THEMATIC WORKING GROUP 2

AFFECT AND MATHEMATICAL THINKING

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Session 1: Introduction

Thematic Group 2 included 18 participants, of whom 12 contributed to papers. At the conference, we discussed the ten prepared papers, and about the same number of presentations, organised to address a number of themes linking the prepared papers. We thus attempted to go beyond straightforward presentations of the papers already circulated. This introduction to the group's work includes background relevant to the papers, and summarises the main issues in them; it is hoped this will stimulate readers' interests, and provide pointers to the papers that follow.

During the conference, the participants had three group activities – one 30-minute activity each day, led by Markku Hannula. The opening session began with an activity, where participants drew pictures of each other – on a sheet of paper held so that they could not see. The resulting funny portraits were used as a stimulus for each participant to introduce him/herself to the group and to tell about one's feelings and expectations for the meeting.

The opening session continued with a discussion of McLeod's (1992) framework, which suggests that affect can be understood as comprising beliefs, attitudes and emotions, and that we can position these along a spectrum running from stability and "cool" on the left to fluidity and intensity on the right, with values between beliefs and attitudes (DeBellis and Goldin, 1997); see Figure 1.

Figure 1 Range of Types of Affect

Beliefs	Attitudes	Emotion
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Stability		Intensity
Values	Mood	

Sources: McLeod (1992), DeBellis and Goldin (1997)

A number of fundamental critiques of this framework were offered:

- Can beliefs always be said to be stable, especially if by that one means 'always in the foreground', as opposed to 'arising regularly'?
- Are emotions always intense? A counter-instance was given, of difficulties of a pupil with his/her family.
- These characterisations of beliefs and emotions may be the consequence of different methods used for research (self-report questionnaires vs. problem-solving interviews): thus, is it necessary to separate the dimensions of stability and intensity?
- This framework appears to assume an *autonomous* individual, as opposed to one who develops within social interaction in a cultural context; and a *monolithic* individual, rather than one who has contradictory, and even irrational feelings.
- Might it be more fruitful to regard an 'attitude', say, as a construct of the observer, rather than a characteristic of the subject?

Session 2: Beliefs and Emotions

Beliefs

In the meeting, the need to clarify the basic concepts was emphasised. In the prepared papers, the central concepts generally had been defined with care. For example, Anu Pietilä and Erkki Pehkonen explored particularly the distinction between knowledge and beliefs, proposing that beliefs be understood as subjective knowledge about, and

emotions towards, objects and their relationships. Paivi Perkkilä reports on her study of in-service primary school teachers' beliefs and practices.

Self-efficacy beliefs, a concept from Bandura's social cognitive theory, are defined as beliefs about one's own ability to achieve specific goals. In our group, Maria Nicolaidou & George Philippou related fifth-grade pupils' self-efficacy beliefs to their attitude towards mathematics and problem solving achievement. Charalambos Charalambous and George Philippou studied the enhancement of teachers' teaching efficacy beliefs during teacher training.

Emotions

Another fundamental concept in the field of affect is emotion. Participants approached this concept from different angles. During the conference, three different aspects of emotions were specified: a *physiological reaction*, a *psychological experience* or *feeling*, and *socially communicated* emotion, one form of behaviour.

Wolfgang Schlöglmann uses recent findings in neuroscience to understand better the nature of affective responses and their functioning in the human mind. Neuroscience argues that most of our neural processes are unconscious; thus, we have only little direct control over our emotional reactions, while they can influence perceptions, memory, and attention, as well as higher order thought processes. Jeff Evans, using a version of discursive practice theory, sees emotions as a 'charge' attached to a signifier (a meaningful term or image), in a chain of signification. Thus emotions are constituted by the context - that is, by the interplay of discourses / practices in which the subjects are *positioned*, and individuals have a limited freedom to choose from positions that are available in the situation. Peter Nelmes proposes to study emotions as 'the dynamic transaction between individual and social context', and thus as an important aspect of the internal monologue of students; he argues that the meaning of the Zone of Proximal Development can be expanded to incorporate also emotional factors.

