INHERITANCE PROBLEMS IN ARABIC ALGEBRA TREATISES.
CAN THEY STIMULATE FUTURE TEACHER'S BELIEFS ABOUT MATHEMATICS?

Jordi Deulofeu and Lourdes Figueiras

Departament de Didàctica de la Matemàtica i les Ciències Experimentals

Universitat Autònoma de Barcelona, Spain

Abstract

The experience we describe in this paper has been carried out with future primary teachers in their first year at university during a course on mathematics and history of mathematics. We focus on the students' performance and discussion while they are engaged in solving inheritance problems in old Arabic algebra treatises. Our aim is to discuss how they use the historical context in which these problems were developed to question historical or cultural aspects involving mathematical activity. Results show that algebra induces an instrumental perspective towards mathematics if no reflections on historical or cultural conditions are taken into account.

1. Introduction

Many authors agree that the study of the history of mathematics contributes to change epistemological assumptions concerning mathematics and mathematics learning (Barbin, 96; Fauvel 91). It makes historical subjects particularly suitable for the research on mathematical beliefs within particular teacher training programs.

It may be obvious from our point of view as researchers or teachers that the history of mathematics and human thinking illustrates different epistemological approaches; but nevertheless it remains a complex matter to elucidate how future teachers elaborate their personal approaches to mathematics when they learn about its history, and in no case is it so obvious. We want to describe and highlight which different approaches arise when people participating in our investigation -future primary teachers in their first year at the university- take a course in mathematics based on history.

2. Theoretical background

Research focusing on the development of teacher's beliefs within a training program is not frequently undertaken, although important investigations have been carried out in the field of mathematics and history, as shown in the ICMI study edited by Fauvel and Van Maanen (2000). Also the recent review carried out by Gulikers & Blom (2002) and particularly Philippou and Christou (1998), are some exemples of important references for us. With the last one we share essential features such as: a)

focusing on teacher's beliefs on mathematics and mathematics education; b) having carried out longitudinal studies; c) analyzing possible changes in teachers' beliefs during a course on mathematics based on history. However, and concerning the metioned investigation, we will consider in the following other theoretical and methodological assumptions which lead us to significant differences in the conclusions.

A qualitative approach based on socio-cultural and discursive perspectives was adopted for the analysis and interpretation of data (Lerman, 01). In this paper, we have decided to focus on the micro-analysis of the students' experience while they are engaged in one specific kind of problems, namely inheritance problems in old Arabic mathematics, rather than on a global overview of the course or the methodological aspects concerning the whole research. However, some specific aspects on methodology and global conclusions will be addressed, and in the following section we briefly describe the academic setting in which this research has been developed.

3. Mathematics for future primary school teachers: The Spanish situation

The students' mathematical knowledge when they arrive at university is generally very different from what we expect, and in most cases insufficient. Those who were successful in mathematics at secondary school are more efficient in solving routine exercises than in problem solving tasks. Besides, they show no interest in looking for sense in the mathematics they learn, or in reflecting about its role in our society.

Successive revisions of the education law have aggravated this situation, for many of these future primary teachers have had no contact with mathematics after they finished compulsory levels of education; and during their stay at university, they will only take two compulsory subjects on mathematics education (60 hours per year each). Two more subjects, one in mathematics and one about mathematics education, are optative credits.

Deciding which contents should be considered during their years at the university is undoubtedly a complex question: On the one hand, primary teachers should have a good knowledge of areas, such as mathematics, language or social sciences; but also professional knowledge concerning general pedagogy, psychology, sociology or didactics is needed. The situation nowadays gives more importance to these educational subjects, while school subjects are, in some way, set aside.

These facts suppose implicitly that future teachers' knowledge in subjects such as mathematics have been acquired during compulsory school. Even assuming that this were a reality, it would mean that there is no need for future teachers to know anything more about mathematics than any other citizen, which is doubtful enough in itself.

In order to equilibrate this situation a little, the Universitat Autònoma de Barcelona negotiated a compulsory course of 40 hours on mathematics when the students enter

university. During many years this has been a course guided by a problem solving approach with an explicit intention of sharing with the students the point of view that problems are at the heart of mathematics. This way of proceeding has proven very productive, as it breaks in some way with the habit that exercises follow theoretical expositions on concrete techniques. However, it was not the best approach to stimulate students to reflect from the point of view of a contextual or social activity. Traditional problem solving courses are really efficient for thinking about the methodology of mathematics, but do not question its relevance for society.

