THE PROFESSIONALISATION OF MATHEMATICS TEACHERS' KNOWLEDGE – TEACHERS COMMONLY REFLECT FEEDBACKS TO THEIR OWN INSTRUCTION ACTIVITY

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Abstract. *Mathematical learning processes are more and more understood as active knowledge constructions of the students in which they proceed in a discovering way themselves and gain new insights by means of common reflection. By analogy to this, the teachers' mathematical instruction activity should also expressively be the subject of their common reflection in which among other things reciprocal observations of their own instruction activity (e.g. by means of video recordings) are analyzed. On the basis of experiences gained in a co-operative research project with elementary school teachers, several conceptual basic ideas of the professional reflection of the own instruction activity are developed and explained by means of concrete examples.*

1. Introduction: Mathematics and Teaching – the Essential Components of the Activity of Mathematics Teachers

The problems, which are connected to the profession of *teaching mathematics*, can be described as a contrast between two poles:

- The *instruction activity* is subject to the tension between the teacher's immediate involvedness and critical distance (Seeger 1998; Seeger & Steinbring 1992). During the current classroom events, the teacher is bound directly into the interaction with his or her students and cannot play the role of a distanced observer at the same time.
- As a science, mathematics, and thus eventually also *school mathematics*, is a consistent and ready worked out knowledge stock. In the instructional interaction, however, the mathematical knowledge only *develops* together with the students, so to speak new and independently.

In scientific didactic discussion, it has been tried to describe, to analyze and to productively transform these essential poles of instruction activity between *immediate involvedness* and *critical distance* and of the mathematical knowledge between *ready product* and *interactive process* (Freudenthal 1973) under different perspectives and with different concepts.

In the discussion about the theory-practice-problem, the role of the teacher and of the instruction activity is described as the one of a moderator, initiator and supporter of the children's learning processes (Seeger & Steinbring 1992; Steinbring 1994; Verstappen 1988). In scientific studies about teacher pre- and inservice training, it is referred to particularities of the professional teacher knowledge and of the teacher's activity as opposed to other professions, for example to the big complexity and variety of the involved knowledge components (AG Mathematiklehrerausbildung 1981; Bromme 1994; Otte 1979; Otte, Reiß & Steinbring 1977; Steinbring 1998), additionally to the necessary sensitive reflection of instruction and learning processes (Krainer 1996; Krainer & Posch 1996) and also to the development of conscience of the teachers in their formation and in everyday practice (Selter 1995).

2. A Changed Focus of Attention: Teaching and Not the Teachers

Requests to improve the quality of mathematics instruction come out of different social domains, for example from the formation seminars of the second phase, from the supervisory school authority, from general instruction and school development research, from didactics of mathematics and from the university mathematics teacher formation and last but not least from instruction practice itself. In view of the results of TIMSS and PISA the discussion about concrete amelioration measures becomes more and more explicit. In different catalogues, there are among other things also such requests, which call for a common reflection of the teacher‘s instruction activity.

Having a look at newer publications out of school development research, one realizes that these are mainly about scientifically founded, extensive concepts to the professional aspects of the teacher‘s activity that are independent of the subject matter to be taught; among those, there also are – often as a marginal note – requests formulated for ”feedbacks on the basis of reciprocal classroom observations between teachers” or the reinforced reference to the ”level of acting in the classroom”. In the sequel of the bad results for German students at TIMSS and currently at PISA (cf. Klieme, Neubrand & Lüdke 2001) Baumert, the German PISA responsible, states the following formula as an essential measure for school-internal quality improvement in a media-effective way: ”Teachers who teach the same subject must … visit each other during the instruction. … Why should a teacher not … let his instruction be recorded and discuss the video document with his colleagues?” (Die Zeit, Nr. 50, 2001).

As necessary all these requests out of the domain of formation and school development research are, they contain an essential deficit and only represent one side of the problem. They concentrate exclusively on the side of pedagogical measures and methods of instruction activity and completely ignore the actual subject, the *instruction content mathematics*.

