FULFILLING THE CRITERIA FOR A GOOD MATHEMATICS TEACHER
- THE CASE OF ONE STUDENT

Anu Pietilä

University of Helsinki, Finland

Abstract. *The purpose of the study was to define criteria for a good mathematics teacher and examine how well one pre-service elementary teacher fulfilled these criteria during the first two years of her studies. The study was carried out during the academic years 1999-2000 and 2000-2001 at a Finnish university. The study shows the important role that all elements of one's view of mathematics have in enabling a student to meet the criteria for a good mathematics teacher.*

Introduction

Every academic year a new group of prospective teachers enrols in courses that are designed to help them become good mathematics teachers. Before teacher training these students have had many experiences that are linked to mathematics and its teaching and learning. These experiences have shaped their views of mathematics and thus influence their receptivity to new knowledge during their studies. In addition, their views of mathematics affect their teaching at school in the future and ultimately also their pupils' views of mathematics. (E.g. Ernest 1989b) The results of prior research show that elementary teachers' views of mathematics at the beginning of their studies are not as favourable as they might be from the point of view of their future career. Their beliefs and knowledge of school mathematics seem to be restricted. In addition, many students have a poor attitude toward mathematics, and some of them are even afraid of it. (E.g. Ball 1990)

Those who train future mathematics teachers have an important job to do. They have to figure out how they can best help students to become good mathematics teachers. I have worked toward a solution to this problem by concentrating on pre-service teachers' (N=80) views of mathematics and the experiences that are connected with them (Pietilä, 2002a). I have looked for common themes in their views in order to have an overview that would help me to plan my own teaching. In this study I have made an effort to determine the extent to which different aspects of one student's view of mathematics helped or hindered her in meeting the criteria for a good mathematics teacher.

Criteria for a good mathematics teacher

A good mathematics teacher can be thought to need some qualities that are connected to his or her view of mathematics (cf. Pietilä, 2002a; Pietilä, 2002b). This view consists of knowledge, beliefs, conceptions, attitudes and emotions and it can be divided into two main components (inside the circle in Figure 1):

1) view of oneself as a learner and teacher of mathematics, and
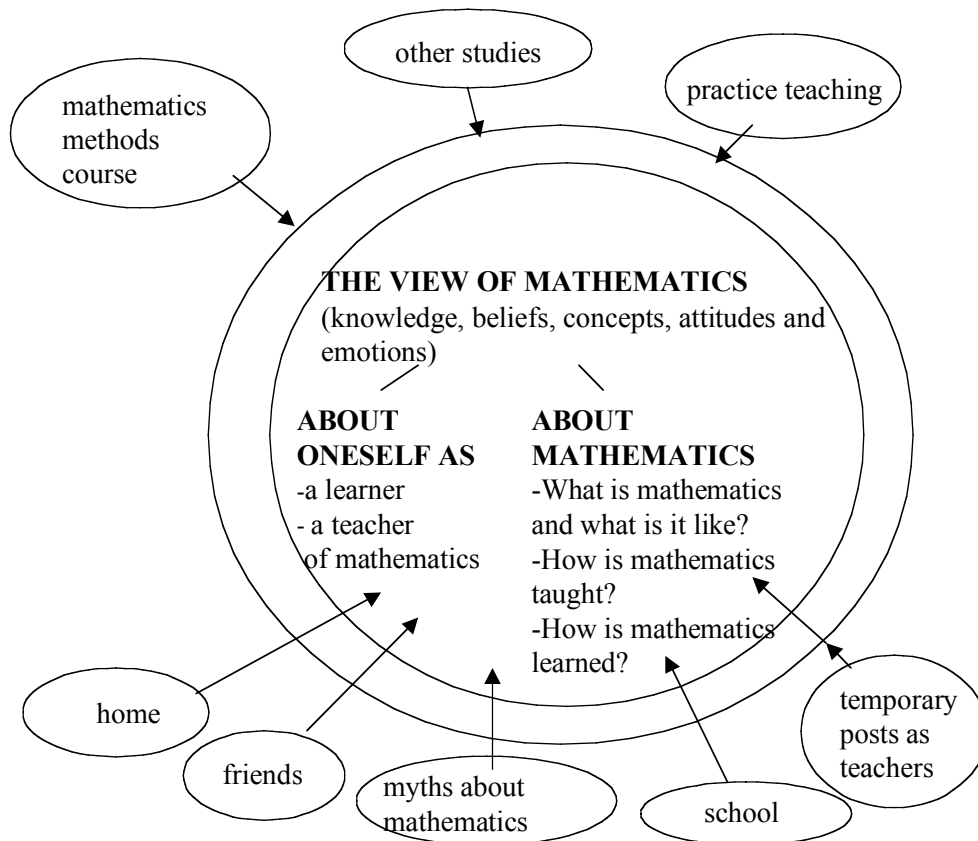2) view of mathematics and its teaching and learning.

