A TEACHING RESOURCE FOR TEACHER TRAINING[1]

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The "Rally Matematico Transalpino" – RMT (the Transalpine Mathematics Rally) is a useful source of information and suggestions for teacher training, both at the initial stage and in in-service training. This study seeks to investigate the influence of the RMT on teacher-knowledge relations.

Il "Rally Matematico Transalpino" (RMT) offre numerosi spunti per la formazione degli insegnanti sia iniziale che in itinere. Nel presente lavoro si indaga sull'incidenza del RMT sul rapporto insegnante-sapere.

Introduction

In our work as university lecturers we are directly involved with the training of future mathematics teachers at two different levels: as being responsible for the Mathematics Teaching Course forming part of the Mathematics Degree Course and as lecturers in the Post Graduate Secondary Education Teacher Training College. The RMT offers a wide range of ideas for the application of the didactic theories dealt with. Furthermore, our positions as co-ordinators of the Parma Section of the RMT gives us the opportunity to assess RMT's impact on in-service training for working teachers whose classes take part in the competition.

Our eight years of experience of this competition has served to convince us that it has had positive effects on training.

Theoretical Framework

The Transalpine Mathematics Rally (RMT) is a competition between school classes which has now been running for 10 years,[2] experiencing extensive development over this time. Currently classes of children involved in their third to eighth year of schooling participate in the event drawn from six different countries (from both Europe and beyond).

[1] The contribution is executed in the sphere of Local Research Unity in Mathematics Education of University of Parma, Italy.

[2] In December 1992 Swiss Romanda magazine *Math-Ecole* proposed an initial comparative study to its readers relating to a number of mathematics problems set for children in the third, fourth and fifth year of primary school to solve as a class. Interest in the initiative spread quickly and in its third edition, in the school year 1994-1995, the Rally crossed its original borders and gained participants from Italy, starting in the provinces of Parma and Pavia, then extending to other provinces in Emilia Romagna and other Italian regions such as Lombardia, Marche, Sardegna, Toscana and Valle d'Aosta. Starting from the academic year 1996/7 the Rally also extended into France (Bourg en Bress), while participation widened to include the Canton Ticino, Luxemburg and the Czech Republic in 1997/8 and Israel in 1998/9. Four years ago it also started to involve classes from middle schools as well as primary schools.

The RMT's goals are explicit: problem solving, interactive working, responsibility of the class group, explicit statement of procedures used to resolve the problem, justification of the solutions identified.

The competition consists of two tests (one in February and the other in April) with a final (in May) involving those classes which have achieved the highest marks. The class is required to resolve the problems set in each test (normally 6 or 7) in 50 minutes, organising themselves autonomously without their teacher being present. At the teacher's discretion, the tests may be preceded by training tests from previous years.

The choice of the problems is of fundamental importance: they must not have been published previously, be relevant at a mathematical level and allow connections to be made with the mathematics programme being taught in class. They must also have the effect of motivating pupils and stimulating their involvement, correspond to the various stages of development, offer the possibility of a range of resolving strategies and opportunities for developing the children's representations (they thus fall within the definition of "rich problems" as described by Hedrén (2002). Furthermore, the RMT problems must as far as possible, be self-validating. The drawing up of such problems must thus satisfy the quality criteria currently required in mathematical learning in a socio-constructivist perspective.

The mathematical competitions between different classes have provoked increased interest both from the children called upon to resolve problems collectively and on the part of teachers and maths education researchers who can analyse the resolving strategies adopted.

The main feature of the RMT is the creation of learning situations permitting pupils to mobilise and put in play their knowledge, at the same time, if possible creating new skills (Grugnetti & Jaquet 1999, Grugnetti, Jaquet & Rinaldi 1998).

This occurs at two distinct stages. The first is during the competition itself when the problem solving is entirely devolved to the pupils who take responsibility for their own activities in the absence of the teacher, replaced by a person unknown to them (Grugnetti & Medici 1996).