One perspective for improving mathematics teaching in which the importance of both components, of mathematics and of teaching is paid attention to, has been worked out in the book ”The Teaching Gap” by Stigler and Hiebert (1999). The particular nature of mathematical knowledge is thematised as a cause of different instruction patterns. Whether mathematics is rather seen as a product or as a process is essential for the kind of the according instruction activity and for the children‘s learning.

In this frame, Stigler and Hiebert formulate the following principle for the amelioration of mathematics instruction: ”Focus on Teaching, Not Teachers” and thus a new orientation from the attention to what makes the so-called ”good teacher” to the conscious conception and common reflection of everyday instruction activities. Teaching is a system and not a mere collection of single abilities and knowledge elements. Understanding teaching as a system means that there are activities that should be regarded as systems on all levels of instruction practice:

The activity of

- one‘s own learning mathematics and didactic subject matter knowledge

- observing and analyzing the children‘s learning and understanding processes
- the (trying out of) teaching of mathematics.

These activities must be carried out actively on the one hand and at the same time consciously reflected on the other hand. One‘s own learning, observing and teaching activities are not to be a blind, recipe-guided action, they must be reflected in a distance, i.e. from a meta-point of view. These reflections are not to be left to the single person as his or her private matter, but a common explicit reflection about these activities is indispensable for professional learning and teaching.

3. The Same Instruction Preparation and Different Instruction Processes – Examples from a Practice-close Research Project

In the frame of an approximately three-year-long project with teachers at an elementary school who have been working together for some time in the planning and preparation of their instruction in 3rd grade classes, possibilities and forms of reciprocal guest observations as well as of common reflection of one‘s own instruction activities in mathematics instruction are to be discovered and worked out. [One elementary school out of the district Recklinghausen as well as the universities of Bielefeld and Dortmund are participating in this co-operation project; it is supported in the frame of the EU program COMENIUS Action 3.1 and also pursues an international comparison with colleagues from Italy and Czech Republic.] A commonly planned and prepared instruction experiment to the transition from semi-written to written strategies of addition and subtraction in the thousands space has been carried out by the co-operating teachers in spring 2002; altogether about 30 lessons have been videographed by the participating researchers. Examples out of these documents served for the common discussion and analysis. At the same time, systematic ways of proceeding for the common analysis and reflection of instruction activities, which are suitable for school practice, were to be developed in this way. One goal of this project is the preparation and testing of a practice-close qualitative analysis procedure of one‘s own mathematics instruction (on the basis of an analysis instrument developed in didactic research). In this way, school-internal processes of critically reflecting instruction processes are to be supported in a positive way and the instruction activity is to be improved in the context of the cultural conditions of school practice.

3.1 Co-operation and Reflection – Elements of Professional Instruction

Besides the teachers‘ co-operation among each other, an intensive exchange with researchers over a longer period of time took place in this research project. The co-operation contained the following domains:

- common instruction planning (e.g. discussion of the mathematical contents; views of different mathematical text books for students and instructional suggestions; discussion with colleagues and researchers)
- reciprocal guest observation in everyday lessons with subsequent reflection
- carrying out and recording instruction experiments (concrete common planning with partly identical exercise sheets and black board images)

- reflection by means of recorded lessons.

The reflection relates in particular to

- the course of instruction in the general comparison
- the children‘s understanding processes
- the teacher‘s coping with mistakes
- language: employment of suitable technical descriptions, the children‘s verbal forms of explanation and justification

3.2 From Semi-written to Written Addition – Didactic Orientations in the Common Preparation

In the planning of the instruction sequence, the following conceptual strategies for the transition from informal strategies to the standardized procedure of the addition were considered important: insightful development of the semi-written strategies, introduction of the written algorithm (first in an extended form under employment of the position table, later in the so-called end form), automatization and extension of the procedure (for instance to the addition of three terms of a sum).