Figure 1. A model for the formation of the prospective teacher's view of mathematics

Based on this model and prior research I have defined the criteria for a good mathematics teacher in this study as follows:

1. A good mathematics teacher needs sufficient knowledge of mathematics (e.g. Fennema & Franke, 1992). He or she needs to have a profound understanding of basic mathematics and to be able to perceive connections between different concepts and fields (cf. Ma, 1999, p. 122).

2. Such a teacher needs to have a sufficient knowledge of mathematics teaching and learning (Brown & Borko, 1992). He or she needs to understand children's thinking in order to be able to arrange meaningful learning situations (cf. Carpenter, Fennema, Peterson & Carey, 1988). It is, for example, important that the teacher be aware of children's possible misconceptions. In addition, he or she needs to be able to use different strategies to promote children's conceptual understanding (cf. Eisenhart, Borko, Underhill, Brown, Jones & Agard, 1993).

3. A good mathematics teacher also needs additional pedagogical knowledge: the ability to arrange successful learning situations (for example, the ability to use group work in an effective way), knowledge of the context of teaching and knowledge of the goals of education (cf. Shulman, 1987; Ernest, 1989a).

4. A good mathematics teacher's beliefs and conceptions should be as many-sided as possible and be based on a constructivistic view of teaching and learning

mathematics (cf. Thompson, 1991; Pehkonen, 1994).

5. In addition, a good mathematics teacher should have a positive attitude toward learning and teaching mathematics (cf. McLeod, 1992). He or she should also have enough confidence in his or her abilities as a learner and teacher of mathematics (cf. Weiner, 1986).

How does one become a good mathematics teacher? What factors influence his or her view of mathematics? Prospective teachers' views of mathematics are affected by many factors, including their own experiences of mathematics (cf. Malinen, 2000), as indicated at the bottom of Figure 1. At school and elsewhere they have experienced how mathematics is taught and studied and how their friends view mathematics. At home, their parents may have encouraged them to study mathematics. Likewise, myths about mathematics (for example the myth that boys are better in mathematics) may have influenced their views of mathematics. Students may have experiences during mathematics methods course, other studies, practice teaching, and temporary posts as teachers that influence their views of mathematics during their first year of study.

Method

Participants

This study is based on my doctoral dissertation study, which was carried out in a mathematics method course during the academic year 1999-2000 at the University of Helsinki. I studied written material produced by 80 first-year pre-service elementary teachers who attended the course.

In this study I concentrate in one student. At the beginning of the studies she was 24 years old. She had had difficulties with mathematics already in elementary school. In upper secondary school she had taken short mathematics, received the lowest grade in her final report and failed the mathematics portion of the matriculation examination. She had very little teaching experience.

Data collection

The research material was gathered from the students (N=80) in the form of written homework as a normal part of their study. The questions were made as broad as possible and they were designed to obtain information about certain themes. Students wrote five different letters during their mathematics studies that dealt with their experiences of mathematics at school and at the mathematics methods course, among other things.

The five letters written by one student are used here, and numerous subsequent discussions and e-mail exchanges with her have further deepened my understanding of her views.