At a second stage, the teachers are able to take up the problems again in class, going over the procedures adopted by the pupils in order to connect the mathematical content with the teaching objectives (Puxeddu 1999).

The preparatory tests and the class discussion are able to give pupils the certainty that the problems set have always solution using the knowledge they already have, even if there is no "standard" procedure available. This understanding then helps them to commit themselves fully during the competition and ensures that "*l'élève accepte la résponsabilité de résoudre les problèmes dont on ne lui a pas einsegné la solution." (Brousseau 1996).*

A further point of interest provided by the RMT from a teaching stand point is the comparison of solutions, the errors and cognitive conflicts revealed by the procedures used by the classes from different countries. It should further be emphasised that the problems set are translated into the different national languages, often having to overcome difficulties due to different traditions, customs and means of expression.

The Problem of Training

As is emphasised in Marchini (2002), a cultural issue relating to the problem of *teacher training* is contained in the Italian word used for this ("formazione"). The Italian word derives from the noun "forma" (form). There is thus an underlying idea that training is an activity which gives form, or models, the pre-existing subjects or contents to make them suitable for a teaching function. In this context it is not clear whether the active subject of training ("formazione") is the post-graduate student training giving form to the contents of the training, or the teacher (artist) who gives form to the post-graduate student. Perhaps the preferable interpretation would appear to consider both parties inter-acting with each other. This is supported by the fact that "formazione" in other meanings is concerned with the act of formation and "formarsi" (to be formed) refers to the way in which things are brought together and made available.

In the context of the inter-action between the teacher and the post-graduate student or, so far as in-service training is concerned, the researcher/teacher, those activities which allow immersion in a teaching environment become of fundamental importance, taking into consideration the activities described by Houdement & Kuzniak (1996) when they indicate three levels of essential knowledge and learning for teacher training in the field of mathematics. These three levels refer to different aspects of the work of a teacher, each increasingly removed from purely discipline-based knowledge, being concerned rather with more general knowledge connected to teaching as a whole (Stegen 2002).

Houdement & Kuzniak add what they describe as the "third area of learning " to those of mathematical and didactic knowledge. "To the mathematical and didactic knowledge (generally theoretical in nature) skills are added which appear better described as "savoir faire" and common sense, without neglecting a set of skills and learning which it appears a teacher can only master through experience".

In the context of teacher training it also appears to us to be of interest to take account of the notions raised by Shulman (1986):

- Knowledge of the contents of the subject, that is, the *accumulation and organisation of the relevant knowledge in the teacher's mind,*
- Knowledge of the pedagogic content *including the understanding of what makes the learning process for a particular subject easy or difficult: the conceptions and pre-conceptions of pupils of different ages.*

- Knowledge of the circular content, that is, *the teacher's ability to connect the content of a given course or a given lesson to questions and subjects dealt with simultaneously in other disciplines.*

Teacher Training by RMT

In this context we have concentrated on the range of potential offered by the Transalpine Mathematics Rally (RMT) in the field of teacher training.

A) Degree Course in Mathematics

1) A number of RMT problems represent (Grugnetti, Jaquet 1997-1998, Grugnetti, Jaquet et alii 1999-2000[RP1]) good practical examples of the socio-constructivism didactic theory and it is for this reason that they are used in the teacher training course. Future teachers have to master the ability to resolve mathematical problems as much as analysis and construction and to introduce them in such a way as to stimulate thought in others (Tsamir, 2000). As a consequence it seems of fundamental importance to us that the students get used to the use of an a priori analysis to identify the concepts at play and to establish if, in which cases and at which levels, the problems can be considered as a-didactic situations[3](either "open" problems or non-standard "reinforcing" problems …) or even as problem situations. Furthermore, the Rally problems can, by their very nature, be used in the identification of didactic variables. The possibilities of working on them are then examined in order to adapt them to the pupils under consideration in the class simulation exercises according to specific objectives.

