

FUTURE MIDDLE SCHOOL TEACHERS' BELIEFS ABOUT ALGEBRA: INCIDENCE OF THEIR STUDIES

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We report about the beliefs on algebra of future middle school teachers expressed at the beginning of a training course on approach to this discipline as a language for producing thought. We analyse qualitatively the collected data and we show some interesting differences among graduates in “mathematics“, “physics“ and “other science“. The analysis also highlights a deep interlacement between affective factors and image of algebra. Finally we give some hints on the attitudes induced in the trainees starting from the discussion of the analysis made.

BACKGROUND, HYPOTESIS AND AIM OF THE STUDY

In the last decade, many studies have been centred on the teacher as the main ‘variable’ of the teaching/learning process (Krainer & al. 1998), with specific attention to different aspects: disciplinary knowledge (Fennema & Fraenke 1992), problem solving (Roddick & al. 2000), the ability to understand students’ mathematical thinking and performances (Even & Tirosh 2002) and also, more in general, teachers’ beliefs, attitudes, emotions and values (MacLeod 1992; Thompson 1992, Vinner 1997). Other studies have underlined the need and the importance of the teacher’ awareness on these aspects (Mason 1998, Jaworski 1998, Malara & Zan 2002). This paper belongs to this frame; it rises in the context of the Italian studies where - for the recent opening of university courses for primary teachers and of post-graduate training schools for teaching in secondary schools – much attention is now devoted to teachers. The paper concerns the beliefs about Algebra in future middle school teachers (pupils’ age: 11-14), which were collected at the beginning of an Algebra training course. In the paper we focus on the differences emerged as to the cultural background of the trainees¹, we reflect on distorted beliefs and we sketch out our choices to correct them.

Algebra is a complex and mani-sided discipline and many are the studies on the problems of its teaching and learning (see for instance Chick & al. 2001), also because school approach to algebra is usually centred mainly on the syntactical study of algebraic objects, instead of the construction of algebraic language for promoting modelization, solving problems and proofs (Arzarello & al. 1993, Malara & Iaderosa 1999, Radford 2000, Menzel 2001). Therefore it is important that teachers become aware of the complexity of Algebra and know the main steps of its historical evolution.

¹ In Italian middle schools the teaching of mathematics and science is joint, so teachers may have a degree in any scientific discipline (mathematics, physics, chemistry, natural science, biology, geology ect).

Our paper is based on the hypothesis that only by bringing future teachers to express their ideas about algebra it is possible to lay bare ground their possible conceptual rigidities, misconceptions, cultural lacks, difficulties. This way they can: a) become aware of their gaps through open discussions; b) reconstruct their knowledge and beliefs through opportune shared experiences; c) promote in the teaching of Algebra a meaningful constructive learning leading the students to understand the reasons of its theoretical study.

The inquiry reported in this paper was carried out in January 2002 through a test made of five free-answer questions (answer time: 4 hours). This test was given to 47 trainees (9 graduated in mathematics, 9 in physics and 29 in various sciences).

THE FIVE QUESTIONS AND THEIR ANSWERS

The test contained the following five questions: 1. What does the word ‘algebra’ evoke to you? 2. Why was Algebra born?, 3. Why must Algebra be taught?, 4. Which are the main difficulties of Algebra?, 5. Which is the best age to start the approach to Algebra? For each question of the test we have hypothesised a set of possible answers on which we have organised the quantitative data (see table 1).

Even if these data can give for each question a first idea of the different beliefs of the trainees, however they cannot say anything about the trainees’ feelings underlying the answers. So we analyse the data qualitatively, reporting also some excerpts of the trainees’ protocols².

The first question

The answers reveal a fragmented and technical vision of Algebra to be strongly dominant. Algebra is often identified with literal transformations, the study of equations, or worse with relative integers. Many graduates in science

Table 1: The quantitative data

Future teachers’ degree	Maths n = 9.	Physics n = 9	Science n = 29	Total n = 47	%
<i>Idea of algebra</i>					
Study of expressions and equations	6	4	18	28	60
Language for codifying relationships	1	3		4	8
Theoretical contents useful for applications	1	2	5	8	17
Algebraic structures	1				2
Other			6	6	13

² To make reading easier, we quote each excerpt by inserting after the author’s initials the letter m, ph o s in brackets (with the meaning respectively of: graduates in mathematics, physics, various sciences) to evidenciate his/her background. In each excerpt we substitute the word “Algebra” with “A.”.