Data analysis

I analysed the research material using a method that was based on phenomenology

and hermeneutics (cf. Giorgi 1997). I read the material through twice in order to get a sense of the whole. In addition, it was important for me to 'bracket', which means to put aside things that I know for example from theory in order to experience students' experiences freshly. After dividing the texts into meaning units (themes of the study), I then grouped the students based on their situation in the beginning of their studies and analysed the research material one research question at a time. I identified and recorded the contents of the meaning units and then recorded the common features and structures for different groups.

Results

In Pietilä (2002a; 2002b) I compared the views of mathematics students at the beginning of their studies with reports made by the same students at the end of the year. Based on the answers given at the beginning of their studies it was possible to divide them into four groups. The criterion for this grouping was students' views of themselves as learners of mathematics, because they had a profound influence on both their descriptions of their experiences and their views of mathematics:

1) mathematics is challenging problem solving (13%).

Students had had very positive experiences in mathematics, and their mathematical self-esteem was high. They thought that mathematics is challenging, clear, logical, rewarding and pleasant.

2) mathematics is important and usually pleasant (36%).

Most of the students had worked hard in order to succeed in mathematics. They thought that they could learn mathematics if they tried hard. Mathematics was, in their opinion, usually pleasant, very important and laborious.

3) mathematics is just one subject among others (20%).

Most of the students had succeeded in mathematics rather well without hard work. They thought that they were quite normal learners, not especially good but not bad either. They were not interested in mathematics but they did not dislike it either. Mathematics was quite indifferent to them.

4) mathematics is difficult and unpleasant (31%)

Students had experienced many failures in mathematics. They had had difficulties already in elementary school or the difficulties had started in high school and they felt that they had completely lost track. In their opinion mathematics was difficult, boring, disgusting or even frightening.

This data indicated that only approximately half of the students were interested in and/or enthusiastic about mathematics and about studying it at beginning of their studies. Some students were even afraid of mathematics (cf. Trujillo & Hadfield 2000). In addition, all students in groups 2, 3 and 4 had a very narrow view of mathematics and its teaching. Most of them thought that mathematics was merely computation and that it was based on rules and procedures that should be memorized

(cf. Ball 1990). In addition, based on my observations during the studies students᾽ knowledge of mathematics was compartmentalized, usually very superficial and not based on comprehension (cf. Hill 2000). Thus only a few students in this group would have met the criteria for a good mathematics teacher mentioned above.

In this report I concentrate on examining the changes in one student's view of mathematics during the period of the study and consider how well she fulfills the criteria of a good mathematics teacher. This student was placed in the fourth group and she did not meet any of the criteria for a good teacher at the beginning of her studies. Some of her views on mathematics can be seen in this slightly paraphrased excerpt:

> *In school I always asked my teacher where do we need mathematics?! Now a few years later I must give answers to very similar questions. I was never good in mathematical subjects at school and I am still not good. In order to be able to teach mathematics well in the future and to be able to give my pupils a good answer to this question, my skills and attitudes have to improve. Based on my school experiences, you either like mathematics or you don't; there is nothing in between. As I understand, as it happened to me, mathematics becomes a detestable subject because it's hard when you do not understand. You do not even give yourself a chance to learn because all the others are far ahead and understand everything. Then negative images of the subject become rooted deep inside.* (A letter written at the beginning of the student's studies)

This student had experienced difficulties in mathematics in school as a pupil, she had failed repeatedly and she felt that she was bad in mathematics. She had not understood the teachers' explanations and could not have done the exercises. Comparing herself to others had increased her sense of failure. Repeated failure had even caused her fear and anxiety. (Cf. Trujillo & Hadfield, 1999) She wished that the teacher had explained where mathematics is needed but the teacher could not do that.

These experiences led the student to adopt the following view of mathematics:

- This student had had difficulties in mathematics already in elementary school. In higher secondary school she had taken short mathematics, received the lowest grade in her final report and failed the mathematics portion of the matriculation examination. Therefore her subject knowledge probably was compartmentalized and was not based on understanding. Her knowledge of teaching and learning mathematics was also weak because it was based only on her own experiences as a pupil.

- Based on what she had written, I came to the conclusion that she had at least the following beliefs: not everybody can learn mathematics, mathematics is not connected to everyday life, and mathematics consists of performing boring calculations.

