THE SPECIALIZED STRUCTURAL SYSTEMS AND MATHEMATICAL PERFORMANCE

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The aim of this study is to investigate the relationship between the Specialized Structural Systems (SSS) (Demetriou, 1993) and their effect on mathematical performance. Four hundred and eight students between the ages of 12 to 18 years were tested with a series of task batteries addressed to the abilities involved in the SSSs: (1) the qualitative-analytic, (2) the causal-experimental, (3) the spatial-imaginal, and (4) the verbal-propositional. The main purpose of the study was to investigate how these SSSs relate to mathematical performance across ages. The results suggest that the SSSs significantly affect mathematical performance independently of the age of the individuals.

INTRODUCTION

This paper presents a study designed to uncover the structure and development of Specialized Structural Systems (SSS) and their effects on mathematical performance. The design of the study has been inspired by experiential structuralism, and especially by Demetriou's theory of organization and development of human cognition (Demetriou, Efklides, & Platsidou 1993). The study is part of an extended research project, which aims to describe the nature and interpatterning of the various systems and subsystems which, according to Demetriou, constitute human development.

Here we consider the cognitive mechanisms available to individuals, which may enable individuals to operate successfully in different mathematical activities. It is a common tradition within the field of mathematics education to consider mathematical achievement in relation to one cognitive function such as spatial ability or mathematical reasoning (Brown & Presmeg, 1993; English, 1997). The underlying assumption is that in order to carry out an in-depth investigation in why one succeeds in mathematics we have to break cognition in small manageable pieces. There is hardly any evidence of mathematics education research looking at the whole spectrum of cognition and how specialised structural systems interrelate and affect mathematical performance. This paper aims to discuss the interrelationship between different specialized structural systems and the mathematical performance of individuals between the ages of 12 to 18 years. It may be that the overemphasis on one of these structures or the interrelationship of some of them may reveal some useful insight into the cognitive development of mathematical concepts.

In the first part of this article, we summarize the general premises of Demetriou's theory, and in the second part we present the structure and the effects of the SSS abilities on mathematics performance.

EXPERIENTIAL STUCTURALISM: THEORY OF THE ARCHITECTURE OF DEVELOPING INTELLECT

According to Demetriou's et al. theory (2002), the human mind is organized into three levels. The first involves a set of environment-oriented SSSs. These are conceived as sets of specialized abilities which enable a person to represent, mentally manipulate, and understand specific domains of reality and knowledge. Demetriou identified five SSSs (1) the qualitative-analytic, (2) the quantitative-relational, (3) the causal-experimental, (4) the spatial-imaginal, and (5) the verbal-propositional. The second level involves a set of higher-order control structures governing self-understanding, self-monitoring, and self-regulation (hypercognitive system). The third level of the mind involves processes and functions underlying the processing of information. This is regarded as the dynamic field where information is presented and processed by the thinker for the necessary time-span, in order to make sense of the information and accomplish the problem-solving tasks.

Although the SSSs, the hypercognitive and information processing systems are considered to be at a constant interplay during development, in this study we only explore the structure and the effects of the qualitative-analytic, causal-experimental, spatial-imaginal, and verbal-propositional SSS on the mathematical performance of students. The first step towards this approach is to present the components of each of the SSSs that are hypothesized to trigger the differences in the mathematical performance of individuals between the ages of 12-18 years.

The qualitative-analytic SSS

The qualitative-analytic SSS specializes on the representation and processing of similarity and difference relations (Demetriou et al., 1993). Its functioning is based on the specification and disentangling of the properties that may co-define the mathematical objects. The abilities required in the qualitative-analytic SSS contribute to the understanding of mathematical concepts that are characterized by the inclusion relations connecting the elements of a hierarchy by the horizontal or intersection relations or by the sequential and dimensional structure .

The causal-experimental SSS

The causal-experimental SSS is applied on causal reality structures. The abilities related to this SSS are: (a) Combinatorial abilities form the cornerstone of this SSS, (b) Hypothesis formation abilities enable the individual to induce predictions about possible causal connections on the basis of data patterns, (c) Experimentation abilities enable the individual to “materialize” hypotheses in the form of experiments, (d) Model construction abilities enable the individual to properly map the results of experimentation with the original hypothesis in order to reach an acceptable interpretation of the data.

