

THEMATIC GROUP 11
INTER-RELATING THEORY AND PRACTICE
IN MATHEMATICS TEACHER EDUCATION

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Introduction

This group was organised by Andrea Peter Koop (Coordinator), Barbara Jaworski, Konrad Krainer and Lurdes Serrazina. It had 26 participants from 11 countries. 14 papers were submitted of which 12 were accepted. One team of authors (Paper 7) could not attend the conference. Each paper was reviewed by three prospective members of the group and papers were revised according to reviewers' comments. A list of accepted papers with authors' names is given below.

1	The dialectic relationship between theory and practice in mathematics teacher education <i>Mercedes Garcia, Victoria Sanchez, Isabel Escudero & Salvador Llinares (Spain)</i>
2	The professionalisation of mathematics teachers' knowledge – Teachers commonly reflect feedbacks to their own instruction activity <i>Petra Scherer & Heinz Steinbring (Germany)</i>
3	A lengthy process for the establishment of the concept of limit <i>Lucia Grugnetti & Angela Rizzi (Italy)</i>
4	Professionals investigate their own practice <i>João Pedro da Ponte, Lurdes Serrazina, Olívia Sousa & Helena Fonseca (Portugal)</i>
5	Patterns of flexibility: Teachers' behaviour in mathematical discussion <i>Roza Leikin & Sariga Dinur (Israel)</i>
6	Inquiry as a pervasive pedagogic process in mathematics education development <i>Barbara Jaworski (UK/Norway)</i>
7	Collaborative work: Tool and context for the professional and metacognitive development of mathematics teachers <i>Ana Cristina Ferreira & Maria Angela Miorim (Brazil)</i>
8	Geometric figures from middle school to secondary school: Mediating theory and practice

	<i>Lucilla Cannizzaro & Marta Menghini (Italy)</i>
9	Aspects affecting pupil's thinking in mathematics during interaction researcher-pupil <i>Jana Kratochvilova (Czech Republic) & Ewa Swoboda (Poland)</i>
10	From research in mathematics education to teachers' training through internet <i>Rossella Garuti (Italy)</i>
11	Linking pre-service and in-service teacher training: Co-operative design of working environments for primary mathematics <i>Bernd Wollring (Germany)</i>
12	Self-reflection and improving mathematics classroom culture <i>Alena Hospesova & Marie Ticha (Czech Republic)</i>

These papers were organised into several sections for consideration and discussion in the group:

1. Teaching approaches in particular curricular areas (Papers 3 and 8),
2. Teaching approaches and their development (Papers 1, 5 and 10);
3. The element of reflection in teacher education (Papers 2, 9 and 12)
4. The role and nature of collaborative work in teacher education (Papers 4, 9, and 11)
5. Inter-relating theory and practice (Paper 6)

These 5 sections formed the major part of group work, following an introductory session and finishing with planning for group reporting and future work. In the introductory session, members of the group introduced themselves and there was a discussion in which initial thoughts on theory and practice were offered. This effectively 'broke the ice' within the group and encouraged everyone to enter into the field of the group. At the beginning of each of the subsequent sessions, members of the group worked together on the material of the papers, either as a whole group or in small subgroups. In each session, following initial consideration of the papers, members of the group addressed questions to the authors of the papers for clarification and authors were given a short time to respond to questions and issues. Finally there was whole group discussion of the main issues arising from this work. In the final session authors in each of the five sections collaborated to offer a perspective on relationships between theory and practice. These perspectives formed a part of the presentation of the work of the group to the CERME 3 conference as a whole. Finally, it was felt that the work of the group might form the basis for a journal special issue in this field and authors were invited to write an enhanced version of their paper for consideration for this special issue.

Below, we present an overview of group discussion – organised according to the five sessions – in which we consider many of the issues arising from the papers. This overview is then followed by the papers themselves in the order listed above.

Overview of issues arising from the work of Group 11 at CERME 3

We organised our work on the papers according to three main questions:

- A. What research took place?
- B. What did it reveal with respect to teacher/teaching development?
- C. What links between theory and practice do we see?

We shall refer again to these questions in our concluding section to this overview.

