

REFLECTION, IDENTITY AND BELIEF CHANGE IN PRIMARY MATHEMATICS

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Reflection is a much used idea in teacher professional development. Many mathematics teacher education programmes have, therefore, formally allocated time to enable teachers to reflect. However, simply allocating time is not sufficient. In order for reflection to take place, teachers need both motivation and the opportunity to distance themselves. In this paper, the concept of identity is used to explore how teacher reflection in mathematics education can be facilitated. This is discussed using a case study of one teacher's changing beliefs. It is suggested that teacher reflection can be facilitated by promoting teachers rich mathematical participation in a multiplicity of roles or identities: as teachers, learners of mathematics, curriculum makers, tutors and researchers.

Reflection is something of a ubiquitous idea within teacher education. Many commentators highlight the crucial role that reflection plays in mathematics teacher education (e.g., see Clarke, 1994). However, this apparent consensus conceals some differences in meaning. Grimmett (1988), for example, lists several different conceptions of reflection, including: "thoughtfulness about action ... to 'apply' research findings to practice, ... deliberation and choice amongst competing vision of 'good teaching', [and] reconstructing experience, the end of which is the identification of a new possibility for action" (p. 12). My focus is on significant change and, hence, I am concerned with the last and strongest of these definitions: reflection as the reconstruction of experience and knowledge.

In practical terms, reflection remains a somewhat elusive concept. It is unclear how teachers' reflection can be facilitated or encouraged. Indeed, there is considerable evidence that enabling teachers to reflect is a far from simple task. Cooney (1994), for example, argues, "no magical way exists to promote reflection" (p.16) Whilst the provision of time may be a necessary condition for reflection, it is not a sufficient one. Both Cooney (1994) and Clarke (1994) highlight the importance of teachers' own motivation to reflect. Yet, Goldsmith and Schifter (1997) argue that motivation is a neglected and poorly understood aspect of mathematics teacher education.

Given the concern with reflection *on* one's own activity, it is unsurprising that several authors use physical metaphors of distance to convey the difficulty of this process. Wood and Turner-Vorbeck (1999), for example, highlight the difficulty and complexity of "decentering." Cooney and Shealey (1997) link this physical metaphor to the motivation of a teacher to reflect:

A precondition for the act of reflection is the ability of the person to decenter and view his or her actions as a function of the context in which he or she is acting. Schön's (1983) reflective practitioner, a notion that enjoys so much credence in the

field of education, cannot exist unless the individual is willing to step out of himself or herself and view his or her actions from a relativistic perspective. (p. 100)

In this paper, I use the example of one teacher's reflections to explore these issues of motivation and distancing, locating them in terms of theories of identity.

CONTEXT AND METHODOLOGY

The research reported here is based on a four year longitudinal study into the professional change of the six teachers involved as teacher-researchers in the Primary Cognitive Acceleration in Mathematics Education (CAME) Project research team. CAME is a thinking skills programme which draws on a substantial body of work in secondary education. At the inception of the primary research, the CAME project had already developed a set of Thinking Maths lessons for secondary pupils (Adhami, Johnson, & Shayer, 1998). The CAME approach has been used in the UK and overseas (e.g. Mok & Johnson, 2000). (See Adhami, 2002, for a more detailed discussion of the project and its background.)

The fieldwork was conducted between November 1997 and July 2001. Data collection was qualitative using multiple methods, including observations of seminars, lessons and PD sessions, semi-structured interviews with individuals and groups, and structured mathematical interviews. My own role was as a participant observer. Initially, the data was analysed through open coding methods (Strauss & Corbin, 1998). As the research progressed, I developed the analysis through writing memos and vignettes, using narrative methods, drawing on both the data and the wider research literature (Kvale, 1996).

The Primary CAME project research team consisted of four researchers, four teacher-researchers and the Local Education Authority mathematics advisor. During the school year 1997/8, the research team met fortnightly to assess the feasibility of the approach and to develop Thinking Maths lessons specifically for primary children aged 9 – 11 (Years 5 and 6 in England).

