HOW DO PROSPECTIVE PRIMARY TEACHERS ASSESS THEIR OWN MATHEMATICAL KNOWLEDGE?

Maria Goulding

University of York, UK

Prospective primary teachers in three universities were invited to self- assess their mathematical subject knowledge, before taking a formal audit assessment. They identified difficulties with shape and space, graphs, terminology and explaining their thinking. Shape and space persisted as a weakness on the formal audit, along with reasoning and proof. The findings are discussed in the context of regulations for primary teacher training in England and Wales.

INTRODUCTION

The *SKIMA* (Subject Knowledge in Mathematics) group is a collaboration between researchers in the Universities of Cambridge, Durham, York and the Institute of Education at the University of London. It grew out of a common interest in primary (elementary) teacher trainees' (the term used by government agencies in England and Wales) subject knowledge in mathematics predating the introduction of the government's (DfEE, 1998) National Curriculum for Initial Teacher Training. This required teacher training institutions in England and Wales to audit a specified body of mathematical subject knowledge and where 'gaps' were found to make sure these were 'filled' by the end of the training course.

Building upon previous work (Rowland, Martin, Barber and Heal, 2000; Goulding and Suggate, 2001) the researchers devised a common procedure for use with over 400 primary trainees on the one year Post Graduate Certificate of Education (PGCE) course at the Universities of Cambridge, Durham and the Institute of Education, London. It involved a self-assessment (the self-audit) of subject knowledge early in the course in October, a period when specific teaching was given and/or trainees could follow up areas of weakness, an audit undertaken in formal conditions in February and a follow up period when peer teaching was put in place. Audit, a term more common in industry and finance, is the term used by government to describe a process or instrument used to identify strengths and weaknesses. This paper concentrates on the early part of this process in which trainees made a self-assessment of their mathematical knowledge using a structured self-audit. It makes some comparisons between this self-assessment and performance on the formal audit. Research from the *SKIMA* group on strategies for supporting trainees, notably with peer teaching, has been reported previously (Barber, Heal and Martyn, 2002) as has an investigation of how primary trainees' mathematics content knowledge is evidenced in their teaching (Huckstep, Rowland and Thwaites, 2002).

THEORETICAL BACKGROUND

The conceptualisation of subject knowledge and its relation to teaching which informed the project has been detailed fully elsewhere (Goulding, Rowland and

Barber, 2002). Shulman's construct of *subject matter knowledge* 'the amount and organisation of the knowledge per se in the mind of the teacher' (Shulman, 1986, p.9) later analysed further (Shulman and Grossman, 1988) into *substantive knowledge* (the key facts, concepts, principles, explanatory frameworks in a discipline) and *syntactic knowledge* (the rules of evidence and proof within a discipline) were influential in our design of the audit instrument.

In auditing trainees' subject knowledge, the group had to work with the content set out by the government (DfEE, 1998), including concepts and processes not in the primary curriculum but deemed to be relevant to it e.g. representing functions graphically and algebraically, understanding gradients and intercepts, familiarity with methods of proof. With previous trainees we had identified some weaknesses in substantive knowledge but also the particular difficulties which trainees in previous cohorts had with generalisation, reasoning and proof (Goulding et al. 2002, Rowland et al 2001). We interpreted these as a weakness in syntactic knowledge, an inability or unwillingness to make and test conjectures by personal investigation. Working with the same requirement to audit and remediate primary UK teacher trainees' mathematical knowledge, but with different audit instruments, Sanders and Morris (2000) found problems in all areas of the curriculum and Jones and Mooney (2002) found particular weaknesses in geometry.

The relationship between subject matter knowledge and the pedagogical content knowledge (Shulman, 1986) required for teaching is still not fully understood although promising insights into the way in which a combination of the two inform teaching are emerging (Huckstep et al. 2002). In the past, Carol Aubrey (1997) has argued for 'the central importance of disciplinary knowledge to good elementary (primary) teaching' (p.33). The Effective Teachers of Numeracy Project at Kings' College, London (Askew et al., 1997) used a much larger sample than Aubrey and also measured pupil progress by test score gains. It found that the most effective teachers believed in the potential of all pupils to become numerate and also had 'knowledge and awareness of conceptual connections between the areas which they taught' (p.3). These so-called 'connectionist' teachers did not necessarily hold advanced *qualifications* in mathematics but they were more likely to have benefited from extended continuing professional development courses. Liping Ma's (1999) comparison of American and Chinese teachers' mathematical understandings, used the term 'profound understanding of fundamental mathematics' to describe the sort of knowledge required for primary mathematics teaching and concluded that no amount of general pedagogical knowledge could make up for its absence.

