TEACHERS' INTERVENTIONS IN STUDENTS' MATHEMATICAL WORK: A CLASSIFICATION

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ABSTRACT. *Teachers' interventions during pupils' engagement with a mathematical task in the classroom affect considerably the mathematical meaning constructed by the latter. In the present study, a categorization of these interventions is attempted and then used to analyze teaching episodes. The results of this analysis indicate that the dominant interventions are of a very directive character and often initiated by the teacher, thus canceling students' initiatives.*

INTRODUCTION

The function of the mathematics classroom is a complex phenomenon, involving at least three fundamental components, that is, the mathematical content, the students and the teacher, but also their interaction. This phenomenon, seen from different perspectives, leads to different interpretations.

The great interest showed in the last few years in teachers' education engendered a considerable number of studies concentrating in particular on one of these perspectives, namely the teaching practices used within the classroom and their implications for students' mathematical learning. However, these phenomena are still far from having been described and analyzed in any complete manner.

Many of these studies present and analyze teaching episodes, focusing on the instructional practices used and especially on the way teachers intervene in order to support or guide pupils' work and generally interact with them. This analysis has identified a number of facts, which appear to be very common in the mathematics classroom, thus appointing some elements of great importance for the understanding of the complex phenomenon of teaching mathematics and for teachers' education.

THEORETICAL CONSIDERATIONS

Two main groups of studies related to teachers' interventions could be distinguished in the available literature. The first group includes studies that look at the consequences of these interventions for the mathematical content and meaning, whereas in the second group belong studies concerned with the consequences of these interventions for the way pupils think and act. For example, with respect to the first group of studies, Steinbring (2001) examined the epistemological nature of the mathematical ideas elaborated in the classroom. In an attempt to clarify whether these ideas are of a general character or concretized to a specific situation, issue or representation, he looked at the meaning of the questions posed by the teacher in specific teaching situations. In a particular episode, the researcher shows how the teacher's demand for a more specific description led the student, who initially referred to a general mathematical idea, to the confinement to specific issues (special situation). This restricted the task to a special case and reduced the mathematical meaning. Specifically, the students of a 4th grade class were given two sequences of four numbers: 65, 35, 55,45 and 35, 65, 55,45. For each of them, they were asked to produce a new sequence by adding successively the numbers of the initial sequence in pairs and carry on like this until they will arrive at a final sum. When they had finished, the teacher asked the class to explain why the final sums of the two sequences were different (380 and 440 respectively). In attempting to do so, Timo argued that this was because of the different position of 65 in the initial sequences of numbers, an explanation that contains aspects of a general interpretation. At this point, the teacher asked Timo "to show this possibility for '65'as a number in the middle in an appropriate example …Here, the rather 'general' interpretation 'if the 65 stands here' (in the second position) is concretized. Timo now only names the readable addition tasks". It is apparent that these types of interventions have certain consequences for the mathematical generated, the pupils' attitude towards mathematics and their knowledge about the mathematical knowledge.

In a similar approach, we examined in previous studies the way in which Greek teachers manage the organization of the mathematical content within the classroom (Ikonomou et als. 1999, Kaldrimidou et als.2000, Sakonidis et als 2001). The main conclusion we came up with was that this management leads very frequently to: a) the homogeneity of the different elements of mathematics within the classroom (i.e., definitions, theorems, properties and problems), thus not allowing pupils to focus on and differentiate them and b) a distortion of the particular features of the different mathematical fields (e.g., algebra and geometry).

Other relevant research studies have identified similar elements of the mathematical knowledge generated in the classroom. Thus, for example, Sensevy (2002) reports that teachers intervening by offering explanations are moved away from the targeted mathematical knowledge, replacing it with processes–techniques. Salin (2002) focuses on the "pratiques ostensives" (presentation/exposition practices) of the teachers, while Margolinas (1999) points out to the many cases, where "ostensive" processes of "local" character appear during the teacher-students interaction.