- She had a negative attitude toward mathematics because she had experienced many failures. She had low mathematical self-esteem and may even have believed that she could not learn mathematics.

During their first year of study, students enrol in a mathematics methods course (4 credits) which is included in the multidisciplinary studies of the school subjects (35 credits). The study block consists of lectures (40 h), group work (44 h), integration projects with other subjects, for example arts (26 h), and individual work (50 h). During the course I tried to influence my students' views of mathematics by using a variety of research-based methods. I tried to increase their subject knowledge, especially of elementary school mathematics (cf. Borko, Eisenhart, Brown, Underhill, Jones & Agard, 1992). Students learned to understand children's thinking by examining their calculations and by analyzing their mathematical thinking (cf. Graeber, 1999). They also practiced in theory how pupils can be taught to understand and learn, for example by using different manipulatives (cf. Quinn, 1998). I tried to influence my students' beliefs by discussing with them their views of mathematics and the experiences that had shaped them. I also wanted to offer students experiences of that kind of teaching I wanted them to do in the future (cf. Cooney, 1999). I combined as much theory and practice as I could in order to help my students understand the usefulness of what they were learning (cf. Pajares, 1992). In addition, I tried to offer them pleasant and successful learning experiences in as safe a learning atmosphere as possible.

My previous research indicated that students' views of mathematics became more positive and differentiated during the studies. The mathematics methods course, practice teaching, holding temporary posts as teachers and other studies combined to make it possible for the students to consider their studies to be meaningful (Pietilä, 2002a). At the end of mathematics methods course, the student I quoted above wrote the following comment:

> *My view of mathematics has always been unpleasant. This is perhaps due, as is often the case, to bad school experiences in mathematics. Hardly any teachers are aware of all the opportunities that teachers have in order to make mathematics teaching more lively, more meaningful and to get pupils interested in solving problems. I see that the mathematics methods course has been good practice and repetition, and I have learned a lot of new things in mathematics, which has always been my weakest link. I can therefore say that my experiences have been quite different than before. It remains in every teacher's conscience how much he or she will use what has been learned.* (A letter written at the end of the mathematics methods course)

This student's experiences of mathematics during her first study year had improved her view of mathematics, such that it had now come to have the following characteristics:

• Student's knowledge of mathematics and its teaching and learning had in her opinion improved. She now understood many basic contents more deeply and probably noticed based on her own experiences that it is important for pupils to understand what they learn.

• Her outlook and beliefs had broadened. She had noticed that mathematics can be

studied in many different ways and that learning can be pleasant.

• She had also a much more positive attitude toward mathematics and probably more confidence in her own abilities.

The student had come closer to meeting the criteria for a good mathematics teacher than before beginning her studies.

As I wanted to verify that my students' subject knowledge was good enough to meet these criteria, I arranged a test. I wanted to evaluate their understanding of elementary school mathematics by asking them to solve the following problem, among others: "We use the base-ten system to mark our numbers. Assume that the system is based on the number of fingers. Simpsons have eight fingers. If everyone had eight fingers, we would probably use the base-eight system. Explain all the qualities of the base-eight system (place value system)."

The student failed similar tests many times and did not therefore receive permission to begin practice teaching as she had planned. As a result, she sent me the following e-mail:

> *It can't be true! My teaching practice is put off till far into the future!!?? Forgive me, but this really destroys everything!! I cannot process this. Because of this, my teaching practice is put off till NEXT YEAR!?? I cannot handle this. My graduation is put off for one year, an addition to the family till who knows when. How can I explain this to others or to myself? No, say that this is not true. There must be some other way. Although I am not the only one who is struggling to cope with this, this ruins everything concerning my studies and also concerning my whole life.*

Failing this test had changed this student's view of mathematics and she was even less able to meet the criteria for a good mathematics teacher than before:

• Her knowledge of mathematics and its teaching and learning had increased during studies but especially her subject knowledge was still too compartmentalized and was not sufficiently based on comprehension.

• Her belief that not everybody can learn mathematics had probably become stronger.

• Failing in tests had caused negative feelings and had made her attitude even more negative. In addition, her self-confidence and her self-esteem had collapsed.