With previous cohorts of trainees at the London Institute of Education an association between the subject knowledge audit score and teaching performance had been found (Rowland et al. 2000, 2001). In particular, trainees with low audit scores were more likely to be assessed as weak mathematics teachers. This does not necessarily contradict the Kings' study, since the audit may have been assessing trainees' current knowledge of mathematics more directly related to primary school mathematics.

Thus 'connectedness' may have been elicited better by the audit than the school examinations taken some years earlier.

In previous work, the *SKIMA* group had not investigated the trainees' own response to the auditing processes. Anxieties seemed to be allayed by peer teaching and interviews with tutors, where talking through errors and misconceptions obliged trainees to think more explicitly about mathematics. This linked with an alternative view of teacher knowledge (Peterson, 1988) involving knowledge of how children think, knowledge of how to enable children's thinking to grow and the teacher's own metacognition. With the common approach in all three institutions, early self-assessment was incorporated into the process in order to encourage the use of metacognitive strategies such as reflection upon and regulation of self knowledge.

METHOD

The Self Audit

Early in the course, all trainees undertook a self-audit (21 items) in their own time. They then consulted a commentary and support materials, and completed a self-report form with judgements of their responses to each item using a five-point scale:

0-I couldn't begin this question without help

1-I attempted this item, but didn't make much progress

2-I made some progress but with significant errors and omissions

3-My response was basically secure, with only minor errors and omissions

4-My response was completely secure

At the end of the form they were asked to 'add any general comments about your mathematical subject knowledge that may be of help to your tutor'. Of the 432 trainees completing the self-report, 274 (64%) added such comments.

This form gave quantitative data from 432 trainees, focused specifically upon the items of the self-audit, but also qualitative data from the 274 trainees' free response to the invitation to comment. Given that the comments were spontaneous, revealing what the trainees felt was appropriate to communicate to their tutors, a range of aspects were identified and so the number of students commenting on some specific aspects is small. This is not to say that some trainees did not have a view on these issues, rather that they did not choose to comment.

The quantitative data (n=432) was analysed statistically. The comments were analysed by two researchers reading, categorising, coding responses, checking the coding and modifying and joining categories in the process. The categories and coding system arose from the data and were not pre-determined. This analysis of trainee comments (n=274) resulted in 9 categories, each with subcategories:

A	Level of confidence	B	Assessment of knowledge
C	Self-audit process	D	The remediation process

E   Help needed and used   F   Specific areas of difficulty

G   Generic difficulties   H   Approaches to learning

K   Miscellaneous

So for instance, the coding for this comment:

> My maths knowledge is patchy at best. I don't feel very confident. I would have trouble explaining lots of these procedures to children. (Durham trainee)

was A4 (Category A, subcategory 4 'Not confident'), B7 (Category B, subcategory 7 'Knowledge patchy – no details given'), G3 (Category G, subcategory 3 'Explaining how to do some/all operations) and K1 (Category K, subcategory 1 'Comments in relation to teaching children').

The Audit

The audit consisted of 16 items on number and algebra, mathematical proof, measures, shape and space and probability and statistics, each marked on a 0-4 scale:

0 - not attempted, no progress towards a final solution

1 - insecure, partial solution, incorrect

2 - secure in parts, insecure in parts

3 - secure, small errors, explanations acceptable but not completely convincing

4 - completely secure with convincing and rigorous explanations (not necessarily using algebra)

This ordinal scale coded responses for the purpose of formative feedback, with a crucial boundary between 2 and 3, since <3 advised further study. Criteria for 0 to 4 specific to each item were mutually agreed, piloted and then refined.

