discipline in its Own Right. In: *Mathematics Education as the Research Domain: A Search for Identity.* In A. Sierpinska & J. Kilpatrick (Eds.), *ICME Study.* Book 1, Kluwer Academic Publishers, Great Britain, pp. 57-71.

Skemp, R. (1979). Intelligence, Learning and Action. *A Foundation for Theory and Practice in Education,* John Wiley&Sons, Chichester, New York, Brisbane, Toronto.

Sierpinska, A. (1998). Three Epistemologies, Three Views of Classroom Communication: Constructivism, Sociocultural Approaches, Interactionism. In H. Steinbring et al. (Eds.), *Language and Communication in the Mathematics Classroom,* National Council of Teachers of Mathematics, Reston, Virginia, pp. 30-64.

Steinbring, H. (1998). Epistemological Constraints of Mathematical Knowledge in Social Learning Settings, In: *Mathematics Education as the Research Domain: A Search for Identity,* In A. Sierpinska & J. Kilpatrick (Eds.), *ICME Study.* Book 2, Kluwer Academic Publishers, Great Britain, pp. 513-526.

Stehlíková, N. (1999). Metody badawcze stosowane przez uczestników praskiego seminarium z dydaktyki matematyki [Research Methods found in Prague Seminar on Mathematics Education]. In: *Dydaktyka Matematyki 21,* Kraków, pp.85-95.

Vergnaud, G. (1998). Towards a Cognitive Theory of Practice. In: *Mathematics Education as the Research Domain: A Search for Identity,* In A. Sierpinska & J. Kilpatrick (Eds.), *ICME Study.* Book 2, Kluwer Academic Publishers, Great Britain, pp.227-242.