The spatial-imaginal SSS

The spatial-imaginal SSS is directed to those aspects of reality which can be visualized by the "mind's eye" as integral wholes and processed as such. This system involves abilities such as mental rotation, image integration, and image reconstruction. This SSS comes out of and directs the activities which are related to location, orientation and experimentation in space.

The verbal-propositional SSS

The verbal-propositional SSS is concerned with the formal relations between mental elements. The main characteristic of this SSS is the ability to differentiate the contextual from the formal elements of a series of statements and operate on the latter. This SSS deals more with reasoning, including induction and deduction and involves the abilities of distinguishing between the contextual and formal elements, discarding irrelevant information, and the abilities of constructing meaning.

In the present study we are investigating the impact of the four SSSs described above on mathematical performance. Therefore we consider mathematical performance as a dependent variable, while the rest of SSSs are considered as independent variables.

THE PRESENT STUDY

The main purpose of the present study is to provide a theoretical framework explaining the mathematical abilities of individuals. To this end, we propose a model, which is based on the theory articulated above. More specifically, the proposed model involves four factors that match the qualitative-analytic, causal-experimental, spatial-imaginal, and verbal-propositional SSSs, respectively, and it is hypothesized that these SSSs exert significant influence on individuals' mathematical performance. This influence is also hypothesized to persist over age, indicating that SSS abilities constitute a decisive factor explaining the mathematical abilities of individuals. More specifically the present study addresses the following questions:

(a) Does the proposed theoretical model adequately explain the relationships between the SSSs and mathematical performance?

(b) Is the influence exerted by the SSSs on individuals' mathematical performance age sensitive?

The first question of this study refers to the validation of the theoretical model explaining the effects of the SSSs on the development of individuals' mathematical abilities. The second, refers to the investigation of how SSS abilities influence individuals' mathematical abilities as they grow older.

The Proposed Model

The questions raised above are addressed through the estimation of a theoretically informed multivariate causal model in which the hypothesized relationships between the factors are decomposed through the introduction of psychological constructs. The proposed model is based on the theoretical assumption that mathematical performance is influenced by the SSSs. The variables that are considered to mediate the proposed relationships between SSSs and mathematical performance are presented in Fig. 1. The proposed model consists of seven first-order factors and two second-order factors. The four of the seven first-order factors are the qualitative-analytic SSS (F1), the causal-experimental SSS (F2), the spatial-imaginal SSS (F3), and the verbal-propositional SSS (F4). The remaining three first order factors represent the individuals' performance on proportional reasoning (F5), algebra (F6), and mathematical relationships (F7). The model also involves two second order factors; the SSS (F8), which is hypothesized to account for any correlation or covariance between the four first order factors (F1-F4), and the mathematical performance (F9), which is hypothesized to account for the correlation of the proportional (F5), algebraic (F6), and mathematical relations (F7). Thus, the second-order factors can be thought of as abstract representations of the overall SSSs and mathematical performance, respectively, since they capture the shared variance across the SSSs and mathematical concepts, and indicate the structure of SSSs and mathematics. Figure 1 illustrates the way in which the various components of the model relate to each other. SSS second-order factor is hypothesized to lead to the mathematical performance second-order factor indicating that SSSs is an independent variable, which affects mathematical understanding.

METHOD

A total of 408 subjects participated in the study. One hundred thirty eight individuals were elementary school graduates (12 years old, group A), hundred and fifty individuals were lower secondary school graduates (15 years old, group B), and hundred and twenty were first year university students (18 years old, group C). The division of the students in the three age groups was necessary in order to investigate whether the relationship between the SSSs and mathematical performance is age sensitive. The 12 and 15-year-old students were randomly selected from schools in Cyprus, while the 18-year-old students were randomly selected amongst the first year students of the University of Cyprus.

