CAN NEUROSCIENCE HELP US BETTER UNDERSTAND AFFECTIVE REACTIONS IN MATHEMATICS LEARNING?

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Research in mathematics education uses affective categories of "beliefs", "attitudes", "emotions" and "values/ethics/morals". However, commonly encountered definitions of these categories do not provide a sharp delimitation between them. The research methods used in mathematics education – both quantitative and qualitative – are not helpful for finding such a delimitation, because the methods, instead, aim to provide more general insights into the problem of mathematics learning. Looking outside the field of mathematics education - in particular, at neuroscience, as this field also deals with affect and cognition - may prove to be fruitful. Many results have recently been discovered in neuroscience using new research methods, and these results could help us better understand central concepts in our field.

1. Affect in mathematics education research

Here I give a brief account of the way affect enters mathematics education research. My goal is to provide a basis for discussing certain questions from the neuroscientific point of view. (For an overview of research into affect, I refer the reader to Evans, 2000).

The categories of affective representation used in mathematics education research are:

> “(1) Emotions (rapidly changing states of feeling, mild to very intense, that are usually local or embedded in a context);
>
> (2) Attitudes (moderately stable predispositions toward ways of feeling in classes of situations, involving a balance of affect and cognition);
>
> (3) Beliefs (internal representations to which the believer attributes truth, validity, or applicability, usually stable and highly cognitive, may be highly structured);
>
> (4) Values, ethics, and morals (deeply held preferences), sometimes characterized as "personal truth", stable, highly affective as well as cognitive, may also be highly structured)”. (Goldin, 2001, p.3)

These categories have been elaborated to varying degrees of sophistication. The most often used – and most developed - concepts are "attitudes" and "beliefs". However, in the literature, a universal, consistent definition does not exist for either of them (D. B. McLeod and S. H. McLeod, 2002; Furinghetti and Pehkonen, 2000). Furthermore, no precise delimitation between the two has been given (Di Martino and Zan, 2001).

I shall now present some important steps in the development of these two categories, beginning with Schoenfeld's definition of belief systems, namely: "Belief systems are one's mathematical world view" (Schoenfeld, 1985).

McLeod (1992) writes, "beliefs, attitudes and emotions are used to describe a wide range of affective responses to mathematics"; and "beliefs, attitudes and emotions also differ in the degree to which cognition plays a role in the response, and in the time that they take to develop. For example, beliefs are largely cognitive in nature, and are developed over a relatively long period of time" (p.578-579).

Törner and Grigutsch (1994) use attitudes as a starting point: "Attitudes represent cognitive and emotional structures. An attitude has three components. The cognitive component consists of knowledge of an object (to be identified as a "conception" or "world view"); the affective component consists of the emotional relationship or connection with an object (to be identified as "attitude"). The behavioural component consists of the observable behaviour together with readiness for or probability of particular type of behaviour (p.213, Translation W. S.).

It may be argued that the category "attitudes" in the sense of Törner and Grigutsch includes, as components, the categories "beliefs", "attitudes" and "emotions" in the sense of McLeod.

Goldin defines "*beliefs*" as "multiply-encoded cognitive/affective configurations, usually including (but not limited to) propositional encoding, to which the holder attributes some kind of *truth value*" (Goldin, 2001; p.5).

Furinghetti and Pehkonen (2000) describe the function of beliefs in the following way. "(a) Beliefs form a background system regulating our perception, thinking and actions; and therefore, (b) beliefs act as indicators for teaching and learning. Moreover, (c) beliefs can be seen as an inertial force that may work against change, and as a consequence, (d) beliefs have a forecasting character" (p. 8-9). (For more on the different conceptualisations of beliefs and attitudes, see Furinghetti and Pehkonen, 2000; Di Martino and Zan, 2001.)