FINDINGS

The discussion for this paper will concentrate upon the trainees' self-assessment of mathematical strengths and weaknesses, together with some comparisons between these and the formal audit taken some four months later. Insights into trainees attitudes, confidence, ways of working and feelings about the whole process have been reported elsewhere (Goulding, 2002). The categories with the largest number of comments were B, assessment of knowledge (223, 81%), F, specific areas of difficulty (118, 43%), D, the remediation process (111, 41%), A, level of confidence (103, 38%), and G, generic difficulties (100, 36%).

1. Areas of perceived weakness

Table 1. Details of the four items on the self audit with the lowest ratings.

Area of mathematics	Detail	Mean rating (n=432)
Algebra	Straight line graph, no context	1.7

Shape and space	Transformations, congruence, similarity	2.0
Number operations	Commutativity, asssociativity	2.0
Shape, space, measures	Constructions, Pythagoras' theorem, ratio.	2.1

For example the number question was

Q.   *To work out 57 + 154 mentally, Sarah says 'I want to know 154 + 57. Now 154 + 50 = 204 and 204 + 7 = 211'. Point out whether (and where) Sarah makes use of the commutative and associative properties of addition.*

Table 2. The most common content-specific difficulties identified in comments.

Area of mathematics	Number of comments	Percentage of trainees making comments (i.e. out of 274)
Graphs	38	14%
Shape and space	49	18%
Transformations	38	14%

Note: These areas were identified from the trainees' own words so although transformations are part of shape and space they were identified separately.

Table 3. The most common general difficulties identified in comments.

Difficulty	Number of comments	Percentage of trainees (out of 274)
Terminology	62	23%
Explaining strategy	30	11%

The vast majority of students were sanguine about their ability to revise or refresh their 'rusty' subject knowledge, although some anxieties to do with remembering and forgetting, emotions and possibly perceptual difficulties.were expressed:

> I found I had forgotten the rules for calculating the areas of shapes and will need to revise shape and space topics (particularly translations etc). In most other areas I think I am basically secure – but a little rusty. (Cambridge trainee)

> I felt fairly secure with most of number concepts and algebra. The 'reasoning and proof' fill me with dread. I don't ever remember doing these type of 'maths' before and would dread having to teach this sort of thing to children. (Durham trainee).

> I often do a sum 3-4 times over - just to check I have the right answer. This is mainly due to the fact that the numbers move. (London trainee)

There is a clear correspondence between the trainees' comments and their rating of the individual items. Surprisingly, the items which trainees identified as particularly

difficult on the self-audit did not correspond to those areas of weakness in the formal audit that we had identified in earlier *SKIMA* research (Rowland et al. 2001; Goulding and Suggate, 2002). In particular, the items on reasoning and proof were not commonly raised in the trainees' comments. The harder of these with a mean rating of 2.4 was:

Q   *In each part of this question, justify your conclusion in an appropriate way. (The parts are not connected).*

  a)Can you find a whole number that leaves a remainder 1 when you divide it by 2 and remainder 2 when you divide it by 4?

  b)Is it true that the sum of four even numbers is always divisible by four?

  c)Is it true that every even multiple of 15 ends in a zero? How can you be sure?

  d)Mary claims that all prime numbers end in 1,3,7, or 9. Gary points out that 21 is not prime, and says that Mary must be wrong. Mary disagrees. Please adjudicate!

Many trainees did not know the terms 'associativity' and 'commutativity' and this almost certainly accounted for the difficulty with the number operations question. Similarly the terminology of transformations may have accounted for difficulties with one of the Shape and Space questions, although there are also predictable conceptual difficulties to do with the axis of reflection and the centre of rotation. In the graph question the word 'gradient' may have been the problem but it also seems likely that the connection between the graph and its equation was a source of difficulty. The fourth item with a low mean rating involved constructing a triangle, using the converse of Pythagoras' theorem, and the ratios of lengths and areas in similar shapes. This complexity may obscure the sources of difficulty.

There were some differences when this is compared with the picture that emerged from the formal audit.

Table 4. Details of audit questions with the lowest mean scores.