1. Teaching approaches in particular curricular areas (Papers 3 and 8),

Both of these papers came from Italian authors. Paper 3 addressed research into students' conceptions of limit and infinity, while Paper 8 focused on research into students' conceptions of geometrical figures. In both cases research revealed significant insights concerning the mathematical topics and ways in which students perceived those topics or had difficulties with the concepts. Such insights are important in linking mathematical knowledge with knowledge of students' conceptions, and in addressing issues for teachers constructing activities for the classroom and analysing students' mathematical understanding with respect to those activities. We saw a progression in Paper 3 from mathematics (concepts of *limit* and *infinity* and meanings associated with these terms) to students' understanding of mathematics terms (e.g. infinity as 'something which has no limits') to teaching methods (emphasising choices made by the teacher in creating classroom activity) to teaching development (in which indications for future research related to the design of classroom activities interfaced with to students' conceptual difficulties highlighted in the research). In Paper 8, progression took us from mathematics (properties of quadrilaterals) through a didactical analysis in which worksheet tasks were analysed according to van Hiele levels of geometrical understanding to teachers' considerations of these analyses. Authors presented scenarios to illustrate teachers' developing awareness of students' thinking as expressed through the van Hiele theory. Research into teachers' use of the worksheets with pupils was planned for the future.

Discussion showed an interest in the progression here from areas of mathematics and mathematical concepts and tasks to teachers' thinking and planning for the classroom and ways in which analyses at different levels impact on the classroom. In the first paper, analysis of students' perspectives of mathematics was planned to lead to design of classroom tasks in which teachers would have a role and the study of such tasks in classroom situations. In the second paper, teachers' conceptions of the van Hiele analyses were to be used as a basis for research into teachers and pupils' activity in classrooms.

The progression here might be seen broadly as:

- Level 1: Mathematics, analysis of the language of mathematics and students conceptions of mathematics
- Level 2: Teachers' (or student teachers') access to analyses at Level 1, study of teachers thinking related to tasks and students conceptions of mathematics. Further design of tasks.
- Level 3 Teachers' (or student teachers') use of tasks, and the results of analyses at Levels 1 and 2, to work with students in classrooms. Study of interactions, students' learning and teaching development.

Of course, each of these levels is complex in its own right as many subdivisions are possible. It is hard for any one research project to address all these layers simultaneously. The two papers here address mainly Levels 1 and 2, with implications for Level 3. Below we will introduce a further level to deal with relationships between research and development.

2. Teaching approaches and their development (Papers 1, 5 and 10);

In this section we had three papers which offered interesting analytical structures related to approaches to teaching, so in this respect we perceived similarity – although what they each offered was very different.

Paper 1 describes a study in a context of curricular analysis of the textbooks. The paper seems to offer different layers of theory related to practice: 1) legitimate peripheral participation – drawing student teachers into the community of practice of teaching – a focus fitting with Levels 2/3 above; 2) The use of technical and conceptual tools: – the theory here involves “different typologies of multiplicative structures and different perspectives of analysis (...)”, a focus largely at Level 1. The paper concludes with questions about the two roles played by the authors – teacher educators and researchers – and issues of compatibility between these roles. Such issues take us into Level 4, about which we shall say more below.

Paper 5 presents a study focused in teachers' flexibility in the course of a whole-class discussion. Different patterns of flexibility are described as well as ways in which different types of teacher knowledge influence teacher flexibility. The study used from Simon's Mathematical Teaching Cycle the notions of hypothetical learning trajectory and teachers' flexibility. As conclusions it seems that the patterns may help teachers plan and analyse their own flexibility and develop teachers' mathematical knowledge and teachers' knowledge of students. These considerations operate largely at Levels 2 and 3.

Paper 10 describes a project involving teachers and university researchers to design and organize materials for pre-service and in-service teacher education through the Internet. Some criteria that have been elaborated and used are presented and discussed. Issues arising in discussion of this paper ranged widely across all four levels.

Discussion highlighted the differences between the three papers. Each paper was discussed by itself and not related with the other two. So, in Paper 1, relationships between the various layers of theory and their relation to practices being analysed were stressed as well as assumptions that have to be recognised in such analysis. Paper 5 seemed to offer a very interesting model and the discussion centred around aspects of this model. Where the Paper 10 was concerned, discussion focused on how results from the research would be communicated to teachers and how teachers' practice might benefit from the research. In response, the author commented on the Italian research tradition of teachers as researchers.

We believe that the focus in this section was mainly at Level 2 above, although the discussion and question asked about how teachers engage with the ideas/concepts/analyses of the research and how this impacts on classroom practices were at Level 3. Relationships between the researchers and teachers seemed important to these questions, and these take us into the Level 4 which we see as dealing with relationships between research at the different levels and the forms of development which can be related to this research. This level encompasses issues in relationships between teachers, educators and researchers engaged in both teaching and research.