All the above-mentioned papers implicitly seem to attribute a certain feature to both "attitudes" and "beliefs", namely: these categories are based on a stable mental system acting in the background. The mental system arises as a consequence of learning processes active when the student is faced with a particular element of curricular content. (Here, I use the term "learning process" in a very broad sense, which includes actions, as well as contexts in which a person is confronted with a particular element of curriculum.)

The category "emotions" is inter alia revealed through qualitative research methods. "Emotions" are particularly clearly revealed in research where students undertaking non-routine mathematical problem solving are videotaped (DeBellis and Goldin, 1997). This is because in such problem solving, strong emotional reactions often appear, combined with bodily expressions. This is the sense Goldin alludes to when he describes emotions as "rapidly changing states of feeling, mild to very intense, that are usually local or embedded in context".

McLeod uses the following characterization: "Emotions, on the other hand, may involve little cognitive appraisal and may appear and disappear rather quickly, as when the frustration of trying to solve a hard problem is followed by joy of finding a solution" (McLeod, 1992; p.579). While McLeod sees low cognitive involvement in emotional reactions, DeBellis and Goldin disagree: "[although] we agree with most of McLeod's analysis, we differ with his assessment that the level of cognitive activity involved in emotions during problem solving is low (at least compared to attitudes and beliefs). Our studies suggest it is very high, though the cognitions interacting with fleeting emotions may be difficult to identify" (DeBellis and Goldin, 1997, p.211).

If we consider descriptions of emotional reactions – puzzlement, curiosity, frustration, confidence, anxiety and so on - emotions could be seen as reactions that find expression through a "general feeling system" that reacts in its own specific way to local and contextual situations. These reactions are not really specific to mathematics: they undoubtedly arise in many nonmathematical situations as well.

Another issue concerns the stability of emotions. All authors see emotions as fleeting phenomena: they "may appear and disappear rather quickly" (McLeod) or they are "rapidly changing states of feeling" (Goldin). However, one may ask, what does "stability" mean? In my view, it means that within a certain period, the same reaction is produced in the same situation. Now, let us postulate that mental systems create affective reactions. These systems are the result of learning processes; and long term learning processes, in particular, lead to stable mental systems, hence to stable reaction patterns.

What can we say about reactions of the following kind, reported in the Internet discussion group, "adult numeracy", by Bonnie Fortini?

"Has anyone run into a case like the student I have had who seems unable to do any math that has unknowns or variables in it? She is mid 40s, very bright, English major going on to a Masters program. She can do all sorts of computations including fractions, percentages, ratios, and word problems are some of her favourite things to do. But as soon as you give her something like 4+2x-6+5x=95, she is totally frozen. She can't get past go when trying to combine like terms, and reacts physically (anxiety, tears, etc.)."

A very intense reaction, embedded in a special context, is described here; but this reaction is stable in the sense that the same situational context repeatedly produces the same reaction. This implies emotions can be stable.

McLeod uses "beliefs", "attitudes" and "emotions" as the categories of affect. DeBellis and Goldin (1997), in contrast, ask: "Do the three components of emotions, attitudes and beliefs adequately capture the spectrum of affective responses in mathematical problem solving?" They argue a fourth category is required, "one that includes aspects of a solver's values, morals, and ethical judgements that interact with problem decision-making" (p.212). This complex value/moral/ethical system is "one of the most powerful motivators of human beings" (p. 212) and is a product of a development process that begins in childhood. Also, Clarkson and Bishop (1999) emphasize "the deep affective qualities "of values in mathematics education, and refer to the close relationship between values and attitudes "with values occupying a more central and deeply held position than attitudes, which are often considered to be reflected in our patterns of response to particular situations" (p.1). "Values, ethics and morals", too, are strongly connected to some mental system active in the background.

The above brief overview of affect in mathematics education may be summarized as follows. Research has led to the affective categories of "emotions", "attitudes", "beliefs", and "values/ethics/morals", which are to be used to comprehend a complex reality. Some research methods typically used in mathematics education manifest some of these categories better than others. Quantitative methods reveal stable and less intense categories, while qualitative methods are able to grasp quickly changing and very intense reactions. Nevertheless, it seems that our research methods cannot establish a distinction between the above categories. Consequently, as argued above, no commonly shared definitions for the categories exists. Furthermore, the qualities "stability" and "intensity" consolidate the description of the categories, but these alone are not sufficient to solve the problem of distinguishing between categories.