For the domain of semi-written calculation, there was a consensus in the whole group about the fact that the students were to develop their own strategies, to compare and discuss different strategies and to use these in a meaningful way. With the written algorithms the teachers had not really encouraged the comparison of the procedure with other ways of calculating and the discussion about this procedure so far. It is familiar that the application of the procedure and its fluent mastery often are in the foreground. The understanding of the procedure is then only of minor importance (cf. Ma 1999) and the connection to semi-written calculation is not sufficiently thematised either. This connection between semi-written and written calculation, the development of a written procedure out of the semi-written strategies, is not at all a trivial transition and not possible for each semi-written strategy either (cf. Pepper 2002).

Considering this background, it was commonly considered what the essential conceptual requirement with the transition from the semi-written to the written addition consists in. Within the group, it was planned to put the insight into the underlying mathematical structure into the foreground when introducing the procedure, in order to make explanations and justifications of the algorithmic procedure possible for the children. The procedure of the written addition presents a particular closeness to the strategy ”positions extra”, which had been intensively employed so far, so that these two ways of proceeding were opposed to each other.

Besides, the utilization of the ciphers in the position-by-position representation as well as the suitable interpretation of the transitions were identified in the planning as further central aspects and possible difficulties.

On the one hand, these conclusions had the consequence that the children were already supposed to pay attention to their language, to the use of the correct designations, and here especially the designation of the positions, during the exchange about the semi-written strategies. Furthermore, the transition problem led to a way of proceeding differing from the textbook in which numbers were already to be represented flexibly in the position table before the introduction of the written algorithm.

The students were given a number written in the position table containing also two-digit numbers for one place value (Fig. 1). The task for the students was to notate the given number outside the position table.

H	T	O
2	47	14

Fig. 1: Blackboard Presentation

98 An Normally, that would be called twenty-four-thousand-seven-hundred-fourteen.

Ahmed (class B) first names the combination of the ciphers without paying attention to the above notated positions, but when asked, he realized the necessity of changing places values:

102 Ah Ähm, this, at the ones, there is written fourteen. There, I would, ähm, ten, ähm, take away ten.

Later on he talks about »changing« the ten ones, which he wants to treat at the tens position. Here, the difficulty of a flexible interpretation becomes obvious, namely interpreting at the same time these ten ones as one ten (and not as ten tens).

123 Ah Ten plus forty-seven makes fifty-seven. [points at the ones column and the tens column]

Such flexible interpretations had often to be supported by a representation (usually the thousand-book).

In earlier years teaching in a third grade class, this kind of the instructional realization had not yet been thematised or treated in the form of autonomous exercises by the teachers involved and in the planning phase, it had been judged as a not very time-consuming activity, which, however, proved to be a wrong estimation during the later carrying out.

Subsequent to this planning concerning the basic orientation, the teachers have then developed concrete exercises and produced common work sheets.

3.3 Reflections of the Lessons Performed

The examples of carried out lessons used are not analyzed here according to a scientific procedure, but they are rather supposed to show in which form this material has given rise to initiating reflections within the group. First common reflections and discussions of instruction episodes already led, first in an unsystematic way, to insights and conclusions related to two central dimensions: The teachers‘ primary focus was their *own instruction activity* and *their different roles* on the one hand. On the other hand, the *students‘ activities* and *interactions* were perceived more sensibly.

In the following, exemplary aspects to both dimensions, which crystallized during the common discussion, will be explained and, if necessary, illustrated by means of parts of the lessons transcripts. The first dimension, mainly is illustrated by the last five points, whereas the second dimension by the first four points.

- *Unexpected Student Contributions also with Routine Topics*

Even though the written addition was estimated as a routine topic, especially by the colleagues who already dispose of a longer instruction activity, the teachers expressed the fact that there resulted a series of new and surprising realizations for them. These realizations especially concerned the changed content related approach (”magic numbers”; interpretation of the relation of the positions among each other) and unexpected feedbacks and contributions of the children in the course of their learning processes.

- *Flexible Interpretation of Numbers and Ciphers*

At many points of this instruction topic, it could be observed that an essential requirement to the children consisted in changing flexibly between a uniform conception of the whole number and an interpretation of the number as a composition of single positions.