I was frequent in contact with her and tried to encourage her to study hard. I explained that she should try to understand what she is studying and not learn by heart. I let her borrow some teachers' manuals so that she could get extra practice. Fortunately she pulled herself together and studied very hard. She contacted me several months after her last failure in the test by e-mail:

> *Hello Anu! I have studied with others and alone. BUT I still do not understand the base-eight system. I understand it somehow but what is the point of using it?! So, when can I come to see you? I would like to ask some questions about the base-eight system among other things. And*

if you have some exercises for me that I can practice with I would be happy. I have more or less waded through the 5th and 6th grade books. And I am thankful that you always help me.

The student came to see me and we went through the main themes. She showed me that she understood what she had learned. She came to the test and passed it clearly. She then sent me the following e-mail:

UNBELIEVABLE BUT TRUE! It was good that I failed previous tests because it was not until now that I understood the contents. It was nice to be in the test and also after it because I knew that I would be able to pass it. Perhaps I did not believe that I could do this well! You also deserve thanks for this, THANK YOU!

Success changed this student's view of mathematics again:

- The student's subject knowledge and knowledge of teaching and learning of mathematics had improved greatly. She understood elementary school mathematics deeply and she was able to explain how she could teach children so that they would understand.

- Her belief that everybody can learn mathematics had become stronger. She had seen that it is possible to learn mathematics by working hard and by understanding the subject matter.

- Her attitude toward mathematics had again improved as well as her trust in her abilities as a learner and teacher of mathematics.

This student now fulfilled the criteria for a good mathematics teacher better than ever before. I believe that her own difficulties have helped her to become a good teacher, because she can put herself into the position of weak pupils. She would probably try to observe her pupils' learning on a regular basis so that she can spot their difficulties as soon as possible. In addition she would probably try to help pupils to understand in a variety of different ways. Through practice teaching, the student may be able to further improve her view of mathematics and acquire more of the qualities of a good mathematics teacher.

Discussion

There is no doubt that use of word 'good' to describe a teacher with desirable skills raises many questions. It could be argued that there are many kinds of good teachers – what may count as good in the eyes of a department head may not be the same as what counts in a student's eyes. Similarly, teachers may be good at different aspect of their teaching – stimulating the gifted students may involve different skills than ensuring that the weaker ones don't fall off the bottom of the ladder. However, I think that the criteria I have used in order to give content to this word permit me to explore aspects of being a successful teacher, a teacher that for example gets pupils to understand the topics presented and to be enthusiastic about mathematics.

This example illustrates how important all elements of one's view of mathematics are in becoming a good mathematics teacher and how the elements affect each other. Previous experiences of mathematics are of central importance in

one's view of mathematics (cf. Malinen, 2000). Students' knowledge and emotions form the foundation for this view. Beliefs and attitudes are formed on the basis of knowledge and emotions and they influence students' reactions to future contacts with mathematics (cf. Daskalogianni & Simpson, 2000). Their view of themselves as learners of mathematics is a central element of their view of mathematics.

The fact that students come from a variety of backgrounds creates challenges for teacher trainers. Successful mathematics teachers can come from many different backgrounds, using many different routes. Teacher trainers should plan their work so that all students can broaden their views of mathematics and become better teachers. In order to succeed, teacher trainers need to get a picture of students' views of mathematics (cf. Malinen, 2000).

It would be interesting to see how well this student meets the criteria for a good mathematics teacher in the future, for example when she enters practice teaching and when she obtains a post as a teacher. In addition, it would be interesting to study what experiences have had the greatest influence on this development, for example, the mathematics curriculum, her colleagues, the size of her classes, the choice of textbook and teaching materials, in-service training, and the like.

References

Ball, D. L. (1990). The mathematical understanding that prospective teachers bring to teacher education. *The Elementary school Journal*, 9 (4), 449-466.

Borko, H., Eisenhart, M., Brown, C.A., Underhill, R.G., Jones, D. & Agard, P.C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education* 23 (3), 194–222.

Brown, C. A. & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (ed.) *Handbook of research mathematics teaching and learning*. New York: Macmillan, 209-239.