In recent years, many new results have been found in neuroscience, using new research methods. An increasing number of studies are investigating affect and the interrelation between affect and cognition. Perhaps these neuroscientific results can help us gain insight into the reality.

2. Some neuroscientific results apropos affect and cognition

Research in mathematics education regards cognition and affect as two different fields, with their own concepts and research issues. The interrelation between affect and cognition in mathematical learning processes is typically only considered in research into affect. The influence of affect on mathematical performance has been studied particularly intensively (Fennema, 1989; Evans, 2000). In the following, I outline some neuroscientific results relevant to the purpose of the present paper. As in mathematics education, definitions of terms in neuroscience are not uniform. Moreover, the term "emotion" is often used instead of "affect".

•The brain contains all of an individual's knowledge, not only of the world outside the body, but of all processes inside it, too.

•Neuroscience distinguishes between different brain systems that fulfil specific functions. These systems are located in different parts of the brain. The brain systems are not isolated: they are connected to other systems, exchange information with each other, and mutually influence each other. Many of these connections act in both directions. It is important to bear in mind that at the neuronal level, all processes are unconscious: we are only conscious of a small subset of the end results of these processes.

•Different systems exist for cognitive processes and for emotional processes. These systems are located in different parts of the brain. Both systems are the product of an evolutionary process. On the one hand, this evolutionary process has led to mechanisms that decide how inputs are processed, and to "basis patterns" that regulate reactions in special situations. (For instance, they create the imperative to flee from danger.) The goal of this phylogenetic basis is to help the individual survive. On the other hand, the two systems are also able to learn from experience, which means that reactions related to personal experience are possible, too. The two systems have their own memory with their own specific features. "It is now common to think of the brain as containing a variety of different memory systems. Conscious, declarative or explicit memory is mediated by the hippocampus and related cortical areas, whereas various unconscious or implicit forms of memory are mediated by different systems. One implicit memory system is an emotional (fear) memory system involving the amygdala and related areas" (LeDoux, 1998; p.202). Below, I emphasize the fear system (amygdala system) in connection with the emotional system, because a rich body of research results exists for it; and fear or anxiety in connection with mathematics is an extensively discussed topic (Evans, 2000).

•Connections exist between the two systems, which allow exchange of information, as well as mutual influence. For the purposes of the present paper, it is important to note that an emotion system has strong connections to the body system, and that an activated emotion system can lead to bodily reactions (arousal, facial movement, increased blood pressure, sweating hands, as well as actions such as flinching, and so on). Furthermore, although the emotion system works unconsciously, we are able to recognize some end results of an activated emotion system - for instance, what we call "feeling". In the light of neurology, Damasio writes (1999, p.37): "For the purpose of investigating these phenomena, I separate three stages of processing along a continuum: *a state of emotion*, which can be triggered and executed nonconsciously; *a state of feeling*, which can be represented nonconsciously; and *a state of feeling-made-conscious*, i.e., known to the organism having both emotion and feeling. I believe these distinctions are helpful as we try to imagine the neural underpinnings of this chain of events in humans. A single event can lead to two different remembrances and therefore to two different "learning effects". On the one hand, an image of the event is stored in the explicit (hippocampus) memory region.

Also stored there is the feeling connected with the event (the state of feeling-made-conscious); but the image of this feeling is a so-called "cold fact" (LeDoux, 1998, p.202). On the other hand, an image of the event is also stored in an implicit emotional memory system. It is important to note that this system is able to react to a suitable input, even if the explicit memory has forgotten the reason for the reaction (LeDoux, 1998, p.203).