Area of mathematics	Detail	Mean score
Shape and space	Transformations	2.3
Reasoning and proof	See example below	2.6
Measures	Area, perimeter, Pythagoras theorem	2.6
Reasoning and proof	Finding and justifying a relationship	2.8

Here the two low scoring items on reasoning and proof accorded with our previous research. The one with the lower mean score was:

Q   *120 square tiles can be made into a rectangular mosaic. The sides of each tile are 1 cm. The shape of the rectangle can vary. For examples, it might be 10*

tiles by 12 tiles. State whether each of the following three statements is true or false. Justify your claims in an appropriate way.

a) The perimeter (in cm) of every such number is an even number.

b) The perimeter (in cm) of every such rectangle is a multiple of 4.

c) No such rectangle is a square.

Trainees may not have addressed reasoning and proof adequately because they had focused upon the topics rated more difficult in the self-audit. The item on transformations similar to that on the self-audit also had a low score, even though trainees had earlier identified this and the associated terminology as difficult. This accords with research outside our group (Jones and Mooney, 2002). The fourth low scoring item involved Pythagoras' theorem, also involved in the fourth low rated item on the self-audit, although rarely mentioned in the comments. The difficulties with terminology in the number operations, identified by trainees on the self-audit, did not seem to cause problems on the formal audit, and the graph problem set in a 'real life' context was tackled more successfully. In both cases this later success may have been a feature of trainees' improved understanding of terms found difficult in the self-audit or a feature of the item itself. This is a very mixed picture. In some cases, self-assessed difficulties seem to have been resolved and in others they persisted.

2. Areas of strength

The four questions with highest ratings on the self audit involved two on number, reasoning in a money context and a pre-algebra, 'think of a number' item. On the audit the questions with the highest mean scores included three number questions and one algebra question involving the strategy for finding a general term of a sequence. The strengths in number may reflect the priority accorded to it in schools and correspondingly in course teaching, or the degree of difficulty in the items themselves. It is also noteworthy that generality and reasoning which were found difficult in other items on both audits were present in some of these secure items.

CONCLUSION

Weaknesses in reasoning and proof identified in our earlier research have been confirmed by the audit, and both audits highlight new difficulties in shape and space, some of which had not been probed previously. Are these weaknesses features of the items and associated terms or indications of shaky understanding? Some problems with terms were later resolved, and in general seemed easier to overcome. Are some of these insecurities irrelevant for prospective primary teachers? Items on reasoning and proof in the audit needed very little specific content knowledge but they did require the ability and willingness to investigate a situation, to look for general patterns, and to make and justify conjectures i.e. expertise in syntactic knowledge. This is certainly required in the primary curriculum but an identifiable group of trainees did not identify, anticipate or resolve these weaknesses. Weaknesses in the shape and space items in both audits involved Pythagoras' theorem and

transformations, both elements of substantive knowledge. Of the two, difficulties with transformations could create lasting problems for primary teachers. Here the trainees did anticipate problems after the self-audit but had not resolved them at the time of the audit.

The weaknesses identified, the fact that some were not anticipated and/or resolved has significant implications for the training of primary teachers. We are particularly concerned that the difficulties with reasoning and proof were neither anticipated or fully resolved. Although follow up work was done and students technically passed. we are still concerned about their syntactic knowledge. This requires a willingness to figure something out rather than memorising facts or techniques, an orientation we would wish to inculcate in pupils from the earliest stages of their mathematical education. Syntactic knowledge is in the National Curriculum for England and Wales in the first attainment target 'Using and Applying mathematics', but has been given a lower profile in the more recent National Numeracy Strategy which priorities the 'content' areas of the NC (Hughes, M. 1999). Weaknesses in teachers' syntactic knowledge, therefore, may not seem important because of this lack of emphasis in current primary practice. We feel that this is a blinkered view.

This process of investigating subject knowledge in order to ensure that the foundations for pedagogical content knowledge are sound has had mixed success. The audit has been useful to 'surface and challenge' (Ball, 1990) trainees' assumptions about mathematics but the groups are still debating future approaches. The degree to which understanding is probed depends on the activities used; trainees may be lulled into a false sense of security by success on relatively easy items. Trainees' comments concentrated on what they needed to do to get through the audit, and there was some complacency about weaknesses identified. Trainees may still not be convinced that non-elementary mathematics is relevant to the primary curriculum. Tutors certainly need to make these links more explicit, at the very least by using evidence of primary pupils engaging at a high level with areas of mathematics which some trainees have found problematic (Goulding, Suggate and Crann, 2000).