2) The students in the teaching course become directly involved in the competition at three points of fundamental importance.

   - They are involved (along with other teacher-researchers) in the final reading of the texts of the RMT problems and the related analysis, just before the fair copy stage. This activity imparts a better understanding of the role of the teacher, getting them used to not accepting the problems to be set acritically and learning to adapt the problems themselves, with an a priori assessment of the difficulties the children will encounter both from a linguistic and mathematical point of view. « ... l'analyse a priori n'est pas dévolue aux auteurs seulement, elle concerne chaque maître qui envisage de proposer une activité. Et cette analyse a priori dèpend de facteurs liés à la classe, aux élèves, à la progression aux sein du programme... » (Jaquet, 2001).

[3] la situation d'interactions dans laquelle les élèves se trouvent, nécessite une médiation de leurs connaissances, leur mise en jeu dans le groupe afin de choisir la solution qui leur semble le mieux convenir pour obtenir le plus de points. Les rapports avec le milieu permettent aux élèves de passer d'un niveau de connaissances à l'autre. Par ailleurs les problèmes sont proposés de manière à ce que les élèves ne soient pas guidés par la lecture des intentions du professeur, mais par la logique du milieu proposé. Ces différentes particularités nous amènent à penser que les situations du rallye ont un caractère essentiellement a-didactique. Grugnetti, Jaquet & Tièche-Cristinat (2002) preprint).

- They assist in the classes participating in the Rally when involved in the problem solving activities. In this way they have the chance to observe group dynamics "in the field" and to listen to the "live" discussions, identifying the manners of reasoning and arguing typical of the age concerned.
- After each test the students actively participate in group correction, working in groups where they have direct contact with in-service teachers. This interaction provokes discussions in which all participants propound their own ideas on the a posteriori analysis of the problems: interpretation of the working papers, comparison with a priori analysis, assessment and awarding of marks.

  Such activities, taking on the character of a kind of "laboratory" have proved to be profitable both for the students and the teachers:

  - it gives the former a concrete opportunity to apply a number of aspects of mathematics teaching theory covered in the course lectures and to put themselves in the teachers' shoes,
  - it gives the teachers the possibility to interact with maths teacher training students who have been trained to conduct an a posteriori didactic analysis which ought to give rise to a more objective assessment of the children's work particularly with regard to the meaning of a response and the consistency of a line of argument which the teacher, for reasons extraneous to knowledge of the subject, tends to over-estimate, attributing a clarity of ideas to the children which in reality they have not achieved.

3) At the end of the mathematics teaching course (made up of two six-monthly modules) each student is required to produce a small thesis connecting the three levels of knowledge previously referred to, deriving from the results of practical training activities in one or more classes. The design and realisation of this activity (choice of level within the school system, the subject and type of intervention) are all the responsibility of the students themselves.

B) Post- Graduate Teacher Training Colleges - SSIS

As things stand at present with the organisation of SSIS courses, initial and general "in-service" training overlap in that, for the greater part, the post-graduate students are working contemporaneously as teachers (supply teachers).

For the moment, given the nature of in-service training, the greatest effort in this situation is concentrated on encouraging the students to explicate their particular teaching models (put into practice on a daily basis in the supply teaching) in order to analyse them critically and constructively (Marchini, 2001).

Current conditions are thus fairly favourable for training in the three areas of knowledge: relating to the subject itself, teaching methods and the "third area".

Once again, the particular features of the RMT problems mean that they lend themselves well to this type of training, including some conceptions with regard to the pupils' knowledge.

C) Working teachers.

The teachers participating in the various stages of the RMT are able to take advantage of in-service mathematics teacher training and learning.

In general they are able to be involved in the following:

- Observation of pupils (in their own or another class) in problem solving activities, in the form of group work;
- The analysis of the work produced by the pupils (of their own or another class) and their organisational abilities, comparing the a priori and a posteriori analyses of each problem;
- The discussion of solutions and justifications, developing them further in class;
- The collective discussion of both expected and unexpected errors and possible reasons for their occurrence;
- The receipt of new elements for their own teaching methods which have been subjected to collective discussion.