I conclude this section with some comments on the interrelations between the two systems. "There are in fact numerous connections from the hippocampus and the transition regions, as well as many other areas of the cortex, and the amygdala" (LeDoux, 1998, p.203). Much stronger are the connections from the emotional system to other systems. "Areas of the amygdala project to a wide variety of cortical areas. Included are projections to all stages of cortical sensory processing, to prefrontal cortex, and to the hippocampus and related cortical areas. Through these projections, the amygdala can influence ongoing perceptions, mental imagery, attention, short-term memory, working memory, and long-term memory, as well as the various higher-order thought processes that these make possible" (LeDoux, 1998, p. 287). It is also important to bear in mind that "we have little direct control over our emotional reactions" (LeDoux, 1998, p. 19).

3. Affect in mathematics education, from a neuroscientific point of view

The aim of this section is to understand models used to describe affective reactions in mathematics learning processes. To do this, I use a model constructed by M. Hannula (Hannula, 1998) to explain changes in attitudes. This model is quite suitable for my purpose because it, too, uses neuroscientific results, and has a process character as well. Hannula sees the "the landscape of mind" continuously changing. The process of change is stimulated by information arriving from the senses, as well as by mechanisms and structures within the brain. One main principle of the model is the distinction between dynamic and static representation systems. Dynamic representations include all systems that are activated at a given moment. Static representations encompass all the information stored in the memory systems of the various representational systems. The structure of the static representations – cognitive and affective schemata – is crucial for all processes. The schemata representing the static representations are changeable via learning processes.

The model describes the path from a situation to an action that is caused by processes within the brain. Each situation faced by a student leads to sensory information that acts as a stimulus for the activation of certain affective and cognitive systems. That means that the different systems activate information and schemata - stored within each memory system - that are utilized for the processes of planning and execution, following which an action is executed.

In the following, I attempt to consolidate the model in the light of neuroscientific results. First, in all situations of interest to us, many brain systems are activated (Damasio, 1999). These activated systems influence the sensory systems. Let me give an example. In class, a teacher explains the solution method for solving equations in one variable; then he or she writes an equation on the blackboard. However, from the student's viewpoint, the information constituting the situation might be different. A successful student sees the different signs on the blackboard and grasps the principles – the structure of the terms - necessary to solve the equation. Another student who is not as apt at handling variables might see the same equation as a set of signs with no structure. The student's fear, present as a background emotion during mathematics lessons, might be intensified, thus preventing a successful learning process. Consequently, the sensory information characterizing the situation for a particular individual is influenced by the dynamic representations that comprise the starting point of the observation of a situation. Furthermore, the activation process - that arises because of sensory information constituting the new dynamic representations with respect to the situation - is mostly not controlled by the conscious self. This activation process is a consequence of biological processes that are determined genetically as well as by the "necessary structure of the systems" (LeDoux, 1998). (In her thesis, M-L. Malmivuori describes the complexity of this structure-building process within the individual, as well as the dynamic via which this structure regulates further learning processes (Malmivuori, 2001)).

Let us consider dynamic representations more closely. These bear the imprint of characteristics of all systems, affective and cognitive, activated in a given situation. As discussed in Section 2, affective systems work as emotional systems unconsciously, and often accompany activation of arousal systems, which produce bodily reactions. An important fact is that emotional systems are physically connected to cognitive systems, and have a strong influence on the person's attention (LeDoux, 1998). Therefore, emotional systems working unconsciously influence the choice of schemata activated in the cognitive systems as well as the sensory input (Ciompi, 1999). Each system of dynamic representation can lead to an action (which could, for instance, comprise of a thought), though it can also lead to changes in static representations. These changes can entail intensification of an existing scheme, an increase in some scheme's complexity, or an entirely new scheme.

Dynamic representations contain unconscious as well as conscious shared elements. However, we have feelings, as well as remembrance of feelings felt in a particular situation. This means we can also receive information consciously about processes that occur unconsciously. All researchers in neuroscience distinguish between explicit and implicit memory systems that store both cognitive as well as emotional content (Damasio, 1999; LeDoux, 1998; Roth, 2001).