History as a context for mathematical problems

This situation motivated us to design a new approach for a first course on mathematics which was also problem-based, but focusing mainly on historical problems within a socio-historic frame. We largely restricted ourselves to measurement problems up until the 17th century, and especially to the very productive Arabic mathematics.

When we propose to the students to reflect on some problems from the point of view of their historical context, our intention is to provide them with a setting in which different beliefs concerning mathematical activity may be recognized. Especially interesting problems or mathematical activities were those where the statement or solution is highly affected by its historical context: Surveying problems, numerology in old Chinese mathematics, geometric models for describing the shape of the Earth, or inheritance problems derived from Arabic laws are illustrative examples. Some references from the History of Mathematics literature concerning these problems may be found in (Martzloff, 97; ben Musa, 1831; Vogel, 83). From our theoretical background, these problems acquire their full meaning when they are immersed in their historical setting. It is this feature which makes them especially interesting, by stimulating the students to elaborate their own perspective, and providing a context for their mathematical activity.

All of the students had to reflect about the contents and method of the course either individually in writing, or by engaging in group discussions. Approximately half of the students chose either approach. Written reports and transcripted discussions were considered as main research data. Each group met four times, and the rest of the students also wrote four reports.

4. Inheritance problems in Arabic mathematics

Inheritance problems belong to ancient Arabic law rather than to mathematics. They explain in detail enunciations of the laws of inheritance in complicated cases. Sometimes the given solutions are mathematically incorrect; it is not that the equations are solved incorrectly, but they are contradictory to the data, or sometimes even arbitrary. Only by situating such problems in context is it possible to understand this fact. Let us now consider one of these problems.

A man dies, leaving four sons and his wife; and bequeathing to a person as much as the share of one of the sons less the amount of the share of the widow. (Ben Musa, 1831, p.95-96)

When we try to solve this problem we realize that something is wrong: In Spain, the widow usually gets as much as all the children together. So it makes no sense to bequeath to a third person the fraction of money set out in the statement. But let us consider the context in which the problem was posed. Arab law imposed among others the following constraints:

Concerning the order of distribution, the stranger's legacy precedes the division among direct heirs.

A wife receives 1/8 of the legacy.

The remaining legacy is shared among the sons.

Under these assumptions the problem could be easily solved as follows:

Let x be the fraction of the legacy that goes to the stranger. The widow then receives 1/8 th part of the residue, that is $(1 - x)/8$; each son gets $1/4 \cdot 7/8 \cdot (1 - x)$.

The stranger's share, x, should be a son's share minus the widow's share, in other words,

$x = (7/4 - 1) \cdot (1 - x)/8 = 3/4 \cdot (1 - x)/8$

from which we get $x = 3 / 35$. A son's share turns out to be 7/35, and that of the widow is 4/35.

5. Research questions

Some authors affirm that, in some way, the complicated law which governed inheritance was an important stimulus for the emerging of Algebra (Boyer, 69) but such contextual facts were removed in the Latin translations of Arabic treatises. For example, a detailed discussion on inheritance problems in Al-Khuwarizmi' *Algebra* was not included in its translations into Latin, probably because of its high dependence on context.

Within this framework, **we** could consider inheritance problems as a clear illustration of cultural and historical influences on the development of mathematics, and in this sense it could be interesting to let the students think about this fact. This could be our intention as teachers, but what significance do **the students** give to inheritance problems? Three more operative questions were developed in our analysis:

Do the students use the setting generated by the course to connect different mathematical activities?

Do their beliefs about mathematics move in a range of cultural and historical relativisms?

Which role do their beliefs on arithmetic and algebra play in relation to the above questions?

6. Case study and discussion

The students' experience was gathered from the discussions groups and the reports they wrote. We selected for the whole investigation the case of four people participating individually, and two of the discussion groups. There was a significant number of situations concerning movements of the students' beliefs towards a relativist conception of mathematical activity, expressed through assessments such as: "It depends on the culture you are embedded in, the problem could be simple or much more complex" or "They are not simple problems where only numbers are used, but there are also laws affecting them"; but we have chosen to discuss here those cases which distance from our main intention as teachers, namely inducing the students to reflect on the influence of culture and history in the development of mathematics. With this selection of cases, we would like to emphasize the deviation from what could be considered as expected results. Our intention here is not to highlight a special dynamic of change in students beliefs, but to emphasize different approaches to mathematics in a specific moment of the course. Selected cases are summarized in table 1.