<i>Reasons for the birth of algebra</i>					
to solve practical or economical questions	1	2	2	5	10.5
to mathematize and to study complex situations	1	1	3	5	10.5
to generalise – to abstract - to systematise	1		5	6	13
to support the study of other disciplines	1	1	2	4	8
I do not know*	3	3	15	21	45
Other	2	2	2	6	13
<i>Reasons for studying algebra</i>					
to acquire technical abilities		1	4	5	10.5
to acquire a language useful for science	1	1	4	6	13
to acquire useful theoretical knowledge	1	2	3	6	13
to matematise – to apply algebra to other sciences	1	3	8	12	25.5
to generalize – to abstract – to produce thought	4		5	9	19
other	2	2	5	9	19
<i>Difficulties in Algebra</i>					
Technical – calculating difficulties	3	3	6	12	25.5
difficulties of translations into formula			3	3	6.4
logic – interpretative difficulties	4	1	4	9	19.2
generalisation – abstraction difficulties	2	3	7	12	25.5
other		2	9	11	23.4
<i>'Best' age to approach Algebra</i>					
9-11	5	2	6	13	27.5
12-14		4	12	16	34
=15	4	1	7	12	25.5
I do not know*		2	4	6	13

* This item has added owing to the high number of answers received.

express this idea, but, paradoxically, also some graduates in mathematics

TB(s) writes: *A. as art of doing calculations, solving equations in an appropriate and speed way according to the rules of this discipline.*

FZ(m) simply writes: *Algebra = calculations, formulas, expressions.*

Among these, however, a conception of the discipline aimed at order and organisation prevails

CF(m) writes: *the term 'A.' makes me think of all that is number and calculation. I conceive A. as a well ordered, formalised discipline with a high grade of abstraction, which allows to solve problems and can be applied to concrete situations.*

The graduates in physics and science show a different idea of Algebra, focused on relational vision, and consider it as language for mathematisation

BB(ph) writes: *$a+3b=7$, well, A. evokes to me a relationship between mathematical entities through opportune symbols.*

CB(ph) writes: *The solution of a problem of physics, chemistry or geometry presupposes the knowledge and the mastery of A., it is the language of scientific disciplines.*

CC(s) writes: *Imagine A. as 'formalisation' or modelisation of real situations, as a representation.*

There are very partial visions, for instance several graduates in science identify Algebra with the study of relative integers or numerical ambits, for instance

BP(s) writes: *A. is the part of mathematics which concerns relative numbers.*

Beyond the partial or distorted visions, we also found out an idea of Algebra as a set of theoretical facts which produces useful techniques for applications. We also detected a static vision of Algebra, heir of the nineteenth-century tradition, therefore quite far from the new trends which privilege a linguistic, in progress approach of Algebra according to the process-object dialectics.

Relationship between affective aspects and image of algebra

The most interesting thing emerging from the protocols is the affective dimension related to the first approach to Algebra. Affective factors appear to be intertwined with the image of the discipline. For all the trainees except one (who conceives algebra as the study of algebraic structures), the discipline is associated to the first years of secondary school; they evoke the classroom atmospheres or the teacher

EB(ph) writes: *the memories connected with A. go back to the first years of upper secondary school and middle school. The thoughts coming across my mind are pages and pages of equations (in the worst case also LITERAL ONES), decompositions and fundamental theorems.*

Methodicalness, order, and precision are a constant reference. These ideas are associated with feelings of pleasure or frustration:

AM(ph) writes: *Thinking about A. I recall the pleasure of listening to my teacher and the pleasure of starting to solve expressions.*

GM (s) writes: *“the word A. evokes me the feeling of frustration arising when the calculation did not succeed.*

Some trainees state they have lived Algebra as a personal challenge, for instance

EC(s) writes: *I saw myself in front of A. as in front a ‘riddle’, a rebus of a puzzle-weekly, the solution of which gives me an unbelievable satisfaction.*