Session 3: Attitudes, and Relationships among the Dimensions

Attitudes

Pietro Di Martino's introduction highlighted the variety of definitions of 'attitude' in the literature. Two types of definition are most frequent:

- A 'simple' definition of attitude as the positive or negative degree of affect associated to a certain subject (as regards mathematics education, see e.g. McLeod, 1992; Haladyna et al., 1983).

- A multidimensional definition recognising three components: an emotional response, the beliefs regarding the subject, the behaviour toward the subject (as regards mathematics education see e.g. Hart, 1989). A variation of this definition doesn't explicitly refer to behaviour (Daskalogianni & Simpson, 2000), in that behaviour is seen as a consequence, and not as a component, of attitudes.

The choice of a definition is linked to the choice of measurement instruments – even if in most studies about attitude this link is not so clear – but, above all, it depends on the problems that the researcher deals with. For example the 'simple' definition appears to be inadequate to face some typical problems of research in the field of affect, such as the relationship between attitude and achievement. And, whereas the simple definition of attitude leads to a 'natural' characterisation of a 'positive' attitude, this characterisation is not so easy in the case of the multidimensional definition. On the other hand, the problem of clarifying the relationship of attitude with beliefs and emotions is trivially solved by the multidimensional definition, assuming attitude as a construct defined by the researcher, that explicitly refers to beliefs and / or emotions.

This problem of the type of definition is one that results from mathematics education being a field of research which uses and fits contributions from other fields in order to address its own problems.

The discussion about these issues highlights some crucial points:

- Accepting the idea that 'attitude' is a construct created by researchers (rather than being a quality of the observed subjects), is this construct really necessary / useful?
- How can the construct guide researchers' observation?
- In particular, which kind of definition is more useful, if 'attitude' has to help researchers to explain mathematical behaviour? Which kind of definition is more useful, if 'attitude' has to help researchers to predict mathematical behaviour?

Relationships among the various dimensions of affect

The need to clarify the relationships among the various dimensions of affect has been underlined by most researchers. Accepting McLeod's classification, this means to clarify the relationships among emotions, beliefs, and attitudes (see above).

Rosetta Zan describes the link between emotions and beliefs in the cognitivist approach to emotions. For example, according to Mandler's theory (one of the most popular in mathematics education):

- Visceral arousal follows the occurrence of some perceptual or cognitive discrepancy, or the interruption of some ongoing action.
- Such discrepancies occur when the expectations of some schema are violated.
- The combination of that arousal with an ongoing evaluative cognition produces the subjective experience of an emotion.

Therefore it is not the experience itself that causes emotion, but rather the interpretation that one gives to the experience. This interpretation is influenced by an individual's beliefs; further, beliefs play an important role also in causing perceptive or cognitive discrepancies.

Ortony et al. (1988), describing 'the appraisal structure' (i.e. of emotion-inducing stimuli), are able to develop a theory which differentiates the various emotions according to their cognitive source. They distinguish three main types of emotions, classified as affective reactions to:

- *objects:* variations of liking and disliking, influenced by the subject's tastes; typical examples are love and hate;
- *events*: reactions of being pleased and displeased, influenced by the subject's goals; typical examples are joy, hope, fear;
- *agents*: reactions of approving and disapproving, influenced by the subject's beliefs and values; typical examples are pride, shame, admiration, reproach.

From these three classes derive other, more complex, emotions like anger, in which the reaction to an unpleasant event is connected to a factor considered to be responsible for this event. The role of beliefs in the theory of Ortony et al. is further crucial, in that beliefs influence affective reactions to not only to agents, but also to events, in that they influence the subject's goals.

A different way to take into account the various dimensions of affect is presented by Anu Pietilä. According to this approach, knowledge, conceptions, emotions and beliefs together constitute an individual's *view of mathematics*. This can be seen as consisting of a hard core, which contains the persons' most fundamental views, and a protective belt, which contains more flexible views. Mathematics experiences are of central importance in the formation and development of the view of mathematics: according to the model, mathematics experiences need to penetrate to the hard core in order to change the view of mathematics in a fundamental way.