Carpenter, T. P., Fennema, E., Peterson, P. L. & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education* 19 (5), 385–401.

Cooney, T. J. (1999). Conceptualizing teachers' ways of knowing. *Educational Studies in Mathematics* 38, 163–187.

Daskalogianni, K. & Simpson, A. (2000). Towards a definition of attitude: the relationship between the affective and the cognitive in pre-university students. In T. Nakahara & M. Koyama (eds.) *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education*. Hiroshima University. Vol 3, 217–224.

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D. & Agard, P. (1993). Conceptual knowledge falls through the cracks: complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education* 24 (1), 8–40.

Ernest, P. (1989a). The knowledge, beliefs and attitudes of the mathematics teacher: a model. *Journal of Education for Teaching* 15 (1), 13–33.

Ernest, P. (1989b). The impact of beliefs on the teaching of mathematics. In P. Ernest (ed.) *Mathematics teaching. The state of the art*. London: The Falmer Press, 249–254.

Fennema, E.& Franke, M. (1992). Teachers' knowledge and its impact. In D. A. Grouws (ed.) *Handbook of research mathematics teaching and learning*. New York: Macmillan, 147-164.

Giorgi, A. (1997). The theory, practice, and evaluation of the phenomenological method as a qualitative research procedure. *Journal of Phenomenological Psychology*, 28 (2), 235-260.

Graeber, A. O. (1999). Forms of knowing mathematics: What preservice teachers should learn.

Educational Studies in Mathematics 38, 189–208.

Hill, L. (2000). Theory, practice and reflection: a pre-service primary mathematics education programme. *Teachers and Teaching: theory and practice*, 6 (1), 23-39.

Ma, L. (1999). *Knowing and teaching elementary mathematics. Teachers' understanding of fundamental mathematics in China and the United States*. New Jersey: Lawrence Erlbaum Assosiates, Inc.

Malinen, A. (2000). *Towards the essence of adult experiential learning. A reading of the theories of Knowles, Kolb, Mezirow, Revans and Schön*. University of Jyväskylä: SoPhi.

McLeod, D. B. (1992). Research on affect in mathematics education: a reconceptualisation. In D. A. Grows (ed.) *Handbook of research on mathematics teaching and learning*. London: Macmillan Publishing Co, 575–596.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research* 62 (3), 307–332.

Pehkonen, E. (1994). On teachers' beliefs and changing mathematics teaching. *Journal für Mathematik-Didaktik* 15 (3/4), 177-209.

Pietilä, A. (2002). *Luokanopettajaopiskelijoiden matematiikkakuva: matematiikkakokemukset matematiikkakuvan muodostajina*. Helsingin yliopiston opettajankoulutuslaitoksen julkaisuja 238. [Pre-service elementary teachers' views of mathematics: mathematics experiences in forming the views of mathematics]. [In Finnish].

Pietilä, A. (2002b). The role of mathematics experiences in forming pre-service elementary teachers' views of mathematics. In A. D. Cockburn & E. Nardi (Eds.) *International Group for the Psychology of Mathematics Education. Proceedings of the 26th Annual Conference 21-26 July 2002 in Norwich,* UK, Vol 4. University of East Anglia, 57-64.

Quinn, R. (1998). The influence of mathematics methods courses on preservice teachers' pedagogical beliefs concerning manipulatives. *Clearing House* 71 (4), *236–238.*

Shulman, L. S. (1987). Knowledge and teaching: Foundation of the new reform. *Harward Educational Review* 57 (1), 1–22.

Thompson, A. G. (1991). The development of teachers' conceptions of mathematics teaching. In G. G. Underhill (ed.) *Proceedings of PME_NA 13 conference.* Volume 2. Blacksburg (VA): Virginia Tech, 8-14.

Trujillo, K. M. & Hadfield, O. D. (1999). Tracing the roots of mathematics anxiety through in-depth interviews with preservice elementary teachers. *College student journal* 33 (2), 219–232.

Weiner, B. (1986). *An attributional theory of motivation and emotion.* New York: Springer-Verlag.