With respect to the studies that focus on how teachers intervene in pupils' work, a number of features of these interventions have been identified. Sensevy (2002) classifies the types of questions raised by the teacher in the course of solving a problem: immediate, step-by-step, allowing for the elaboration of the pupils' answers and encouraging students to formulate the important questions themselves. Tzekaki et als (2001) focus on teachers' management of the pupils' errors and note that the immediate correction or the adjusting intervention, even before the error is made and in expecting it, constitutes a common practice in the mathematics classroom and lead to closely directive interventions. Comiti et als. (1995) track down more general teaching practices such as addressing the competent students, in order for the teacher to secure and support the development of the lesson according to the initial planning and without having to deal with the difficulties met by the rest of the students.

The above two categories of studies are certainly not totally independent from one another, since the two issues on which they focus are interrelated: the type of action determines the quality of the content of the generated knowledge and the content or the way it is organized and presented determines the kind of pupils' action that is feasible. For example, in Steinbring (2001), despite the fact that the focus of the researcher is on the mathematical content and the teacher's questions are also concentrated on it, the latter resulted in the modification of the student's mental action. That is, the student, while using the same words, he signified different meaning, e.g., in the episode quoted previously, the student referred originally to 65 as the number determining the one to be placed next to it, whereas later to 65 as a number, making no difference to the choice made next. Furthermore, as pointed out by Tzekaki et als (2001), although the teacher's interventions determine the action taken by the pupils (e.g., no application of control procedures), their outcome concerned the content of the mathematical knowledge finally shaped.

Concluding, it could be argued that all these types of interventions function as external indicators often misinterpreted by the pupils, who adapt them to their existing system of knowledge (Brousseau, 1997), to their old, familiar knowledge (Steinbring, 2001), this requiring less effort. As a consequence, the mathematical content of the task is simplified and possibly distorted (Diezman et als, 2001), its cognitive value is reduced (Henningsen & Stein, 1997) and the students' knowledge of the nature of the mathematical ideas and the way they are constructed is falsified (Steinbring, 2001).

The above suggest that it is very important for the mathematical knowledge elaborated within the classroom and for the teachers' teaching practices to undertake to systematically identify "types" of critical teaching phases and their management by the teachers. That is, to examine teachers' behavior(s) and attitude(s) towards the variety of the pupils' behaviors and attitudes, when dealing with mathematical tasks. This will offer an essential insight into the function of the mathematics classroom and will make it possible to consider valuable approaches to the teachers' education.

In teaching approaches that rely in particular on the constructivist theoretical framework (where students are seen as participating actively in the elaboration and the construction of the mathematical meaning), the way in which certain situations and problems giving rise to specific mathematical ideas are perceived by the pupils and are handled by the teachers is of great importance. This is because these approaches are based to a great extent on the degree to which the specific tasks can guide pupils through search and speculation to the formulation of the new or the extension of the existed knowledge. Every intervention that modifies this search changes the character of the pupils' function and consequently reduces the targeted learning outcome; it finally changes the form of the adopted teaching practice.

The avoidance of this type of intervention appears to be a very difficult task for the teachers. An example of a teacher who suddenly became aware of her 'improper' intervention is cited by Gonffrey et als (1999, p.166) "So I erased their lines and said, 'Look, this figure is too complicated…'. It was so funny. They didn't ask anything and I went there and erased their work… So, clearly, I was meddling in matters that didn't concern me". Some researchers attribute this difficulty to the way teachers perceive their teaching role within the mathematics classroom (Arsac et al., 1992, Jaworski, 1994, Sakonidis et al, 2001), whereas others relate it to what Brousseau (1977) calls "devolution" of the task by the teacher to the students: "devolution is the act via which the teacher makes the student accept the responsibility of a learning situation (a-didactique) or of a problem and accepts him/herself the consequences of this transfer". This is a very essential aspect of the formation and management of the teaching environment in the sense that the kinds of interventions adopted shape and often distort this environment, allowing for the learning to take place either as an adaptation or as an active construction.

On the basis of the theoretical concepts and the results of the relevant research literature reported above, an attempt is made in the study presented below to classify teachers' intervention practices in the development of a task, that is, during the pupils engagement with it and particularly when a difficulty emerges or the course of development does not follow the path intended by the teacher.