The participants were tested with a series of task batteries which addressed two aspects: (i) abilities involved in the SSSs and (ii) mathematical performance. What follows is a brief description of the task batteries used:

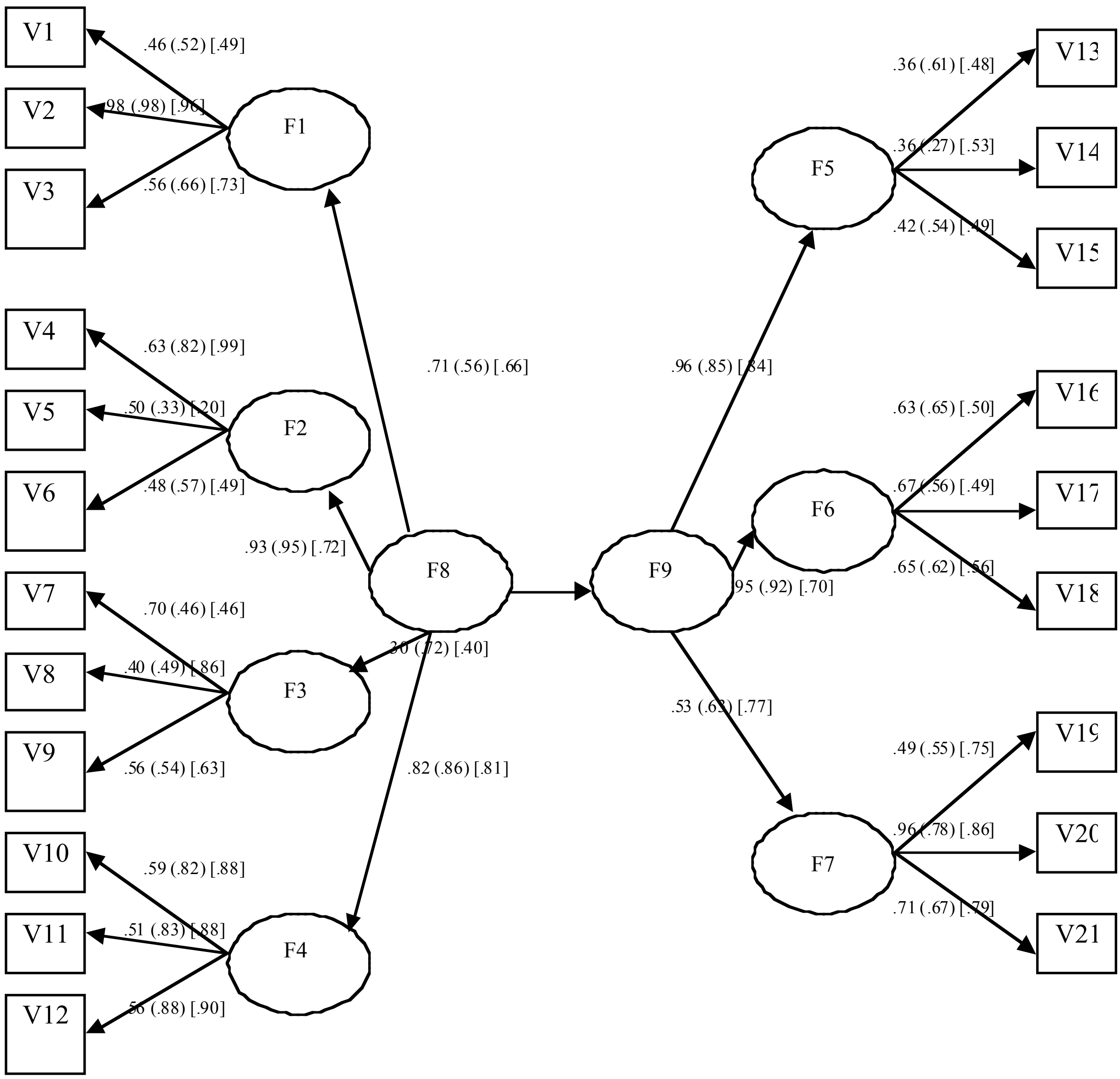
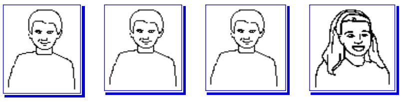


Figure 1: The proposed Model

F1= Qualitative-analytic SSS, F2=Causal-experimental SSS, F3=Spatial-imaginal SSS, F4=Verbal-propositional SSS, F5=Proportional problems, F6=Algebraic proplems, F7=Patterns, F8=SSS, (higher order factor), F9=Mathematical performance, (higher order factor); V1-V21=the tasks in the questionnaire.