3. The element of reflection in teacher education (Papers 2, 9 and 12)

The theme here is 'reflection' and the papers offered examples from practice and research. We might ask "What is 'reflection' - in theory"? What literature are we drawing on? How do we conceptualise reflection? How do we see reflection in practice? How do the examples reveal key ideas about reflection that allow us to make some kind of generalisation?

Paper 12 addressed "systematic reflection on practice in order to improve it". Researchers sought an improvement of mathematics classroom culture through self reflection of teachers and collaboration between teachers and researchers with a focus on students developing conceptions of mathematics. The research described was situated in the mathematical topic of 'fractions' and the design of a classroom task to enable students to develop deeper understandings of this mathematics. Authors used Krainer's two pairs of contrasting elements of action/reflection and autonomy/networking to analyse development of teaching relative to teachers' thinking and activity.

Paper 2 also took up notions of action and reflection with video documents from classroom activity acting as a basis for reflection and analysis in discussion between teachers and researchers. The video document encouraged teachers to distance themselves from their immediate classroom involvement. Elements of mathematics and children's activity with mathematics were central to discussion and reflection on the content of the video.

In both of these papers we can see largely a focus at Levels 2 and 3 above in which research tries to link developing teaching in classroom situations with elements of

theory which allow conceptualisation of the (reflective) processes involved. However, both papers take also a mathematical focus in which reflections on teaching and learning relate to the area of mathematics being considered – fractions in Paper 12 and semi-formal approaches in connection with place value in Paper 2. Considerations of researchers' and teachers' inter-relation with respect to teaching development indicate elements of a Level 4 focus.

Paper 9 focused on researcher reflection on interaction between the researchers and a student whose thinking is the focus of activity. The authors analysed the researcher-student interaction from both student and researcher perspectives using a model that considered cognitive and social aspects as well as aspects of language and emotion to provide insights into processes of conceptualising learning and teaching. Here focus was mainly at Levels 3 and 4 providing insights into the learning-teaching interface through a close study of interactive communication.

With the focus here on reflection we seem to have moved more into Level 4 with teacher thinking about teaching, and the developmental process in which research plays a strong part. Discussion focused on reflection as a way of becoming aware, of seeing things that provide one with insights into differences for action – particularly where the differences seen through reflection on events are greater than people might have expected. Here the teacher might be seen to become an 'intuitive' researcher. In a study of the discourse of interaction we recognised the enormous complexity from a 30 second interaction and the value of analysing interaction in such depth.

4. The role and nature of collaborative work in teacher education (Papers 4 and 11)

While it needs to be noted that the papers in the section on 'reflection' all involved collaboration between teachers and researchers in some ways the discussion of Papers 4 and 11 shifted the group progressively from a focus on mathematics and learners' appreciation of mathematics towards teachers' thinking and the ways in which teachers' analyses of classroom activity lead to development of this activity. The key question was, how collaboration could enhance reflective activity.

Paper 4 discussed the ways in which the authors are researching their own practice. Two examples of such research from the activity of a study group on teachers' research that included teachers of all grade levels and teacher educators were presented. One of these examples referred to mathematics teaching in the middle grades and the other to pre-service teacher education. Both examples were theoretically grounded in a perspective that emphasizes the role of research as a process of knowledge construction, both in mathematics and in learning how to teach mathematics. The activity of the study group provided a setting for each author to develop a written account of her/his experience, revise and refine it so that it would provide a description of the experience and stress out the key emerging findings. In this process, at the beginning, there were some theoretical ideas, a distinctive professional experience and a mix of interests and concerns. At the end, when a book

with a collection of papers was published, some new professional and practical competences had emerged, as well as new theoretical insights regarding tasks, pupils, classroom work and the role of the teacher, as well as new projects to develop in the future. That is, this process began with some theories and some practices and ended up with new theories and new practices developing from the former ones.