With the introduction into the written addition, the (known) strategy ”positions extra” was opposed to the written addition by means of using the position table (Fig. 2).

338	+	257	=	595
8	+	7	=	15
30	+	50	=	80
300	+	200	=	500

	H	T	O
	3	3	8
+	2	5	7
	5	8	15
	5	9	5

Fig. 2: Blackboard Presentation

The students were requested to name and explain striking features: Dina states the similarities which she has noticed and focuses first on the ones, yet her conclusion is restricted to the result of the addition of the ones, which also appears as a whole in the ones column of the written algorithm. According to the teacher‘s demand, Dina points at the further ones and is to state the according ones exercise.

29 Di There is a one [*points at the 15 in the ones column*] and there. [*points at the 5 in the ones column*]

30 T Where else are the ones?

31 Di There [*points at the 7 in the ones column with the pointer*] and there. [*points at the 8 in the ones column with the pointer*]

32 T Good. Because what's this ones exercise called?

33 Di Eh, eighty-seven and plus ... [*points at the 7 and at the 8 in the ones column with the pointer*]

Dina (class C), however, has a rather local point of view to the ones column and the two ciphers 8 and 7 are connected to the number 87 (line 33). An integral view onto the whole position table with its relations of the positions and thus also of the single ciphers among each other does not succeed here. Another student has to help in the following.

39 Ca Seven plus eight or eight plus seven.

- *Adequate Use of the Technical Descriptions*

Often the children were asked to justify the single steps of the written addition with suitable technical descriptions. The common analysis of such scenes made it obvious that this requirement was not merely mastered in a schematic way, but that the designations had to be consciously reflected by the children again and again and that thus a further insight into the relations between the positions was supported.

In class A, the already stated procedure (Fig. 2) was chosen for the introduction of the written addition as well, and Anne comments on the communities of semi-written and written addition:

10 An You have now always the eight ones, [*points at the 8 in the informal calculation*] you have then here with (...), eh, how was it here now? Have you written with the eight ones there. [*points at the 8 in the ones column of the position table*] Then the seven ones you have also written with the seven ones. [*points at the 7 in the informal calculation and at the 7 in the ones column of the position table*] Then you have calculated eight plus seven here, that makes fifteen, then you have written the fifteen there. [*points from top to bottom at the ciphers in the ones column*] And that's the same here, because, because three hundreds plus two hundreds are five hundreds. [*points from top to bottom at the ciphers in the hundreds column*] And here, too, because plus five, eh, three tens plus five tens are, eh, eight tens. [*points at the 8 in the tens column*] Then you still have this one, eh, put this one ten to the others. [*points at the 1 of the 15 in the ones column and moves the pointer down to the 9 in the tens column*] Then these here were only five [*points at the 5 in the ones column]* and these five *[incomprehensible*] (*remains that as the result*) [*points at the 5 in the hundreds column*]

Anne begins with the ones of the first number and produces a relation to the semi-written part calculation. Then she remains with the written algorithm and immediately goes on to explain the calculation within one column, i.e. within one position. She explains the procedure also for the further positions and also explains the worked out transition by means of employing the correct designations for the positions.

- *Danger of Mere Cipher Manipulation*

Already with the introduction of the written algorithm, but especially in the course of its extension and automatization a growing tendency of the children to carry out a mere cipher manipulation and to speak less of the meaning of the single positions could be observed.

	H	T	O
	1	8	4
+	1	4	8
	2	12	12

Fig. 3: Blackboard Presentation

In the following phase (follow-up lesson after introducing the written algorithm), Katrin works out a written addition at the blackboard (Fig. 3) and explains her calculation.

