IMPROVING TEACHERS' ABILITY TO COPE WITH PROBLEMATIC LEARNING SITUATIONS - THE CASE OF ETI

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The paper describes the process of change that Eti, a pre-service mathematics teacher, undergoes during a year-long academic course. The course was aimed at enhancing teacher awareness of students' ways of thinking and improving the teacher's ability to identify, analyze and cope with problematic learning situations in geometry. It was designed on the basis of findings (e.g., Gal, 1998) pointing to lack of pedagogical knowledge and lack of awareness of children's thought processes. The case study outlines Eti's progress from not foreseeing difficulties, not being able to analyze them when they occur and handling them intuitively to using gained theoretical knowledge to identify, analyze and cope with problematic learning situations.

BACKGROUND

Years of observation of geometry lessons in junior high school classes for low- and intermediate-level students exposed me to countless situations in which teachers had difficulty helping their students to overcome barriers or pitfalls. Upon identifying and analyzing these Problematic Learning Situations (hereafter PLS, after Gal & Linchevski, 2000), I discovered that teachers tend to be unaware of their students' difficulties. When they do recognize them, they often fail to cope with PLS due to lack of pedagogical knowledge or inability to use existing knowledge (e.g., Gal & Vinner, 1997).

Part of the solution could be to enhance teachers' pedagogical knowledge - especially knowledge about students, which is critically important (Borko and Putnam, 1996) - as recommended by the paradigm of Cognitively Guided Instruction (Franke, Fennema, & Carpenter, 1997), which largely applies to elementary school teachers. While the teachers I observed were unfamiliar with some of this knowledge (e.g., visual perception), they had, in fact, been exposed to some theories (e.g., van Hiele) during their studies. But they had not applied the knowledge during their training, and they did not use it in classroom instruction. As Borko and Putnam claim, for knowledge to be useful for teaching, it must be integrally linked to, or situated in, the contexts in which it is to be used. I therefore sought to present the needed knowledge in a manner that made it relevant to teachers and motivated them to use when planning and giving lessons. I decided to introduce real PLS to junior high school teachers and familiarize them with relevant theories they could use to analyze the PLS. In this way I hoped to enhance their awareness and understanding of students' ways of thinking.

With this in mind, I designed a year-long academic course for future and in-service mathematics teachers dealing with PLS in geometry. The course combined theoretical pedagogical knowledge with its practical application. The practical aspect included

video clips exhibiting PLS and interviews with teachers documenting how they coped with PLS in geometry. The theoretical aspect integrated general topics, such as concept formation, with topics specific to geometry, like visual perception and van Hiele theory. Among other assignments, course participants were asked to analyze PLS for which the theoretical subjects they had learned could provide an explanation (for more details about the course, see Gal, Linchevski & Cockburn, in press).

METHODOLOGY

Qualitative research was carried out on a sample comprising two groups. The first included 20 pre-service teachers studying in a college towards a B.Ed. in mathematics teaching for junior high schools. During this year they taught actual classes as part of their training. The second group was made up of 23 in-service teachers studying at a university towards an M.Ed. in mathematics education. The paper focuses on the progress of Eti, who belonged to the first group. Eti was a third-year undergraduate, studying towards a degree in mathematics and physics teaching. During the year in question, she taught two weekly hours of geometry to low- and intermediate-level eighth graders. At that time, I was her tutor.

Data was collected by taping course activities (such as analysis of PLS) and copying written assignments, such as lesson plans, questionnaires and recorded reflections about course activities and their own classroom experiences. The study examined changes in course participants over the year, with special attention to their ability to identify, analyze and cope with PLS. It focused on change in both the "laboratory setting" (i.e., environments in which the teachers do not have to make real-time decisions, such as when they analyze PLS presented in a video clip) and actual classroom instruction. This paper reports on changes in the laboratory setting only. (A future paper will deal with classroom instruction.)

FINDINGS

The analysis refers to three aspects of change (all in the "laboratory setting"):

1. Changes in the teacher's *awareness* of difficulties (anticipating difficulties before they happen and/or recognizing them when they do) and ability to *identify* them.
2. Changes in the teacher's ability to *analyze* difficulties by means of cognitive theories and to exhibit awareness of the student's ways of thinking.
3. Changes in the teacher's ability to suggest effective ways of *coping* with encountered difficulties.