During Phase 2 of the project, over the school years 1998/9 and 1999/2000, a further cohort of teachers from seven more schools joined the project to begin implementing the Thinking Maths lessons more widely. In Phase 2, the teacher-researchers continued to develop lessons, in addition to leading professional development sessions for the new cohort of teachers and acting as tutors to the new group of teachers by supporting them in teaching the lessons with the teachers' own classes.

Alexandra

Alexandra (a pseudonym) participated as a teacher-researcher throughout the four years of the research. During this period, the change in her beliefs about mathematics and mathematics education was very significant.

Firstly, Alexandra's orientation towards knowing mathematics changed from one of *external authority*, a belief in knowledge as validated by experts, to one of *author/ity*, an understanding of mathematics as negotiated and co-authored with others (Povey,

1997). Drawing on the work of Belenky et al. (1986), Povey links author/ity explicitly to the notion of authorship in mathematics:

Author/ity links back together two words that have a common root, but which have come to be read very differently from each other. An author is one who brings things into being, who is the originator of any action or state of things. Authority is linked with power and the validity of knowledge. Linked together they lead to the construction of an epistemology which recognises each of us as the originator of knowledge. (p. 332)

In contrast to the position of external authority, where authoritative sources, textbooks or expert mathematicians, for example, are relied upon and largely unquestioned, in the position of author/ity such authorities are critically evaluated.

Secondly, Alexandra developed what Askew et al. (1997) call a *connectionist orientation*, a set of beliefs about the teaching and learning of mathematics as a discipline with “a rich network of connections between different mathematical ideas” (p. 1). In a study of primary mathematics teaching, Askew et al. found teachers with a connectionist orientation to mathematics teaching and learning to be more effective teachers than those with either discovery or transmission orientations. This perspective has similarities to Ma’s (1999) *profound understanding of primary mathematics*. She extends the notion of connections by identifying four aspects of teacher knowledge: an understanding of the *connectedness* between simple and more fundamental ideas in mathematics; consideration of *multiple perspectives* and different approaches to mathematical ideas; knowledge about the *basic ideas* underlying the mathematical curriculum; and, knowledge of the entire elementary, or primary, mathematical curriculum and its *longitudinal coherence*.

I note however that these significant changes to Alexandra’s beliefs did not appear to be accompanied by equivalently substantial changes to her knowledge of specific mathematical concepts. (See Hodgen, 2003, for a discussion on this point.)

IDENTITY, DISTANCE AND MOTIVATION

Underlying this analysis is Wenger’s (1998) conception of identity as located in communities of practices and Holland et al.’s (1998) notion of identity change in terms of authorship and improvisation. Schifter (1996) conceives of teacher change in terms of teachers constructing “narratives of professional identity” that draw on of their experiences in local communities (p. 2). She stresses the plurality of teacher professional identity:

These teachers enact multiple identities: as mathematical thinkers, as managers of classroom process, as monitors of their students’ learning, as colleagues, and as members of the wider education community. “identities” in this sense – more a matter of what one does than who one thinks one is – are constructed in and realised through practices. (p. 2)

Thus, in contrast to a notion of identity as rooted in membership of a distinct community, this conception of a teacher's identity might be as a mathematical thinker, for example, which could be enacted in a variety of distinct communities, including the classroom, planning sessions with colleagues, the wider school, community, professional communities and more. This multiplicity of identity offers a potential for distancing and reflecting on aspects of one's identity. Wenger (1998) argues that this "combination of engagement and imagination" is very powerful:

Such a practice combines the ability both to engage and to distance – to identify with an enterprise as well as to view it in context, with the eyes of an outsider. Imagination enables us to adopt other perspectives across boundaries and time ... and to explore possible futures ... [and thus] trigger new interpretations. In turn, engagement provides a place for imagination to land, to be negotiated in practice and realized into identities of participation. (p. 217)