Post script: The new regulations (TTA, 2002a) do not specify a body of knowledge or require an audit, but one of the standards is 'a secure knowledge and understanding of the subject(s) [the trainees] are trained to teach.' The non-statutory handbook (TTA, 2002b) suggests that the source of evidence for this standard 'is most likely to be found in trainees' teaching.' (part 1, para. 2.1, p11).

REFERENCES

Askew, M., Brown, M., Rhodes, V., Johnson, D. and Wiliam, D. (1997) *Effective teachers of Numeracy.* London: King's College.

Aubrey, C. (1997) *Mathematics Teaching in the early years: an investigation into teachers' subject knowledge.* London, Washington D.C.: The Falmer Press.

Ball, D.L. (1990) The Mathematical understandings that prospective teachers bring to teacher education, *Elementary School Journal,* 90,4, pp 449-466

Barber, P., Heal, C. and Martyn, S. (2002) Primary teacher trainees' mathematics subject knowledge – progress in developing systems of support? Paper presented at the Annual Conference of the British Educational Research Association, University of Exeter.

Department for Education and Employment. (1998) *Teaching, High Status, High Standards: Circular 4/98.* London: Her Majesty's Stationery Office.

Department for Education and Employment. (1999) *The National Curriculum: Mathematics.* London: DfEE.

Goulding, M., Rowland, T., and Barber, P. (2002) Does it matter? Primary teacher trainees' subject knowledge in mathematics, *British Educational Research Journal,* 28(5) (in press).

Goulding, M. (2002) Primary teacher trainees' self-assessment of their mathematical subject knowledge. Paper presented at the Annual Conference of the British Educational Research Association, University of Exeter.

Goulding, M. and Suggate, J. (2001) Opening a can of worms: investigating primary teachers' subject knowledge in mathematics. *Mathematics Education Review,* 13, pp. 41-54.

Goulding, M., Suggate, J., and Crann. O. (2000) Developing student teachers' understanding of algebra and proof, *Mathematics Teaching.* 172, pp. 18-21.

Huckstep, P., Rowland, T., and Thwaites, A. (2002) Primary teachers' mathematics content knowledge: what does it look like in the classroom? Paper given at the Annual Conference of the British Educational Research Association, University of Exeter.

Hughes, M. (1999) The National Numeracy Strategy – are we getting it right? Psychology of Education Review, 23(2), 3-7.

Jones, K. and Mooney, C. (2002) Trainee primary teachers' knowledge of geometry for teaching. *Proceedings of the British Society for Research into Learning Mathematics.* 22 (2), pp. 95-100.b

Ma, L. (1999) Knowing and Teaching Elementary Mathematics: teachers' understanding of fundamental mathematics in China and the United States. Mahwah, NJ: Lawrence Erlbaum Associates.

Peterson, P.L. (1988), 'Teachers and students' cognitional knowledge for classroom teaching and learning,' *Educational Researcher,* 17(5), 5-14.

Rowland, T., Martyn, S., Barber, P. and Heal, C. (2000) Primary teacher trainees' mathematics subject knowledge and classroom performance, in Rowland, T. and

Morgan, C. (eds.) *Research in mathematics education Volume 2,* London: British Society for Research into Learning Mathematics.

Rowland, T., Martyn, S., Barber, P. and Heal, C. (2001) Investigating the mathematics subject matter knowledge of pre-service elementary school teachers. In M. van den Heuvel-Panhuizen (ed.) *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education, Volume 4* pp. 121-128. Utrecht, The Netherlands: Freudenthal Institute, Utrecht University.

Sanders, S and Morris, H. (2000) Exposing student teachers' content knowledge: empowerment or debilitation? *Educational Studies,* 26(4), pp. 397-408.

Shulman, L. (1986) Those who understand, knowledge growth in teaching. *Educational Researcher,* 15, 2, pp 4-14.

Shulman, L and Grossman, P. (1988) *Knowledge growth in teaching: a final report to the Spencer Foundation.* Stanford, CA: Stanford University.

Teacher Training Agency (2002a) *Qualifying to teach. Professional Standards for Qualified teacher Status and requirements for Initial Teacher Training.* London: TTA.

Teacher Training Agency (2002b) Handbook of Guidance on QTS standards and ITT requirements. London: TTA