Note that this order is not necessarily chronological, and change in one aspect does not necessarily effect change in another (e.g., we found progress in coping even before there was progress in analysis). Nevertheless, these three aspects are probably somewhat interdependent.

In the following analysis, I emphasize the main characteristics of change over time.

The Starting Point - Beginning of the Course

Awareness of difficulties and ability to identify them

In the first week of the course, Eti could not foresee the difficulties typically encountered by slow or average junior high school students of geometry. When asked about expected difficulties, she referred to: using Latin letters (rather than Hebrew characters); breaking complex figures down into components; and abstracting by looking at characteristics and not just the shape. She disregarded such common difficulties as: limited concept image of geometrical objects; difficulties identifying non-prototypes; problems originating in visual perception, such as limitation of gestalt principles; problems resulting from a non-suitable level of instruction (van Hiele theory); and pseudo conceptual and pseudo analytical behavior.

In the first week, Eti was asked to prepare three lesson plans about assigned subjects (e.g., perpendicular lines). Analysis of these plans indicated that she not only disregarded many of the common problems faced by these students, but she did not always take into consideration those difficulties she herself raised:

Visual perception. Eti did occasionally anticipate difficulties in decomposing configurations into components (e.g., she foresaw the problem of identifying equal parts when proving congruency of triangles), but not in all cases they could be expected. She did not mention other topics related to visual perception, such as mental transformations (e.g., difficulties identifying congruency when a mental transformation of one of the triangles is needed) or decomposing complex configurations (e.g., she asked students to identify a right angle only among "basic angles").

Van Hiele levels. In the opening questionnaire Eti indicated familiarity with van Hiele theory, though she had not used it yet: "I understand that there are levels of development in geometry and I'll try to take these levels into consideration." Unfortunately, there is no sign of this in her initial lesson plans for the course. Some of her lessons were at van Hiele level 4 (e.g., proving the orthogonality of diagonals in a rhombus formally), yet the plans were supposed to be written for slow to average students, whom we may assume to be at level 1 or 2. Eti thus seems to be unaware of the difficulties arising from an inappropriate level of instruction. Moreover, she used Latin letters in her formal proof, even though she expected them to be "problematic".

Concept formation. Eti did not consider concepts like the bisector of an angle in a triangle (Gal, 1998) or perpendicular lines (Gal & Vinner, 1998) to be problematic; when she used these terms in her lesson plans, almost nothing was mentioned about possible difficulties. Nor did she take into account, directly or indirectly, the issue of "concept image" or the problem of a limited concept image. This is especially important with lower-level students, whose perception of figures tends to be visual rather than logical-analytical.

Prototypes. Despite a certain exhibited awareness of the need to avoid the sole use of prototypes, Eti did not consistently put this into practice. For example, she drew a non-prototype rhombus in a lesson plan, but when presenting the plan to colleagues in

the course, she drew a prototype rhombus on the board, although she then apologized: "Sorry for drawing a prototype." Further, she directly referred to the "difficulty in identifying a right angle which is not a prototype," yet presented the concept of a right angle via two prototype examples only.

Ability to analyze difficulties according to cognitive theories

Eti did not try to analyze any of the difficulties she mentioned in her initial lesson plans. Nor did she make any effort to follow the hypothetical thought processes of students.

Ability to suggest ways of coping with encountered difficulties

Eti suggested didactic solutions intuitively and not analytically. Some of the solutions she proposed can be considered inefficient and may even frustrate the students. For example, when difficulties arose in one proof, Eti suggested an alternative formal proof. She did not take into account that formal proofs of any kind are inappropriate for students at van Hiele level 1 or 2.

Sometimes her solutions seem superficial; she seems to be trying to follow some rules not because she understands their importance, but because she was told to do so. For example, although aware of the difficulty of a limited concept image when using only prototypes, all she did to enlarge the concept image was to draw a *single* example of a non-prototype figure, which is clearly not enough. Moreover, she did not give reasons for drawing the non-prototype figure or even mention that using prototypes exclusively could be problematic.

Other solutions she suggested could be effective. For instance, in order to identify easily equal parts in triangles that have to be proved congruent, Eti suggested using a transparency on which one figure is copied. Putting one shape on top of the other can help to identify the corresponding parts. She also suggested the use of colors, relying intuitively on the principle of similarity (Anderson, 1995).