I will argue that this combination of distance and engagement afforded by the fractured nature of identity provides the *possibility* but not necessarily the motivation for reflection. Research within mathematics education that seeks to understand and theorise motivation is limited. (See Middleton & Spanias, 1999, for a review.) Often, where motivation is considered, it is treated somewhat simplistically in terms of individual factors or external rewards (e.g. Earl et al., 2000) or even more naïvely as a matter of individual choice. Middleton and Spanias (1999) argue that even relatively sophisticated theoretical studies of motivation in mathematics education tend to treat motivation as a given and unchanging individual factor and do not explore why or how individuals are motivated, how motivation changes over time, or how motivation can be integrated within social theories of learning. In contrast, I draw on the work of Holland et al. (1998) who argue that human activity is a necessary but constrained response to the social world.

ALEXANDRA'S REFLECTIONS

In the following example, I explore a series of reflections that enabled Alexandra to begin to transform her beliefs and knowledge about school mathematics. (See Hodgen, 2002, for a more extended discussion.) The starting point for these reflections was the following Whisky and Water problem which she presented to the research team:

I have two glasses. One glass contains whisky, whilst the other contains water. If you pour half of the whisky into the water, mix it up, then pour half of that quantity back into the original whisky glass, which glass now has more whisky?

In January 1998, when the Whisky & Water problem was presented to the research team, each of the academic researchers attempted to solve the problem using an algebraic solution. (For example: If the two glasses originally contain X whisky and Y water, the final mixtures in the respective glasses are $\frac{3}{4}X$ whisky + $\frac{1}{2}Y$ water, and $\frac{1}{4}X$ whisky + $\frac{1}{2}Y$ water.) On the other hand, Alexandra had previously solved the problem using diagrams. (See Figure 1 for an example of a diagrammatic solution similar to Alexandra's.)

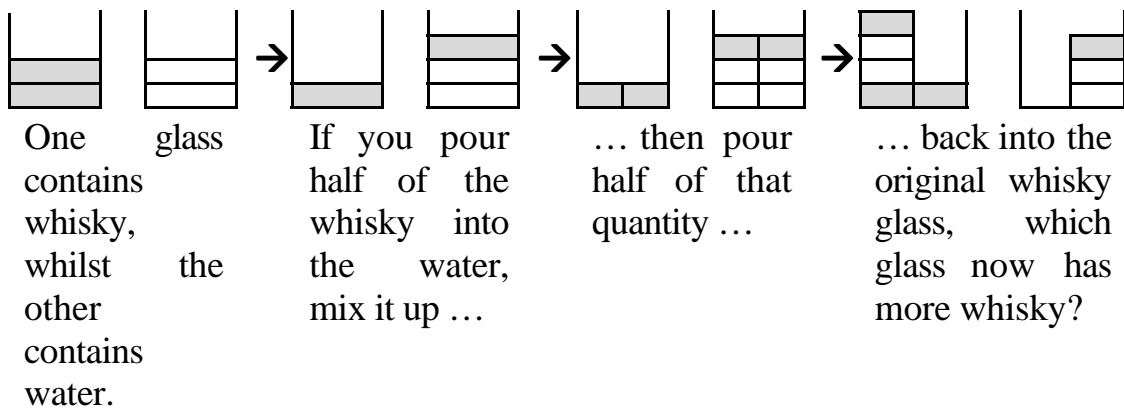


Figure 1: A diagrammatic solution of the Whisky & Water problem similar to Alexandra's approach.

Although her solution was judged by both teachers and academics as a mathematically better solution, Alexandra was insistent that her solution was “not scientific”. Her belief was that her solution, although perfectly appropriate for everyday problem solving and despite producing a convincing solution, was not truly mathematical, because it used diagrams.