4 Ka I first started calculating with the ones, and four [*points at the ones column*] plus eight makes twelve (...) [*writes 12 under the ones column*]

5 L Twelve ones, aren‘t these?

6 Ka Yes, twelve ones. Eight tens and four tens [*points at the ciphers in the tens column*] makes again twelve tens (...) [*writes 12 under the tens column*] and one plus one makes

7	L	Stop. You have said it so well in the other case. Please, say it more precisely.
8	Ka	One hundred plus one hundred makes two hundreds (...) [*points at the respective ciphers and writes 2 under the sum line in the hundreds column*]

Katrin is able to assign the meaning of the ciphers to their corresponding positions; however, she does not always name the particular place values, but must be encouraged explicitly. Other children, who named the ciphers without the corresponding place value, struggled and became unsure, when asked to specify the ciphers' meaning.

- *Individual Instruction Styles and Subjective Learning Theories*

In spite of the common planning of single lessons even with identical work sheets and corresponding blackboard presentations, different instruction interactions and thus different courses of instruction and learning processes resulted. The teachers had already become aware of existing differences during the reciprocal guest observations, but they were only identified more exactly with the comparing analysis with the help of the video material or the transcripts. The instructional personality styles were stated as the first approach of an explanation, besides, aspects of subjective learning theories were formulated.

One of the teachers for example attached great importance to the *understanding of the children among each other,* whereas for another teacher the understanding processes are documented in the *conversation with the teacher*.

- *Changed Understanding of Instruction Reflection*

With the reflecting comparison of the different lessons it is not about a judgment which of the lessons passed better or worse and whether and to which degree another teacher attitude could have avoided certain difficulties.

Such form of analysis of lessons in the sense of a *judgment* of the instruction activity certainly represents a tradition of teacher formation. This one-sideness of analysis that focuses primarily on judgment should be broken open in any case.

- *Extension of the Room for Manœuvre*

The difference in the course of the lessons in spite of the same planning also led to a productive conclusion of the teachers during the discussions: The comparison of their instructional proceeding with the one of their colleagues extends their own possible room of acting.

For example, the introduction of the written addition was chosen identically in classes A and C. In class B, however, the written algorithm was developed on the blackboard by the children themselves. Both ways are possible and meaningful.

- *Necessity and Meaning of the Video Documents*

Altogether, the participating teachers stated to the project and analysis works so far that, even with long professional experiences, they now for the first time had the possibility to reflect their own professional activity in a systematic way by means of the video documents. An essential difference can be seen between the reflection by means of video documents on the one hand and the reflection in the form of a conversation or after a common guest observation: The subject of the reflection in the first case is a copy of the instruction while it is restricted to subjective memories and statements in the latter case.

- *Involvedness and Distance*

The common reflection of the lessons made a problem of the respective teaching persons obvious: On the one hand, the teaching person is *involved* with the reflection relating to her lesson, on the other hand, she must take a *distanced* point of view. One teacher, for instance, stated that it is difficult for her to regard herself on a video document in a neutral way, as she often tries to remember what she has thought and intended in the according situation. Here the common discussion can help to make this tension field productive.

4. Concluding Remarks: Learning Mathematics and Teaching as Reciprocal Processes of Activity and Reflection

Learning and teaching mathematics are no processes of the delivery of ready knowledge from the knowing teacher to her still ignorant students. This position means that learning of mathematics should be understood as an active, social and discovering process of the children (Müller & Wittmann 1995). In addition, it becomes more and more obvious that these learning activities carried out by oneself always have to be accompanied and guided by explicit reflections about the mere doing. This complementarity of activity and reflection plays an increasingly important role on all levels of the students‘ learning mathematics as well as in all phases of mathematics teacher formation at the university, in the study seminar and in the professional practice.

In everyday practice, guest observations with subsequent common reflections of one‘s own instruction are a rare exception so far. The requirements for an effective, long-term improvement of mathematics instruction focus the view to the professional activity of the teachers‘ own instruction as a core problem. Like other professional activities, teaching also is subject to the complementarity of *action* and *reflection*. As a matter of course, commonly co-operating teachers have always reflected about mathematics teaching, for example during the common preparation of teaching, or in their reports about events out of their own instruction. What must be increasingly tied into these reflection processes is the direct observation of the instruction activities, partly also the videographical documentation of instruction, so that a new and productive distance to their own activity is made possible for the teachers.

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