THE STUDY

As already mentioned, the study looks at teachers' interventions in the development of an activity or problem within the classroom. More specifically, it focuses on teaching instances, where a task has been suggested to the students and the teacher intervenes in the development of the students' work and actions in the context of this task.

For the analysis of the data, a classification of the interventions was pursed based on evidence offered by both the actual data and the relevant literature. That is, the adopted typology of interventions includes categories either identified in the data or suggested by the relevant literature. The basic criterion used in order to formulate these categories was the 'degree of freedom' provided by each of the interventions under them to the students. It is important to note that in general, the type of intervention made by the teacher is related to the students' attitude and actions with respect to the suggested situation or problem. However, the examination of the teaching episodes indicated, as it will be explained later, that the teacher often intervenes independently of the pupils' action, in moments which him/herself considers in need of an intervention.

The above approach gave rise to the following three distinctive classes of intervention (it should be clarified that all three categories refer exclusively to situations where the solution of a problem or the processing of a task has been assigned to the pupils):

Category 1: **Re-setting the problem**. This category concerns those cases where, following the presentation of the problem and the initial dealing of the pupils with it, the teacher realizes that they interpret its content or the required outcome differently and in reality they deal with another problem. The possible and effective reactions of the teacher, if s/he does not wish to simplify or distort the problem, are:

- Re-setting the problem, anticipating elements for the checking of the outcomes (as described in the problem of the puzzle's enlargement of Brousseau, 1977, p.177);
- Re-setting the problem in another framework, which appoints the required outcome;
- Re-setting the problem providing some guidance (which, however, does not answer or distorts the problem).

Category 2: **Providing clues and help for the solution**. This category refers to cases where, given the students' difficulty to deal with the problem, the teacher simplifies or changes it, providing some help, but leaving enough space for their contributions. The kinds of help given by the teacher can be summarized as follows:

- Concretization-Specialization (restriction of generality, reference to examples, as described by Steinbring, 2001);
- Focus on the technique, process, algorithm, representation (as reported by Kaldrimidou et als, 2000);
- Simplification with provision of helping clues (as used by Diezaman, 2001).

Category 3: **Imposition of the solution**. This category includes cases where, finding out that the students face difficulty in dealing with the problem, the teacher offers him/herself the required solution. The cases classified under this category are:

- Simplification by breaking down the problem to sub-problems and referring to already existed knowledge;
- Coherent guidance (step-by-step) via questioning or by providing clear guidelines (as reported by Sensevy, 2002);
- Demonstration of the problem's solution (as described by Salin, 2002).

The data of our study came from a large project focusing on the mathematics teaching in the nine years of the Greek compulsory educational system (6 – 15 year olds) and aiming to investigate the possibility of applying alternative, pupil-centered mathematics teaching approaches in the Greek school. The data collected consisted of 48 mathematics lessons (28 primary and 20 secondary) given by 23 teachers (11 primary and 12 secondary), observed in various classes of the last two grades of primary school and of all three grades of high school for over a month in the northern part of Greece.

The analysis attempted to check the validity of the interventions' typology adopted, given our view that the systematization and categorization of the types of teachers' interventions in the students' activity and their consequences for the features of the emerging teaching environment constitute a very significant research issue. Thus, examining the recorded lessons, we tried to confirm the existence of the above categories, test the functionality of the specific categorization, make an effort to pin-point appropriate examples and to identify possible dominant tendencies or rare practices.

ANALYSIS OF DATA

In the following, certain instances of teaching episodes are presented for each category of intervention, which confirm its existence.

Category 1: Re-setting the problem

No episodes of intervention or behavior were found in the data that could be classified under this category. It was often noticed that students modified the given problem or dealt with a problem different from the one set, but no occasion of a teacher who tried to re-set the problem without simplifying or distorting its meaning was tracked down. This was to a certain extent expected as, according to the literature, the devolution of the responsibility of a learning situation from the teacher to the student constitutes one of the most crucial (and therefore rare) moments of the teaching process.