In fact Alexandra's diagrammatic solution is both mathematically elegant and rigorous. In her solution she “imagined” that, although the liquids are mixed completely, she could still separate out the whisky and water in each glass in order to solve the problem. This is exactly the same reasoning step that is needed for an algebraic solution. Indeed, in many ways her diagrammatic solution mirrors the algebraic solution, using an area model to illustrate the multiplication of fractions. In this particular example, in which the original quantities of whisky and water are equal, the diagrammatic solution is a much more efficient than an algebraic solution in generating answers to related questions. For example, the ratio of whisky to water in each glass, and hence the strength of the two mixtures, can simply be visually read off the final diagram. In contrast, the algebraic solution requires further manipulation to answer this second question. (I note, however, that the algebraic solution is more general in that it covers cases where the original amounts of whisky and water are different.)

Alexandra was both pleased and excited at the positive reaction to her solution. This did not result in a fundamental shift in her mathematical thinking. Indeed, she subsequently described her solution as “just my little way of doing it”, a description which further suggests that she did not value her method as a *mathematical* solution. However, the experience did appear to provide the basis for a further reflection.

A year later, following a tutor visit to a Phase 2 school, Alexandra appeared to experience a sudden insight about the mathematical validity of diagrams. Alexandra had taught another CAME lesson, Pegboard Reflection, with the Phase 2 teacher. In this lesson, children explore number relations in the context of a reflection in the line $x = 5$, using pegboards to model the Cartesian co-ordinate system. The transformation is, then, represented algebraically. Hence, like the two fractions lessons, connections are

made between algebraic and diagrammatic representations, although the context in this case is relationships between numbers.

After the lesson, Alexandra had had a long discussion with the Phase 2 teacher in which she had to justify the context of the co-ordinate system in a CAME lesson in response to the teacher asking: “What’s difficult about co-ordinates?” A particular focus of Alexandra’s response was to emphasise “counting the zero” in identifying the co-ordinates of a point.

Later the same day, at her own school, Alexandra initiated a discussion with another teacher about number lines and their mental images of numbers with: “You know, the way I picture numbers is in steps. Steps of 1 up to 20, then steps of 10 up to 100, then steps of 100.” I then suggested that this linked to her discussion with the Phase 2 teacher about the co-ordinate system. Alexandra responded as follows:

But it’s different isn’t it. On the number line you’re counting steps, but with the co-ordinates you’re counting the zero, aren’t you. So it’s different. You’re counting steps on the number line and you’re counting points with the co-ordinates [Long pause] No, it isn’t. They’re the same thing really. I’ve just realised that. Counting the zero means you’re counting the steps. ... Co-ordinates are like a 2D number line.

This appeared to be a very intense experience for Alexandra. My fieldnotes record it as follows: “It felt like ideas slotting into place there and then ... an ‘ah-ah’ moment.”

Whilst she expressed this in a slightly clumsy way, the connection Alexandra made between number lines and the co-ordinate system is a very significant one, since, as she recognised, Cartesian co-ordinates are formed by two perpendicular number lines. Although the immediate prompt for this was my comment, Alexandra’s discussion with the Phase 2 teacher was I suggest more crucial. The Phase 2 teacher had confronted her with a problem for which she had not set response. Yet, as a CAME tutor, she expected herself to be able to respond. However, in constructing her response, she drew on earlier research seminar discussions. This link between Cartesian co-ordinates and the number line had been made very explicitly during these seminars. Yet, despite these prolonged discussion in which she took an active part, it appeared that she had not fully grasped this connection until this point.

This first reflection itself prompted Alexandra to reflect further:

Alexandra: Thinking about that it was something no-one really made clear to me at school. You know that something like quadratic equations have a spatial meaning. No-one made the connections between the spatial and the number system.

Jeremy: A bit like Whisky and Water.

Alexandra: Yes, like at school we just did fractions using fraction notation, you know using the procedure to multiply and add fractions. No-one ever made it clear that diagrams were just as mathematical.