Process of Change

Raising awareness: Getting a shock

In the second week of the course, the teachers watched three video clips presenting PLS. One presented difficulties in identifying the right angle between the square's diagonals (Gal & Vinner, 1997). Another showed a student having difficulties using theorems of congruency and identifying the relevant parts in a simple congruency problem. The third one presented a student having considerable difficulty with the concept of the bisector of an angle in a triangle: although able to recite a definition of a bisector, he was unable to locate the equal parts formed by it.

As was expected, when asked to respond to these PLS, the course participants registered surprise. They were shocked to discover these difficulties in topics they assumed easy to learn: "I need to overcome my shock," said one; "All the lesson plans I wrote are useless," stated another. These reactions to real PLS suggest the very beginning of change in teachers' awareness of difficulties.

Progress in similar situations - Impact of the shock

After 2-3 weeks in the course, Eti explicitly noted the impact of the PLS she was observing on her lesson plans. She could now foresee difficulties in situations similar to those introduced in the PLS. Still, she could not focus on why these difficulties occurred. In other words, there was no change in analysis. In terms of coping with PLS, although she used visualization more than formality, there was still no connection between the reasons underlying the difficulty (which she could not explain yet) and the suggested solution. Her solutions were frequently instrumental.

Beginning of overall change - Using "new" theories and evoking "known" ones

The first theoretical subject covered in the course was visual perception. This subject was covered until the end of the second month and had an immediate influence on Eti's lesson plans. Eti criticized her own lesson plans written one month before, indicating difficulties involving visual perception she hadn't noticed before and explaining difficulties of this kind that she was unable to explain in the past. There was also a clear change in her suggestions for coping with difficulties involving visual perception; they were now based on theory and no longer intuitive. For example, in regard to the difficulty in identifying three angles within two angles with a common ray, she stated:

I was expecting difficulties in recognizing more than two angles [in the case of a common ray], but then I didn't know how to explain it. The difficulty originated in the principle of proximity [one of the gestalt principles (Anderson, 1995) taught in the course in the context of visual perception]. Now that I know that this is the problem, … I'll erase the ray in the middle [the common ray] … and by that I'll help them to recognize [the third angle].

… And there are more solutions: we can give the students transparencies, draw one angle on each. The student can place one angle at a time or both at once and be convinced that the existence of one angle does not interfere with the other.

This theoretical instruction not only helped Eti to reflect on her own earlier lesson plans, but also influenced her new lesson plans. This time she foresaw difficulties concerning visual perception. She used theoretical issues taught in the course to foresee and explain difficulties. For example, Eti expected difficulty in recognizing a simple form in a complex one (explained by gestalt principles), as well as difficulty in naming figures with letters (which could be explained by various representations for verbal and visual information; see Gal & Linchevski, 2002). Moreover, when she analyzed new PLS in the course, Eti applied the theory she had just learned.

Interestingly, Eti also began to refer to van Hiele's theory (though not consistently), even though that was not being taught at this stage of the course. For instance, she referred to this theory to explain why she did not use a formal definition and why she starts with sensory and especially visual assignments. This suggests that the increased awareness of difficulties and the new tools (theory of perception) offered to explain them stimulated in Eti's mind known theories that can also help to analyze or foresee

PLS. Nonetheless, while she was not using level 4 any more, Eti rarely referred to van Hiele when she analyzed given PLS. Apparently, it is harder to acquire the ability to analyze PLS in which problems originate in a gap between level of instruction and student level than it is to take van Hiele levels into consideration when planning a lesson. In other words, *it may be easier to prevent difficulties of that kind than to classify them into the right category of difficulty after they occur.*

It should be noted that, at this stage, Eti referred only to one dimension of difficulty (e.g., visual perception, van Hiele theory) at a time. As we shall see, it will take her more time to refer simultaneously to several dimensions. It should also be noted that Eti was beginning to become aware of her own progress. As she described in the monthly feedback (6th week): "When I was asked to pinpoint a new subject I learned [in the course], it was difficult to choose because I learned many new things. … Thinking back, the new thing that I got is a changing attitude and point of view." As we shall see later, these feelings will deepen in the following months.