Graduates in mathematics or physics see Algebra as a sure and reliable discipline,

BB(ph) writes: *Emotionally I have to do with something exact which leads to an absolute truth in the field, then I can say that A. leads myself to a security and certainty: given things cannot be otherwise.*

Graduates in science see Algebra as an intricate and complex discipline, but also hermetic and full of fascination

MV(s) writes: *I associate the word A. to the image of a ‘BRAIN’: because A. gives me the idea of calculation, of reasoning and of elaboration.*

GM(s) writes: *The word A seems to me charming, perhaps because I do not know its meaning.*

The reading of the excerpts shows how the trainees’ feelings are conditioned by the approach of their secondary school teacher. Hence the different images of Algebra: as an abstract, difficult and less meaningful discipline vs. as an interesting discipline, also in its theoretical part. It is therefore necessary that future teachers become aware of their responsibility.

The second question

As to this question more than half of the trainees having a science degree states they do not know how to answer. Quite frequently the answer was: *To tell the truth, I do not know how A. was born* - unfortunately also among graduates in mathematics. The awareness of their cultural gap is testified by the fact that they refer to the responsibility of their teachers, FM(m) writes:

I do not know how A. was born because none of my teachers has explained it to me and moreover because I have never taught it, so I have not asked myself this question.

However, there is someone who has a clear idea of its historical evolution. As to the reasons of the birth of Algebra, a few of the graduates in mathematics or physics vaguely refer to practical needs (for instance trading) which involve complex calculations. Someone states that Algebra was born for representing geometry, others for representing reality. Among the graduates in science there is a distorted vision of the evolution of Algebra which reflects the inclusion of the various set numbers (Naturals, Relative Integers, Rationals etc). Several have a static vision of the discipline. Most trainees change ‘why’ with ‘when’ in the question, they (sometimes wrongly) declare the time of the birth of the discipline and refer to the cultural heritage by the Arabic. Each answer reflects the idea of Algebra of its author. Sometimes they are

appropriate and composite, but more often they are partial and confused, especially among the graduates in science; however we have to underline the freshness of their images and the expressive effort even in the naivety of the results. In some trainees there is the idea of algebra connected with the arrangement and rationalisation of facts accumulated in time.

The different texts (fragments of conceptions, intuitions, hypothesis, mixed with naivety and imprecision) create a multifaceted but enough pertinent frame of the reasons of the birth of Algebra and constitutes a good ground for the trainer when it comes to opening to the historic-epistemological dimension.

The third question

As to the third question only two or three trainees motivated the study of algebra as a tool to modelize and solve problems or make proofs. The main reasons given are vague: 1) to organise and synthesise one's thought (mainly among the graduated in physics); 2) to train logical reasoning (mainly among the graduated in mathematics); 3) to generate flexibility and mental openness (only among the graduated in sciences). A common idea (above all among the graduates in science) is that pupils need to study Algebra for learning techniques of calculations or theoretical facts to be applied on modelling reality. For supporting this thesis RS(ph) resorts to the analogy with music.

She writes: We cannot think of studying something without the tools; an example is given by music. Before we can play an instrument at a certain level we need to know how to read music, we have to know how to pitch the note and the arrangement of the instrument, this series of notions requires a certain time to be assimilated. So before we know how to play an instrument with a certain mastery we need to acquire the tools that allow doing it. Similarly if we wish to set out a theory, a model of equations, a theorem before we have to master the basic notions of A.

Among the trainees there is someone who, in tune with his/her idea of Algebra, states it has to be studied for improving attention, precision, reflection and order. However, in general the trainees spend few words on the question. This confirms the hypothesis of non-conceptualisation of their algebraic experiences on the side of the meanings.

The fourth question

Among the trainees there is the common idea that the difficulties in Algebra are due to the need of attention in doing calculations. Beyond this there is a big differentiation among the trainees' beliefs. Again, the ideas of mathematics or physics trainees are different from those of the graduated in science. For the graduates in mathematics the difficulties are of logical-interpretative kind, even if they also consider difficulties in calculations and of generalisation/abstraction.

GA(m) writes: we meet the main difficulties in logic, because it is difficult for the pupils to connect common language and symbolic language.