Session 4: Theoretical considerations

A discussion on theoretical frameworks followed. Jeff Evans's approach to discursive practice theory draws specifically on Critical Discourse Analysis and is informed by insights from poststructuralism and psychoanalysis; for him feelings, as well as thinking, need to be understood in context – that is, through tracing an interplay of discourses in which the person has a *positioning*. In a relatively compatible approach, Peter Nelmes explores Vygotskian theory as a useful framework, and particularly a version of the idea of Zone of Proximal Development, expanded to include the emotional.

Wolfgang Schloglmann documents the claim that cognitive and affective issues are dealt with in different parts of the brain, though there are interconnections. More specifically, humans have two memory systems, both relating to affect. The implicit memory system produces the initial emotional reaction based on prior examples, while in the explicit memory system we have remembrances of our prior experiences. The distinction is important not only for theory, but also for methodology.

Session 5: Methodological Issues

The second group activity led by Markku related to methodology and multiple interpretations of data. The activity was based on a classroom incident described and analysed in Jeff Evans's paper (see below). Participants created plausible emotions and internal monologues for students, and then role-played different versions of the hidden affective dialogue between students.

The aims of the session on methodology were to consider the basis on which we had used different approaches, and the relations between 'quantitative' and 'qualitative' methodologies. What is notable is that many of the projects reported here produced data from multiple sources. Paivi Perkkilä described how her study of in-service primary school teachers' beliefs and practices had used both quantitative methods, Likert-type questionnaires on beliefs, and qualitative methods, classroom observations (video-taped), documents (lesson plans), and interviews. Maria Nicolaidou and George Philippou used quantitative analysis (statistical modelling of affective outcomes) of scales aiming to measure attitude to mathematics, self-efficacy beliefs and problem-solving performance of primary school pupils, and to explore their inter-relationships. Charalambous and Philippou used questionnaire and interview data to test a model of the main bases for one's efficacy beliefs.

Anu Pietila asked a sample of students to write five letters on the themes of her research (various aspects of their 'view' of mathematics) over the period of their course, and subjected the material to a 'phenomenological' reading; see her paper on

the development of the sense of teaching ability of one student teacher below. Jeff Evans analysed the effects of two different research methods – classroom observations and research interview with adult students – on the nature of the data produced. Classroom observations appear to produce fewer expressions of emotions, he argues, because of the different positions available to persons in the classroom and in the interview situations. Markku Hannula draws attention to the double gap between the subject's experiences and the researcher's experiences, as reflected in his/her evaluations and interpretations, on the one hand, and the reader's world, on the other. To make up for this inaccessibility problem, he proposes a new technique, 'fiction writing' based on interviews with the subject and observations as the researcher edits and analyses them.

On the relationship between quantitative and qualitative methods, a number of points emerged in discussion:

- It is important to distinguish *the form of the data* – 'quantitative' data are numerical, apparently more 'exact', while qualitative data are textual, apparently more 'open' to different interpretations – from *the type of analysis* – numerical and using tables, as opposed to more descriptive 'case study' analysis, respectively.
- The boundary between qualitative and quantitative must therefore be seen as 'fuzzy': for example, Evans (2000) distinguishes three approaches: a quantitative survey-based approach (using standardised scales based on self-report measures), a qualitative descriptive case study approach, and a hybrid approach using qualitative data (from semi-structured interviews), but analysed using systematic categorisation and basic statistical analysis, for example, cross-tabulations.
- Whether one sees oneself as working within a quantitative or a qualitative approach, there is still a concern with 'validity' or 'dependability', or 'credibility'.
- Whether working in this area or another of educational research, we need to recognise the emotional charge that our methodological commitments may have: specifically, what anxieties are kept at bay by designating oneself as a 'quantitative' or a 'qualitative' researcher?

Session 6: Group Activity and Discussion of Findings

In the third group activity led by Markku, participants were positioned as 'students' in a mathematics class with a very strict 'teacher'. The number system was changed so that even the basic arithmetic was not automatic for participants, and the 'students' were given a task that seemed quite simple, but was challenging because of the

distorted number system. In the discussion we reflected on our different experiences of such 'pressure'.