How do we get conscious information in a particular situation? LeDoux uses the concept of "working memory" to explain how consciousness is generated. The working memory (probably located in the prefrontal cortex (LeDoux, 1998, p.276)) integrates all the information that comes from systems that are activated in a particular situation (LeDoux, 1998, p.273). This information contains sensory input from sensory systems, information from the cognitive long–term memory, together with information that comes from activated emotional systems such as bodily reactions or feelings, et cetera. The content of the working memory is a representation of the immediate conscious experience, and is suitable principally for being stored in the explicit long–term memory (LeDoux, 1998; p.204).

The concept of working memory also helps us understand another observation. If we ask adults to describe their relationship to mathematics, they often recount nasty situations in the classroom where they could not handle some task, and other students laughed at them, or the teacher was angry etc.; but the interviewee does not mention (let alone argue against) specific mathematical points.

Evidently, the working memory integrates sensory information about a situation with information about feelings experienced in the particular situation. This means a very close connection exists between situational information (stored in the episodic memory, a submemory of the explicit memory) and feelings experienced in that situation. Presumably, a strong connection also exists between the episodic memory and the emotional memory, since many of the adult subjects in the above survey manifested bodily reactions during their recollection.

Knowledge of feelings in relation to certain objects or situations allows humans to handle emotions in a "rational" way. (This is only possible if the situation is not traumatic, as is, for instance, the case reported by B. Fortini in Section 1 above.) DeBellis and Goldin (1997) and Goldin (2001) use the term "meta–affect" to denote the process of monitoring and regulation of emotions.

Many adults (and also students) develop strategies to handle negative feelings in relation to mathematics (In (Schlöglmann, 1999) I called these strategies “replacement strategies” because understanding is replaced by other strategies). A widespread strategy is "avoid using mathematics", and is particularly commonly used in situations encountered in everyday life. Many adults avoid using mathematical methods even if their use would be very important. The avoidance strategy is not possible, however, when one is forced to learn mathematics - for instance, in retraining courses – in which case another strategy is sought. I believe this also happens with many students at school. Underpinning the search for a strategy for coping with mathematical tasks is the student's belief that mathematics is not understandable for them, together with the imperative that they must eventually pass mathematics exams. Mathematical exams often involve problem-solving tasks, and the student develops their strategy on the basis of the problem-solving tasks they encounter. The strategy can be: "Always use the algorithm you learned last time, and put in all the numbers you find in the problem definition." "The first number in the task instructions must always be the first to be inserted into the algorithm." "Always try to write something even if you don't know what." The last strategy here has an equivalent in oral exams: given some question, students often say whatever they have on their mind, even if the answer has no connection with the question. All these strategies have one aim: to handle a highly emotional situation (Schlöglmann, 2002b).

Finally, a brief look at the research methods used in mathematics education. All methods based on linguistic information work with the explicit memory system. The following are cognitive facts: remembrances of a certain element of curricular content, event etc.; and knowledge of emotional reactions aroused by that particular element of curricular content or event. If we later contemplate a particular situation, the explicit memory receives an emotional coloration through an implicit arousal of emotion (Le Doux, 1998; p.203). This emotional coloration is always activated when the cognitive remembrance is activated, and has consequences for the "mathematical world view of a person". Humans also have a tendency to insert meanings that are constructed on the basis of personal experiences within a meaning system that is shared by a group. (Damasio distinguishes between primary or universal emotions, secondary or social emotions and background emotions (Damasio, 1999, p.50-51). This explains, on the one hand, observed situational outbreaks of emotion (Goldin, 2000) similar in all cultures (Ekman, 1980); and, on the other hand, the observed expression of emotional states dependent on the person, culture or group.) Therefore, in research into affect, gender, social class and group effects have been observed. For a discussion apropos the problem of "credibility" of explicit memory content, I refer to Schlöglmann (2002a).

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