Someone mentions also the difficulties due to the mental rigidity induced by a certain teaching of mathematics. Not by chance, just the trainees graduated in science express a wider range of difficulties. Among them the prevalent idea is that the bigger difficulties are due to the inability to generalise and abstract

AB(s) writes: *I believe that the passage from the number to the letter or, in any case, from specific to generic or general is not so obvious and foregone and so it can be represent a difficulty.*

Some trainees see the biggest difficulties in managing the negative numbers. Others correlate the difficulties with the teacher's behaviour

TB(s) writes: *learning difficulties of A. depend on the way followed by the teacher in facing the topic, if in a convincing and also amusing way, putting questions which enjoy the pupils.*

Many statements refer to the difficulties due of the lack of meaning for the pupils

SA(s) writes: *The main difficulty in learning A. is to understand why one has to study it. At the beginning it is presented as something abstract, especially during middle school. Therefore one finds it difficult to learn something of which he does not find links with reality.*

This is also confirmed by this excerpt which refers to a practice of Algebra teaching as blind application of rules

AMA(m) writes: *I can say why it was so difficult for me in school to understand the rule of the square of a binomial: because I wanted only rules like the ones I had learned at compulsory school: my study was mnemonic. It was unknown for me to try to understand, to reason, to wonder why.*

This shows the relapse of their own experience on the way they conceive teaching. Among the graduates in mathematics or physics, some state that Algebra is not difficult and some of these, at the first experience of teaching, are amazed by the difficulties picked out in the pupils. Among the graduates in science there is someone who transfers his/her negative experiences to the pupils

GM(s) writes: *I believe that the pupils are terrified just by the same word, because they do not know it and they wonder with fear "what happens about us?"*

These sentences confirm the central role of the teacher, who must lead the student: a) to conceive abstraction as "multiconcreteness" by offering various experiences of generalisation, recognition and justification of regularities, observation of relationships and analogies; b) to become aware of the thinking process underlying algebraic rules and of the reasons of specific conventions; c) to control aim and meaning of the activities faced.

The fifth question

The data show that the more convenient range of age to approach algebra is between 12 and 14. But none of the graduates in mathematics states this traditional range. Almost surely this depends on the fact that those who indicate an early approach refer to ideas learned in previous university courses.

The answers of the graduates in science who support the idea of an early approach have a different origin. They are due to a very poor idea of Algebra, which is identified with relative integers, which formerly was the first chapter of Algebra books and now is a topic in the syllabus for primary school. Still, some of them conceive a meaningful early approach to minimise the difficulty of an ex abrupto approach,

SC(s) writes: *In my opinion it should be started in primary school, perhaps shaped as a “game“, but pupils must be accustomed to managing numbers and letters simultaneously, so they get the abstraction process as familiar as possible.*

These answers show that the time for a revision of the primary teaching in the sense of early algebra is ripe.

SOME GENERAL REFLECTIONS

To sum up, we can say that the inquiry highlights the trainees' fragmented and distorted vision of Algebra. Moreover, even when this vision is pertinent, it still results reductive and hardly adequate to its teaching. Paradoxically, there are more pertinent beliefs among the graduates in physics than in the graduated in mathematics. Among the graduates in science there is a widespread cultural poverty, despite the presence of pertinent visions and interesting ideas for limiting pupils' difficulties. Several trainees state the importance of giving meaning to this teaching and of conceiving an early and friendly approach which gives room to reasoning. A few are afraid they might not succeed in teaching because of its difficulties.

The test is extremely productive because of the requested metacognitive reflection. Many protocols are read and discussed in the meetings and this reading allows an open and free confrontation among the trainees. In the sharing, the single convictions are reflected and interlace each other, constituting in the mind of trainees a mosaic of many little elements of awareness, that all together magnify themselves as the fragments of glass in a kaleidoscope. Moreover, it comes to create in the class an atmosphere of full co-operation, far by the scheme of the university lessons to which the trainees are accustomed. From a disciplinary point of view, there is a widespread feeling of curiosity, interest and participation for the things that we progressively get to propose, particularly in the workshops.