In contrast, Paper 11 outlined a concept of co-organising teacher training and school-related research in the form of classroom experimentation. On the one hand, this research involves pre-service teachers as teacher-researchers who can experience practice-oriented investigative learning and obtain research results by specifically organised teaching experiments. On the other hand, these teaching experiments are designed to support the teachers of the classes involved in the experiments in developing constructivist-related approaches to mathematics teaching and learning at elementary school level. The 'triad' of a university researcher, pre-service and in-service teachers who are co-operatively involved in the classroom experiments allows the integration of teacher pre-service and in-service education following the idea that theoretical and methodological approaches which are "brought in" via the university based partners in the form of the teaching experiments meet with teachers existing beliefs and classroom practice. Issues arising are then discussed and further developed in collaboration between the partners from the school and the university.

Focus in these papers developed across the four levels, with a strong Level 4 focus related to Levels 1, 2 and 3.

5. Inter-relating theory and practice (Paper 6)

The one paper here was a theoretical paper looking at how aspects of inquiry might be seen to permeate research into learning and teaching at all the levels indicated above. Thus, inquiry was offered as a theoretical element to describe a variety of aspects of practice in which questions are asked and investigations undertaken. These might be questions into the nature of mathematics and students' understanding of mathematics; they might be questions into classroom tasks or teaching approaches; they might be questions into elements of interaction in the process of teacher reflection and the development of teaching to improve learning in the classroom. Thus, this paper offered a possibility to look across the various levels and, although not designed to do so, might be used to analyse relationships between the different constructs in the papers offered in the other sections.

Discussion moved into considerations of theory and practice and what is distinctive about this group, rather than for example the teacher education group, or the algebra group. Comments included the following:

- Teachers' theories differ from those of teacher educators and researchers – new practices come in, old practices are refined.
- People bring different types of knowledge, theoretical wisdom and practical wisdom, keeping theory and practice at the same level.

- We can think of applying theory and practicing theory.

We moved into considerations of evaluation and impact – whether practice is a theory generator, or whether theory has impact on existing practice. What change or evolutionary effects do we see from practice? Does a theoretical framework allow us to make more sense of what we see from reflection on and analysis of practice? Such questions took us overtly into a discussion of the four levels outlined above, and this will be continued in our final section below.

Conclusions

This section is in no sense a conclusion to the issues raised in the group: we were left with many questions that will generate further thinking and writing and which we expect to feed into work on the proposed special journal issue. However, the section should offer some closure to our work as a group. We will therefore reflect on the questions on which we focused in our discussion of papers and the four levels that emerged from our considerations.

The initial questions were:

- A What research took place?
- B What did it reveal with respect to teacher/teaching development?
- C What links between theory and practice do we see?

In relation to A, it was clear that the research described in each of the papers was different in many respects from that in other papers while being in the domain of mathematics teacher education and development of teaching and focused on issues relating theory and practice. The brief summaries of the papers, given above, provide glimpses of this research and the papers themselves (below) can be read to provide further detail.

The summaries above of discussion in the various sessions relate largely to Question B. Here we see what was revealed in terms of the four levels that have been outlined. The levels provide a means to approach issues relating theory and practice (Question C) as we shall indicate.

The four levels as they emerged from discussion are as follows:

- Level 1: Mathematics, analysis of the language of mathematics and students conceptions of mathematics.
- Level 2: Teachers' (or student teachers') access to analyses at Level 1, study of teachers thinking related to tasks and students conceptions of mathematics. Further design of tasks.
- Level 3 Teachers' (or student teachers') use of tasks, and the results of analyses at Levels 1 and 2, to work with students in classrooms. Study of interactions, students' learning and teaching development.

- Level 4 Relationships between research at Levels 1, 2 and 3 above and the forms of development which can be related to this research. This level encompasses issues in relationships between teachers, educators and researchers engaged in both teaching and research.

Elements of theory and practice are evident at all of these levels. For example, at Level 1 we have theory concerned with ways in which mathematical knowledge is translated into classroom activity for students' learning of mathematics. Such theory concerns the weaving of mathematical, pedagogic and didactical knowledge and research reveals associated issues of practice. At Levels 2 and 3 we see theory in teachers' knowledge and beliefs relating to task design and reflection on classroom experience shifting into teachers' analytical thinking about their students' learning and their overt focus on teaching development. Again research reveals the practical detail of such theoretical areas with many questions and dilemmas to challenge theory and provide indications for teaching development. As we move into Level 4 we become aware of the complexities of both theory and practice in teaching development. Here we recognise inter-relationships between the practitioners at the various levels, the reflective analyses that research encourages and the ensuing implications for developments in both theory and practice.