In the conference a range of findings, and of types of findings, were reported (see the papers). Nicolaidou and Philippou confirmed earlier findings that both attitudes and self-efficacy are correlated with and hence predict mathematical achievement; in this study, the latter construct was found to be a better predictor of achievement than the former. Charalambous and Philippou found that the prospective teacher's efficacy beliefs with respect to teaching mathematics develop during training: the greatest influence is mastery experiences, supplemented by vicarious experiences and verbal persuasion provided by their mentors and tutors. All input information is processed and assessed through self-reflection against the individuals` beliefs and value system. In a study with a related focus, Anu Pietila case study shows a student-teacher gradually improving to better fulfil some of the criteria that she failed at the beginning. Paivi Perkkila concludes that the teacher's mathematics related beliefs regulate the classroom practice as a powerful hidden factor influencing the quality of mathematics teaching and learning.

Overall, our work, including that in small groups (see next section) led to a general agreement about the following points:

- We are not likely to find full agreement on definitions we use in the field of affect (re. beliefs, emotions, attitudes, etc.). Therefore, it is important to make explicit the definition one is using in one's own research, and to carefully consider the connections between the definition and the measuring instruments used.
- Using multiple methodological approaches, in particular combining qualitative and quantitative research methods is especially important for capturing the complexity of the affective domain.
- It is possible to change affect; one typical pathway of change is to have experiences that create a conflict with our beliefs / emotions / attitudes, and then to reflect upon them.
- It is necessary to include affective issues in teacher training, regarding both students' and teachers' affect.

Session 7: Group discussion

In the last session we worked in small groups, to summarise the most important points emerging, to underline open issues and different positions, and to suggest new directions for research.

Our discussions suggest new directions for research:

- dimensions of affect, and measures of these: a need for multiple methodologies
- a deeper study of the relationships between affective dimensions and mathematical outcomes, such as performance
- the need to clarify the role of affect in problem solving episodes
- influences on a person's affective relationship with mathematics: e.g. early experiences with mathematics
- exploring differences in affect over the age-range, and across social groups.
- the possibility / difficulty / modality of changing teachers' and students' affect towards mathematics
- affective measures and policy outcomes: e.g. recruitment of students to mathematics courses at various levels.

Conclusions

The affective is a multidimensional domain consisting of overlapping constructs such as emotions, attitudes, beliefs and values. Despite the fact that research in the field has been carried out for decades, we are still far from an agreed conceptualisation of the meaning of those basic constructs. The papers presented and the resulting discussions among the members of this Thematic Group show our attempts to find common ground or at least convergent lines of thought. There was broad agreement on an extended definition of attitudes that involves affective, cognitive and behavioural dimension (see Session 3 above). Similarly, there was broad agreement on the value of starting one's investigations from McLeod's (1992) analysis of the affective into aspects of beliefs, attitudes and emotions, though many of his model's specific claims were questioned (see Session 1 above). Beyond this, we hope that our discussions and the papers set out below give some appreciation of the range of viewpoints in research into the affective in the field of mathematics education today.

References

Daskalogianni K. & Simpson A., (2000) 'Towards a definition of attitude: the relationship between the affective and the cognitive in pre-university students', in T. Nakahara & M. Koyama (Eds.), *Proceedings of PME 24* (Hiroshima), vol.2, 217-224

DeBellis V. A. & Goldin, G. A. (1997). 'The Affective Domain in Mathematical Problem-Solving'. In E. Pehkonen (Ed.) (1997), *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 209-216), Lahti, Finland.

Evans, J. (2000) *Adults' Mathematical thinking and Emotions: a study of numerate practices*, London: RoutledgeFalmer.

Haladyna T., Shaughnessy J., and Shaughnessy M. (1983) 'A causal analysis of attitude toward Mathematics.' *Journal for Research in Mathematics Education*, vol.14, n.1, 19-29

Hart L. (1989) 'Describing the Affective Domain: Saying What We Mean'. In D. McLeod & V. Adams (Eds.) *Affect and Mathematical Problem Solving* (pp.37-45)., New York: Springer Verlag

McLeod, D. (1992). 'Research on Affect in Mathematics Education: a Reconceptualisation'. In Grouws, D.A. (ed.) *Handbook of Research in Mathematics Education Teaching and Learning* (pp.575-596), New York: Macmillan.

Ortony A., Clore G. L., and Collins A. (1988) *The cognitive structure of emotions*, Cambridge: Cambridge University Press

List of contributions

List of Thematic Groups