We face the mending of the distorted beliefs working on many a point of view: a) *Cognitive*: by promoting the acquisition of models of behaviour in dealing with questions, of basic knowledge and technical abilities; b) *Metacognitive*: by unifying in a wider frame partial visions, constructing a net of links among the various aspects of the discipline progressively clarifying its meaning; c) *Affective*: by operating through a

dialogue which gives room even to the emotions for the *recovery of the confidence* in the possibility they can overcome their own difficulties and teach in a good way .

We cannot talk diffusely about the course here, but we can say that besides the individual effects on each trainee, the more productive result concerns the ‘classroom culture’ (Even & Tirosh, cit.) and the meaning given to Algebra as a tool for thought. A problematic issue is the feeling of insecurity many (mainly graduated in science) trainees have about their ability to enact a constructive teaching in the classroom, according to the model learned in the course, since they fear that the older models, more rooted in school, could prevail.

REFERENCES

- Arzarello F., Bazzini L., Chiappini G.: 1993 Cognitive Processes in Algebraic Thinking: Towards a Theoretical Framework, *proc. PME 17*, v.1, 138-145
- Chick, E., Stacey, K, Vincent, JI., Vincent, Jn. (eds): 2001, *Proc. 12th ICMI Study ‘The Future of the Teaching and Learning of Algebra’*, Univ. Melbourne, Australia
- Even, R., Tirosh, D.: 2002, Teacher Knowledge and Understanding of Students' Mathematical Learning, in English, L. (ed) *Handbook of International Research in Mathematics Education*, LEA, NJ, 219-240
- Fennema, E. and Franke, M. L.: 1992, Teachers' Knowledge and its Impact, in Grows, D. (ed.) *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, 147-164
- Jaworski, B.: 1998, Mathematics Teacher Research: Process, Practice and the Development of Teaching, *Journal of Mathematics Teacher Education*, n.1, 3-31
- Krainer K., Goffree F. Berger, P. (eds): 1998, On Research in Mathematics Teacher Education, *proc. CERME 1*, Univ. Osnabruck, Osnabruck
- MacLeod, D.B.: 1992, Research on Affect in Mathematics Education: A Reconceptualization, in Grows, D. A. (ed) *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, 575-596
- Malara N.A, Iaderosa, R: 1999, Theory and Practice: a case of Fruitful Relationship for the Renewal of the Teaching and Learning of Algebra, in *Proc CIEAEM 50* , 38-54
- Malara N.A., Navarra G.: 2001, “Brioshi“ and Other Mediation Tools Employed in a Teaching of Arithmetic with the Aim of Approaching Algebra as a Language, *proc. 12th ICMI Study on Algebra*, v. 2, 412-419
- Malara, N.A., Navarra, G., 2003, *ArAl Project: Arithmetic Pathways Towards Favouing Pre-algebraic Thinking*, Pitagora, Bologna
- Malara N.A, Zan, R.: 2002, The problematic Relationship between Theory and Practice, in English, L. (ed) *Handbook of International Research in Mathematics Education*, LEA, NJ, 553-580
- Mason, J.: 1998, Enabling Teachers to Be Real Teachers: Necessary Levels of Awareness and Structure of Attention. *Journal of Mathematics Teacher Education*, 1, 243-267
- Menzel, B.: 2001, Language Conceptions of Algebra are Idiosyncratic, in Chick, H. & al. (eds) 2001, *Proc. 12th ICMI Study ‘The future of the teaching and learning of Algebra’*, Univ. Melbourne, Australia, 446-453
- Radford L.: 2000, Signs and Meanings in Students' Emergent Algebraic Thinking: a Semiotic Analysis, *Educational Studies in Mathematics*, v.42, n.3, 237-268
- Roddick, C., Rossi Becker, J., Pence, B.: 2000, Capstone Courses in Problem Solving for Prospective Secondary Teachers: Effect on Beliefs and Teaching Practices, *proc. PME 24*, v. 4, 97-104

- Thompson, A.: 1992, Teachers' Beliefs and Conceptions: a Synthesis of the Research, in Grows, D. A. (ed) *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, 105-128
- Vinner, S.: 1997, From Intuition to Inhibition – Mathematics, Education and other Endangered Species, *Proc. PME 21*, v. 1, 63-80