Multi-dimensional analysis - When analysis and reflection become an inherent need

After 7-8 months, Eti identified and foresaw a broad range of difficulties and analyzed them comprehensively, including multi-dimensional analysis. In one PLS, she simultaneously took into account aspects of visual perception, van Hiele theory and pseudo conceptual behavior (Vinner, 1997): "I would have said that the student identified [the diagonal] in level 1 and gave reasons in level 2 …, but when I go on reading the dialogue [between teacher and student] I think his answers testify to pseudo conceptual behavior." She used insightful ways to cope with these difficulties, applying the learned theories.

Not only was Eti able to identify and analyze PLS as part of an external assignment, she began to see this process as inherent to her lesson plan. At this stage, Eti *started* a lesson plan saying: "Before planning the lesson I'll first consider expected difficulties that might arise among the students, and I'll try to figure out solutions which will be used in the lesson plan." No longer was Eti responding to the question in an assignment, "Which difficulties are expected concerning that subject?" Instead, she saw this process of PLS identification and analysis as meeting her own needs as a teacher.

Eti's lesson plans show that she was now more attentive to students' answers, trying to figure out their misconceptions, their concept image. In a lesson plan she was asked to write in the 7th month of the course about the bisector of an angle, Eti wondered whether to begin the new subject by simply asking students to bisect an angle (without explaining to them how it was done) and asking them to explain why they did so. The advantage of such an approach, according to Eti, was having the opportunity to follow their way of thinking.

At this stage, Eti also reflects on her own teaching without being asked to, criticizing a case in which her way to cope with PLS was ineffective. Eti was no longer responding

to the assignment of reflection (as she was frequently asked to reflect about her actual instruction). Rather, she found reflecting on her own practice to meet her own needs.

Not the same as before

In the last lesson, Eti summarized the course: "I am not the same Eti as in the beginning of the year. … There is a tremendous change in the way I see things. It affects also my instruction of physics." This suggests a transfer to other areas beyond geometry. This change is also reflected in her self-confidence. Compare Eti's answer to the same question asked at the beginning, middle and end of the course ("To what extent do you feel confident with the following (1-5, 5 is highest confidence)?"):

\ Time Category \	Beginning of the course	Middle of the course	End of the course
Knowledge of geometry	**4**	**4-5**	**5**
General pedagogic knowledge	**3**	**3-4**	**4**
Pedagogic knowledge in geometry	**2-3**	**3**	**4**
Success in meeting your own expectations	**4**	**3**	**3-4**
Ability to analyze your actions as a teacher	**4**	**3-4**	**3-4**

It is clear that confidence in knowing geometry, general pedagogy and geometry pedagogy increased during the course. But Eti, who knows more now, is less confident of her ability to analyze her work and even to meet her own expectations. This is not surprising: the more knowledge acquired, the more you recognize your own ignorance.

SUMMARY

With the aim of improving teachers' ability to recognize, analyze and cope with PLS in geometry, I designed a course in which relevant theory and examples were introduced, actual class situations were presented and participants reflected on their own teaching experiences. Analysis of the impact of this course on Eti indicates a significant increase, over the academic year, on her ability to *identify* the student's and teacher's difficulties, *analyze* the reasons for them and *cope* with them in a "laboratory setting."

Several characteristics of change can be pinpointed: (1) There was movement from identifying a single aspect of difficulty to identifying multiple aspects. (2) There was growth from recognizing and analyzing similar PLS to being able to transfer this knowledge to other areas. (3) Growth in awareness helped to retrieve dormant knowledge that can aid in understanding and explaining PLS. (4) It is harder to criticize PLS in terms of van Hiele theory than to plan a lesson taking van Hiele levels

into consideration. (5) "External" assignments (given by the course lecturer) of identifying, analyzing and suggesting how to cope with PLS became a need of the teacher – an "internal" assignment. (6) The teacher came to be attentive and reflective on her own. She was not doing this because she was asked to, but rather as a result of her own enthusiasm.

Rhine (1998) claims that it is unrealistic for teachers to learn about their students' thought processes in all areas; thus, the main goal is to get teachers involved in the inquiry of their students' ways of thinking. Hopefully, the presented course will promote that tendency. Further analysis should consider the other pre-service teachers and especially in-service teachers (the second group). Additional research is also needed to follow changes over a longer period of time.

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