Research Hypothesis

The research hypothesis is that the Rally has positive effects at any training level.

The testing of this hypothesis is difficult at the initial training stage (A and B) in that the students only rarely have the chance to effect an assessment in the context of continuing teaching activities. To assess the affect of the RMT on initial training we have thus analysed the final mini-theses and the manner in which the problems have been used, on a qualitative basis.

So far as continuing training is concerned (C), our research hypothesis has been that teachers will not easily recognise the teaching potential of RMT, under-valuing its positive effect on maths teaching precisely because of a number of features of the problems: the complexity of the conceptual contexts put in play, the elementary nature of the knowledge required to deal with them and the fact that they do not use well known problem solving techniques. What this means in practice, is that the RMT problems cannot be categorised in one of the themes traditionally forming part of the teaching programmes. The risk for some teachers is that the "play" element of RMT takes precedence, considered simply as a competition in which the children involve themselves with enthusiasm – they thus see the potential for improving social interaction but not for the construction and strengthening of the children's mathematical knowledge.

It appears from this that the teachers are not generally in the habit of assessing a problem from the point of view of knowledge of the curriculum content, rather concentrating on the knowledge of the content of the subject. The Rally problems should however, be considered in the context of the teaching of concepts, rather than in accordance with a linear organisation by subjects.

Some Results and their Implications

To assess the affect on initial training we analysed the final mini-theses from the maths teaching course, the written tests and the training reports of the SSIS students.

In 16 of the 31 mini-theses from the teaching course over the last five years, the RMT problems represent the main thread running through the training.

To assess the influence on in-service training, we provided all teachers whose classes took part in the competition with a questionnaire structured in such a way as to assess the affect of RMT on:

- pupil-knowledge relationship,
- teacher-knowledge relationship,
- pupil-teacher relationship.

The analysis of the questionnaire is still on-going. On the basis of the 73 (30 of Primary School Teachers and 43 of Secondary School Teachers) that have been returned however, we can set out some early results (even though partial in nature) in particular in relation to the teacher-knowledge relationship, of specific interest for this paper.

To the question "*State whether your relations with mathematics after the RMT experience have:*

- *changed for the better*
- *changed for the worse*
- *remained unchanged*"

we have had the following replies:

	Primary School Teachers	Secondary School Teachers
Better	26	20
Worse	0	0
Unchanged	4	23

On the basis of these data it appears that the willingness to use RMT for in-service training is greater among primary school teachers.

We would like to highlight in particular the experience of a working primary school teacher who used, among other things, the RMT problems in her thesis for her mathematics degree. The thesis concerned emphasises, by means of the class comparison technique, that teaching the class with a method involving RMT problems has a positive effect on the pupils' abilities to solve traditional problems as well (Bertazzoni 2002).

We have also noted a direct relationship between the years of involvement in the Rally and its affect on training: the results have been set out in the following table:

Primary School

No. of years of participation	Unchanged	Better
Less than 3	3	15
More than 3	1	11

Secondary School

No. of years of participation	Unchanged	Better
Less than 3	18	10
More than 3	5	10

It seems to us that the above data confirm that those participating in the Rally for a longer period have a greater appreciation of its positive effects on their training, that is, not considering it purely as a mathematical competition encouraging pupil involvement.

As further confirmation we should add that we have identified greater constancy in participation among primary school teachers, while Secondary School teachers very often partecipate with their classes for no more than 2 or 3 years. Even so, they still record their satisfaction with the experience gained.

A number of primary school teachers have included the Rally as part of their overall teaching programme.

We consider it to be of fundamental importance that teachers enhance and widen the a priori analysis of the problems in the context of their own classes, taking account of the different conceptual ambits and their connection with their own programmes.

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