The Qualitative-Analytic battery: This battery involved 18 items. The first 6 addressed the participant's ability to understand a verbal analogy such as "ink: pen:: paint ______ (color, brush, paper)". The participant's task was to choose the correct among the three alternatives provided for each missing element. In addition to this, 6 items constituted the answers to pictures which depicted a set of objects. The participant's task was to choose the sentence, which expressed the correct relationship among the objects represented in the pictures as it is shown in Figure 2:



Based on the picture above answer the following question:

Which group is the largest, the group of boys or the group of children?

a) The group of children b) The group of boys c) Equal d) Cannot determine

Figure 2: Example of Task for Qualitative-Analytic SSS

Finally, the remaining questions were made up of a series of drawings or patterns with one piece missing (Raven, 2000). The participant was asked to "fill in the missing piece" as it is illustrated in Figure 3:

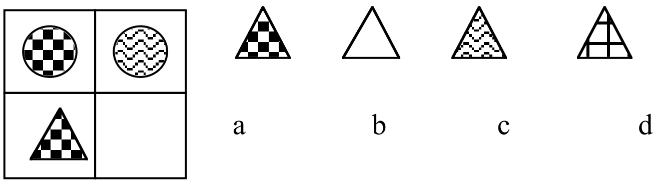


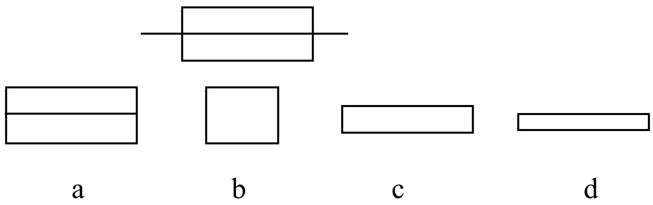
Figure 3: Example of Task for Qualitative-Analytic SSS

Causal-Experimental Battery: The 18 items in this battery examined the combinatorial, hypothesis formation, experimentation and model construction abilities of individuals. An example of this problem: A picture, which illustrates several rods defined by three dimensions, length (long and short rods), width (thin and thick rods), and shape (round and triangular rods) was presented to the participants. The individuals were asked to propose a pair of rods that would constitute a fair test of the hypothesis: "long rods bend more than short rods".

Spatial-Imaginal Battery: The items in this battery addressed the following subconstructs of what we consider spatial-imaginal ability: the visual perception, i.e., the ability to determine spatial relations despite distracting information (The water level tilted task (WLT)), the spatial visualization, i.e. the ability to manipulate complex spatial information when several stages are needed to produce the correct solution (Paper folding activities), the mental rotation, i.e., the ability to rotate, in imagination, accurately two- or three-dimensional pictorially presented objects (The rotation of a three dimensional object and the arrows problem), and the spatial orientation, i.e., the ability to rotate oneself in space relative to object and events and the awareness of self-location (Reber, 1985). Three examples of tasks investigating the abilities involved in the Spatial-Imaginal SSS are illustrated in Figure 4:

(a) A bottle is standing up and it contains some liquid. The surface of the liquid is parallel to the horizontal surface on which the bottle is standing. Draw the surface of the liquid inside the bottle when the bottle is tilted 45°.

(b) What will the piece of paper look like if it is folded around the horizontal axis?



(c) What will the shape on the vertical hand look like if it is rotated and placed on top of the horizontal hand?

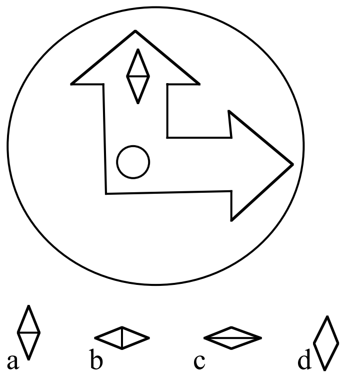


Figure 4: Examples of Tasks for the Spatial-Imaginal SSS

Verbal-propositional Battery: The items in this battery addressed the participant's ability to infer a conclusion from two premises based on logical rules. It involved 12 syllogisms; each of the syllogisms involved two premises and one conclusion and participants were asked to judge the validity of the conclusion (e.g. Birds can fly. Elephants are birds. Elephants can fly). The validity of the conclusion must be based solely on true or false logical terms involved in the argument. The syllogisms involved in the test addressed implication and transitivity.