Mathematics is considered a scholarly subject, and for many of the students it is only useful outside of school to solve arithmetic problems or to be used by scientists. From this assumption, they derive that their commitment as future teachers is learning what they should teach at the primary school; no sense is given to a possible discussion in this setting on the development of algebra, a subject that was an obstacle during their secondary education. They are strongly motivated to think about teaching in the primary school, especially in this first year at university. These facts lead them to perceive all new mathematical activities from the point of view of both their mathematical experience, and its immediate application to the primary school setting.

Because of the constraints imposed by this perspective, problems are evaluated in terms of some didactical rules such as "posing problems close to the children's context", and not in a wide sense involving different perspectives, historical facts, or other people's contexts.

C1	*There is no explicit recognition of the historical or contextual sense of this kind of problems, but they are considered mathematically interesting.*

Inheritance problems are considered adequate for primary levels whenever no reference is made to Arab laws, and whenever they have a solution using easy operations. The same problems including their historical conditions are considered mathematically ``much more complicated'', and useful perhaps for secondary levels of education.

C2	*There is no explicit recognition of the historical or contextual sense of the problems; they are mathematically interesting, but their statements are considered inappropriate to be used in the classroom.*

This could be considered a sub-category of the previous one, as the problems are considered adequate for developing some arithmetic skills, but their statements should be changed to bring them closer to the children's reality: inheritance is not considered a good context because it involves death. Furthermore, when speaking of "dirhams" in some of the problems, they considered that no foreign currencies should be considered.

C3	*There is no explicit recognition of the historical or contextual sense of the problems, and they are considered of no mathematical interest.*

Some of the students manifested that this kind of problems was "the type of problems they had always done at school, and that made them hate mathematics." They referred to them as *typical* problems to be solved when they were "doing equations."

C4	*The contextual role of the problem is recognised, but no mathematical reflection concerning the evolution of mathematics is mentioned.*

This category belongs to the case of a woman with a solid background in social sciences. Inheritance problems impulse her to reflect on the moral values transmitted during the teaching of mathematics. The following is a passage in one of her reports:

"It is important to know how to share something without prejudicing anybody, and trying to give each person the same. This is a fact showing equity. [...] Maybe it seems incredible or surprising, but mathematics can also transmit moral values. [...] In one problem, students could perceive a tolerant attitude while in another they could see how different parties are discriminated because of their condition."

Table 1: Categories C1 to C3, arose in one of the discussion groups; C4 belongs to the case of one of the students participating individually through written compositions. None of them made any reference to the development of Algebra, but consider the problems as complex arithmetical exercices.

Most of the students have a very low level in mathematics which does not permit them to engage in conceptual mathematical discussions. For example, they do not refer to Algebra when discussing the problems, but instead to "everyday operations using letters." It is very difficult to appreciate the cultural influence of Arab laws on the development of Algebra when only such a primary meaning has been constructed. If deeper meanings have not been constructed before, it is very difficult to create the proper setting to do it. This is basically because, as adults, the students clearly manifest what they want to learn, namely, how to teach mathematics to children at primary school, and in their view this involves no algebra.

7. References

BARBIN, E. (1996): The Role of Problems in the History and Teaching of Mathematics. In: Ronald Calinger (Ed.) *Vita Mathematica. Historical research and its integration with teaching,* MAA Notes, 40, 17--25. Academic Press, New York.

BOYER, C. (1969) A History of Mathematics. Spanish edition: Historia de la matemática, 1986, Alianza, Madrid.

FAUVEL, J. (1991): Using history in mathematics education. *For the learning of Mathematics,* 11(2), 3-6.

FAUVEL, J. and Van MAANEN, J. (2000): (Eds.) History in Mathematics Education (Introduction). Kluwer, Dordrecht.

GULBEKIAN, I. and BLOM, K. (2002): 'A historical angle', a survey of recent literature on the use and value of history in geometrical education. *Educational Studies in Mathematics,* 47, 223-258.

LERMAN, S. (2001): Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics,* 46, 87-113.

MARTZLOFF, J.C. (1987): Histoire des Mathématiques. English edition: A history of Chinese Mathematics. Springer, Berlin.

ben MUSA, M. (1831): The algebra of Mohamed ben Musa. Georg Olms Verlag, Hildesheim. (Edition 1986).

PHILIPPOU, G.N. and CHRISTOU, C. (1998): The effects of a preparatory mathematics program in changing prospective teachers' attitudes towards mathematics. *Educational Studies in Mathematics*, 35, 189--206.

VOGEL, K. (1983): Ein Vermessungsproblem reist von China nach Paris. *Historia Mathematica*, vol. 10, 360-367.