Our treatment here is necessarily brief since the analysis we present has emerged from thinking and discussion within the group and our joint analyses relating to the presented papers: i.e., none of the papers address this development of levels. It is clear that we have putative further theory here, but that it needs a clearer relating to practice before it can be offered coherently in the public domain. We see it as being one of the major tasks of a journal special issue to work towards such an articulation of the outcomes of our group's work.

However, we will end our discussion by quoting from four perspectives that were offered in writing by certain members of the group as a result of their reflections on some of our sessions.

1) About theory and practice (in connection with discussion on Paper 2): We could say that *theory* is represented by the world of the (university) researchers and *practice* by the world of teachers and school. In fact, we, as researchers, learn and construct theories about teaching and learning mathematics, and we want to use these theories in the work with teachers. This is not trivial if we look at the evolution of didactics of mathematics: the relation between theory and practice is a recent issue in didactics. ... [As researchers] we more specifically want to apply *theory* in order to favour the growth of teachers' awareness. The question could then be to what extent these theories shall be explicit or implicit (for researchers, or for the teachers involved).

2) Distinction between theory and practice seems to be often a useful tool for keeping under control the complexity of a situation. But, on many occasions, more than to elaborate a theory we are focused on practicing a theory in order to

test or modify a theory as well as to solve practice issues. In other cases we are engaged in theorising practice in order to

- grasp in the practice something to modify;
- find some non-local values or some general clues to be transferred;
- bring in to evidence some clues to confirm, distinguish or elaborate a theory.

To separate the two poles (T and P) brings us towards a clarification and specification; the problem is to cope with a suitable and fruitful level of complexity. The problem is which kind and which level of theory is useful for helping everyday decisions in the teaching process? And which kind and level of practice can be helped by theory and helps theory to grow?

3) Assume that

- theory is the objectives of teaching mathematics that are set by didacticists and didactic processes that in our research proved to be successful in attaining those objectives;
- practice is what is going on in ordinary classrooms.

The constraint and pressure exercised upon teachers by tests (“educational Measurement”) make idle our effort to influence practice. More attention should be devoted to finding ways of assessing students’ performance in the sense that would be acceptable by didacticists and politicians at the same time. Without change in this area we shall remain an isolated group of enthusiasts needed by nobody.

4) What do we mean by theory and practice in the field of teacher education?

Teacher education is an interaction process, mainly between teacher educators and (student) teachers, but also including systematic interactions among teachers aiming at professional growth. At the same time, we can see teacher education as a learning environment for all people involved in this interaction process.

When we talk about planning and evaluating these learning environments we are talking about teacher educators’ practice. It is assumed that teacher educators build this practice on theoretical assumptions about the interaction processes and aim at understanding the effects of their interventions. This means that they do not only teach about e.g. constructivist theories of learning but to put these theories themselves into practice in order to be a role model for their (student) teachers. For the latter such a link between theory and practice allows them to reflect on their own learning process, to anticipate real practice in classrooms and to develop their theoretical understanding further. In this interpretation, the teacher educator is a practitioner who reflects on his or her own teaching in order to improve it and to enrich the complex interrelation between what is possible in theory and manageable in practice. The teacher educator both aims at understanding and improving his or her practice.

If we interpret teacher education as a learning process for all, we more and more overcome the division of labour between theory and practice. Working at universities means more than creating theories, and working at schools more than putting these theories into practice. Situations and problems in teaching are complex and need particular solutions that can only be developed in the specific context of their appearance. There are no general solutions that might be transferred from theory to practice. Also at schools, improving and understanding one's own practice is important. We need more teachers who reflect critically on their teaching, exchange their experiences, and read theory-driven papers in order to broaden their understanding of educational processes. We need more teacher educators who take their teacher education practice as an object of evaluation and research. And we need more collaboration between teacher educators and teachers in order to promote teacher education – as a field of practice and research.

These extracts capture some of the issues that were discussed and which further thinking, discussion and writing of group members can address. They emphasise the complexity in processes of teaching, learning to teach and developing teaching in mathematics. They raise fundamentally a concern with the integration or separation of theory and practice. For example, if we see *theory* as being in the domain of universities and *practice* in the domain of schools and classrooms, can this introduce barriers, explicit or implicit, to a useful interaction between theory and practice? What does it mean, however, to move towards a realistic integration of theory and practice? And in whose practical worlds is such integration possible and realisable? These questions provide a glimpse of the essence of our field and its ongoing debate.

The papers follow.

[List of contributions](#)

[List of Thematic Groups](#)