Here Alexandra linked her earlier insight into the co-ordinate system to the grander notion of linking spatial and numerical, or algebraic, representations. Indeed, in invoking the iconic notion of quadratic equations, she makes the link to algebra very clear. School experiences of learning mathematics were very important to Alexandra. Indeed, she often referred to the absence of a connectionist approach in her own school mathematics. However, up until this point, her references to the notion of connections were largely general and unspecific. When prompted to make a connection with the Whisky and Water problem, she linked her diagrammatic solution very explicitly to the standard procedures for the multiplication of fractions. Her comment that diagrams are “just as mathematical” is very different to her earlier description of this as “just my little way of doing it.” In contrast to her earlier pleasure at her diagrammatic solution being judged acceptable by experts, here she appeared to understand the mathematical validity of diagrammatic solutions for herself.

Alexandra’s ‘new’ mathematical knowledge, in terms of specific concepts and skills, is in a sense relatively small. She had not learnt to use diagrammatic solutions, since she could do these previously. Moreover, during the development of lessons, she demonstrated on many occasions an arithmetical proficiency that would suggest she would have been able to successfully perform the algebraic solution used by the academic researchers. However, in terms of her beliefs about school mathematics, the shift in her thinking is highly significant. Without an understanding of the validity of diagrams in mathematical argument, it is difficult to see how a teacher could promote a connected understanding for children. Moreover, in terms of connections, she had begun to develop an understanding of the importance of multiple perspectives: an appreciation of the “different facets of an idea and various approaches to a solution, as well as their advantages and disadvantages” (Ma, 1999, p. 122).

What appeared to be crucial to Alexandra making the connection for herself, was the necessity to justify the challenge of the lesson in her role *as* a tutor. In her role as a tutor, she had been forced to be communicate articulately with the Phase 2 teacher requiring her to justify the importance of co-ordinates. In this case, the discussion about children’s understandings appears to have created a need to resolve issues within her own learning of mathematics. Holland et al.’s (1998) argument about the necessity of authorship is particularly appropriate in this case: “the world demands a response – authoring is not a choice” (p. 272). As a tutor, she was *able* to step outside and reflect on her identity as a learner and doer of mathematics. In addition, it seems likely that the presence of the Phase 2 teacher enabled Alexandra to vividly remember and thus engage with her earlier experiences in the research seminars. Hence, as in the reflections discussed earlier, the distancing from herself as a teacher afforded by her identity as a tutor, was grounded by a concrete reminder of her previous engagement.

DISCUSSION

Here, I have recast motivation in terms of circumstantial necessity. Alexandra did not choose to reflect in isolation: rather she reflected because circumstances required her to. Yet, as Holland et al. (1998) argue, this imperative nevertheless leaves space for

authorship and improvisation. So for example, it was Alexandra herself who raised her own mental images of number.

Alexandra was able to reflect on her mathematical thinking through the distance provided by her identity as a tutor. I describe in some depth elsewhere (Hodgen, Forthcoming) how teachers' identities as tutors, lesson-developers and researchers can step outside their immediate identity as a teacher or as a learner of mathematics, for example, and thus, 'decentre' and distance themselves.

In my study, this distancing was often accompanied by vivid reminders of the teachers' own early engagement and attempts to make sense. These reminders took the form of written notes, lesson materials or, as in Alexandra's case described here, the presence of another teacher. Alongside the distance afforded by their different identities, these reminders enabled the teachers to distance themselves in time from their previous selves. This combination of distance and proximity enabled the teachers to "imagine" different practices, whilst at the same time "anchoring" these "imagined futures" in terms of their past experiences (Wenger, 1998).

I note, however, that Alexandra's professional development opportunities within Primary CAME were very different from that available to most primary teachers. To replicate such intense experiences for the majority of primary teachers would be an extremely difficult and costly task. The problem, then, is how to offer less intensive experiences, which nevertheless provide imperatives, rather than simply an opportunities, to reflect. My analysis would suggest that a model of teacher education in which teachers not only engage critically with the mathematics curriculum as teachers and as learners of mathematics, but also places them in situations where, as teacher tutors and curriculum makers, they encourage other teachers to engage critically in similar ways. Such a model of teacher education is certainly qualitatively different to that currently available to the majority of primary teachers. It is not of necessity an experience as intensive as the one described here.

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