Category 2: Providing clues and help for the solution

2.1. *Concretization-Specialization* (restriction of generality, reference to examples)

Example 2.1 (class: 3rd high school grade, topic: Solution of linear equations systems)

A problem leading to the solution of a system of two linear equations with two unknowns has been suggested to the class. The students' interest is not concentrated on finding ways of dealing with the system of equations, but on the actual solution of the given problem. In trying to solve the problem, the students play around with specific equations and relationships: "… If we add 60 in the second equation, it would look like the first…"

The teacher deals with the situation in a similar manner, that is, he is confined to the specific and special solutions provided by the students and are related to the particular system of linear equations given. The targeted mathematical knowledge is never given rise to and the solution of a system of linear equations is never generalized; it is only the process of solving such a system that is explained. Thus, when moving to the next, similar problem, the students are not capable of generalizing and fail.

2.2. Focus on the technique, process, algorithm, representation

Example 2.2a (3rd high school grade class, Distance of a point from a line)

The lesson is about defining the height of a triangle as the distance of one of its vertices from the opposite side. The students are trying to identify the heights of the triangle by drawing them and as expected, they encounter difficulties with the heights of non-vertical direction. The teacher provides the definition and as this is not enough for the difficulty to be dealt with, she refers to a process that overcomes the construction difficulty, but does not allow for the generalization of the concept of height.

T(eacher): Originally, we give the definition, what do we call height of a triangle. The height of a triangle is the distance of a vertex from the opposite side (she repeats).… [A little later]

T: Well, we will try it in a practical manner. Take the protractor. The one side will pass through the point, and the other… What the other will do?

Example 2.2.b (3rd high school grade, Factorization)

The teacher works on various cases of factorization with her students. After having elaborated a number of such cases, she confines to the rule, using a language that focuses on the representational features of the algebraic terms of the expressions.

***T**: There are about ten cases all together. We will name them one by one, providing practical rules. If they give us this or that, what do we do, etc. ……… [A little later]*

***T**: First of all, one case that should come to our mind as soon as we are given an algebraic expression is the case of "common factor(s)". Secondly, we noticed that, when powers of the <u>same letter</u> appear in all the terms of the polynomial, the common factor taken out is the power of the letter with the smallest exponent. In the case of grouping, when not all the terms include a common factor, the common factors of each group of terms are taken out of the bracket and what is left inside the bracket is the <u>same</u> for all groups.*

2.3. Simplification with indication

Example 2.3 (3rd high school grade, Trigonometric circle)

Through a series of activities concerning the calculation of the trigonometric numbers in triangles, the students are challenged to generalize these numbers to angles of bigger size and come up with the idea of the trigonometric circle of radius 1. The question to which they are invited to answer is about the most suitable value of the hypotenuse that would made the trigonometric numbers easy to calculate in the system of the orthogonal axes. The teacher doesn't exactly correct the students, but he does not encourage the justification of their answers either. He guides the class through indirect questions; the pursued speculation is reduced to the tracking down of the expected answer.

T. *What should I choose in the place of (the side) OA?*

S(tudent). *10*

T. *One found 10. You;*

S. *The same*

T.*10 again. Anyone else? Which is the most appropriate number that will help us to make fast divisions?*

The students carry on suggesting all of them 10, but the teacher is not pleased…

T. *You say 10 as well and you? ...Maybe this is also suitable, I do not know, but I am saying... Why should this (value) be the appropriate one?*

S. *It Is easily divide by 10.*

T. *It is easily divided by 10, eh? .....* [a little later]

T. *Is there a number that divides all the others even more easily?*

S. *1!*

T. *1, eh? (closes the matter). Then, why shouldn't we divide by 1?*

Category 3: Imposition of the solution

3.1. *Simplification by breaking down to sub-problems*

Example 3.1 (6^{th} primary school grade, Finding the area)

The students have constructed cubes made of paper and they are invited to calculate the area of its surface. They try, search, and measure. The teacher worries that they are having difficulties and breaks down the task to sub-problems.

T. *Shall I help you a little, because I can see you are finding it difficult?*

S. ....

T. *How many faces does a cube have?*

S.(some students). *Six!*

T. *What is the shape of each of the cube's faces?*

3.2. Coherent guidance (step-by-step) via questioning /providing guidelines

Example 3.2 (6^{th} primary school grade, Calculation of the angles of a triangle)

Pupils are invited to measure the angles of a right angled and isosceles triangle, where none of the angles but the right angle is known. The teacher extracts the pursued answer.

T. *Be careful children, there are two characteristics. First of all, what type of triangle with respect to its angles is it Niko?*

S. *Right-angled.*

T. *Right-angled. Now, be careful. With respect to the sides, is it something else? ...Tania?*

S. *Isosceles.*

T. *Isosceles, well done Tania! So, it is right-angled and isosceles, children. And we know the one of the angles, the right angle. Michali?*

S. *b and c angles ...*

T. *Yes...*

S. *It is 45^0 each.*

T. *But why?*

S. *Ehhh…because…*

T. *The triangle is …*

S. *The triangle is right-angled and isosceles.*

3.3. *Demonstration of the solution of the problem*

Example 3.3 (2nd high school grade, Calculation of the surface area of solids)

The students have constructed a number of solids and the suggested activity requires them to calculate the surface area of these solids. Before they even start thinking, the teacher provides part of the answer.

T. *In this activity, you will consider the prism you have constructed and calculate the area of its total surface, noticing though that when you make it stand right up, it stands on its two bases. Now, since most of you have constructed a triangular prism, in order to calculate the area of each of the two bases, you will need the formula for the area of a triangle. So, what is the area of a triangle equal to?*

S. *Base times height divided by 2.*

T. *So, take the ruler and measure on the two bases of the triangular prism: base times height. In the box provided, write the total area you will come up with. All right? …* [The next activity is about the area of a cylinder].

T. *Well, the next activity is about the same thing…we have two bases. We start from there, is the basic thing. Thus, in the fourth activity, which says «the area of the total surface of the cylinder», we must do the same work. That is, to break down the area to the area of ….*

S. *The two bases.*

T. *And the area ...*

S. *Of the surrounding surface.*

T. *This is exactly the case! This is what we are going to do. Whether you need to destroy the cylinder by opening it, in order to see the surrounding area, is your own business. I mean that you can do it. Did you understand what did I say? Because I do not know whether you have done this in the primary school. Did you open it?* […and provides the solution to the problem].

CONCLUDING REMARKS

The first of the intervention categories identified appears to concern very rare behaviors, a finding which is in agreement with the evidence offered by the relevant literature, as this behavior is seen as the exemplar, the "ideal" behavior. The other two categories emerged in a number of occasions, with the frequency of their sub-categories presenting a great variety. The dominant types of interventions from the last two categories of interventions were:

- focus on techniques, processes and representations;
- step-by step guidance;
- demonstration of the problem's solution.

It is important to note that in general, the type of intervention made by the teacher is related to the students' attitude and actions with respect to the suggested situation or problem. However, the examination of the teaching episodes showed that the teacher often intervened for no apparent reason related to the pupils having a difficulty or being stuck, that is, independently of their action, in moments which him/herself considers in need of an intervention (e.g., examples 2.2b, 3.2). This could be attributed to:

- the role teachers attach to themselves, when teaching mathematics. This role seems to provoke concerns of securing the "unmistaken" approach of the pupils to the mathematical knowledge and at the same time underlines teachers' conception of this knowledge as being non negotiable;
- teachers' conceptions about how children learn mathematics, that is, that they cannot discover the mathematical knowledge themselves, but they have to be directed and shown by someone, a position that loads them with anxiety and leads them in adopting an authoritarian attitude to functioning within the classroom;
- the interaction of the above two.

The preceding analysis is far away from providing a complete picture to the question concerning teachers' interventions and their teaching behaviors in general. However, it constitutes a first step in the direction of looking for systematization and categorization of them, a step of crucial importance for the study of the teaching of mathematics and the management of the mathematics classroom.

There are many questions arising directly from such an attempt, which are significant for the completion of the relevant research agenda, but which demand long term research projects: does the teacher exchange his/her intervention practices? If yes, how, when, with what criteria and under which conditions? Can we have a more quantitative analysis of data of the kind reported in this paper or not? The answers to these questions could be also useful for teachers' education, as they would provide important tools for training teachers in identifying and understanding the various types of teaching interventions as well as their consequences for the pupils' learning of mathematics.

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