Mathematical Performance battery: The individuals' mathematical performance was measured based on their responses in three kinds of problems: proportional problems (7 items), algebraic equations (7 items), and completion of numerical patterns (6 items).

The models tested contained both observed (measured) variables and latent constructs. The observed variables are specified as indicators for each of the latent constructs. All the constructs are measured with three indicators. We reduced the large number of raw scores to a limited number of representative scores by selecting the variables with the highest factor loadings. This was done to increase the reliability of the measures fed into the analysis and thus to facilitate the identification of latent variables (Bentler, 1993).

RESULTS

The assessment of the proposed model was based on a multi-sample covariance structures analysis, which is part of a more general class of approaches called structural equation modeling. One of the most widely used structural equation modeling computers programs, EQS 6.0, was used in this study to test for model fitting. In order to evaluate model fit, three fit indices were computed; the chi-square to its degree of freedom ratio (_2/df), the comparative fit index (CFI), and the root mean square error approximation (RMSEA). These three indices recognized that observed values for _2/df <2, values for CFI>.9, and RMSEA values close to 0 are needed to support model fit (Marcoulides & Hershberger, 1997).

The main purpose of the present study was to test the structure of the relationships between performance in mathematics and the SSSs. Although different patterns of relationships can be argued theoretically, there are no empirical studies, which examine the relationships or the causal ordering of SSSs that interact with mathematical performance. Because of the lack of clear empirical evidence supporting the above assumption, the data was analyzed based on the theoretical model appearing in Fig. 1.

In this study, we posited an initial structure and tested the ability of a solution based on this structure to fit the data. To answer the first question of the study we tested the ability of the initial model (see Fig. 1) to fit the data for the whole sample of the study with no invariance constraints. The parameter estimates were reasonable for all groups, in that all factor loadings were large and statistically significant and the patterns of correlations were logical and consistent. Moreover, the goodness-of-fit indices were good in relation to typical standards for the total sample and for each group separately. This is in accord with previous psychological studies (Demetriou, et al., 1993) which indicate that the SSSs can be represented by a second order factor, which influences the mathematical performance of individuals.

These results provide good support for the proposed model but do not address the issue of the invariance of the influence of the SSSs on mathematics performance across ages. To this end, we pursued one more test imposing invariance constraint for the regression of SSS systems on mathematical performance across the three age-groups. The introduction of the invariance of regression resulted in a better model, indicating that the path coefficients among latent factors are the same across groups, and that the latent causal process being modeled is similar in the three groups. This means that the SSSs exert a significant influence irrespective of the age of the individuals. The size of the effect of the SSSs on the mathematical performance of individuals is very high. This is indicated by the loadings of the SSSs on mathematical performance in each group (.806 in group A, .912 in group B, and .901 in group C). These results confirm that in a great extent the variance in mathematical performance can be explained by the influence of the SSSs (65% in group A, 83% in group B, and 81% in group C).

DISCUSSION

So far, research conducted within the field of mathematics education has concentrated mainly on the relationship between mathematical performance and a single cognitive factor such as spatial ability, visualisation, verbalization, reasoning etc (Brown & Presmeg, 1993; English, 1997). There is hardly any evidence of mathematics education research attempting to put forward a more comprehensive theory on how mathematical performance may be affected by various systems that constitute the organization and development of human cognition.

In Demetriou's theory, mathematical performance is part of the SSSs in conjunction with the qualitative analytic, the causal experimental, the spatial-imaginal and the verbal propositional SSSs. In this study, one of our aims was to investigate how the SSSs influence mathematics performance, and thus we considered mathematical performance as a dependent variable, while the rest of SSSs were considered as independent variables. Through the confirmatory factor analysis, it was shown that the qualitative analytic, causal experimental, spatial-imaginal and verbal prepositional abilities are subsystems belonging to one higher order general factor represented in the proposed model as SSS. In the same way, it was found that proportional problems, algebraic reasoning and mathematical relationships are subsystems to another higher general factor, the mathematical performance of individuals. The results suggest that the mathematical performance is significantly affected by the SSSs.

Our second aim was to explore whether the effect of the SSSs on mathematical performance is age sensitive. The findings suggest that SSSs are an important antecedent of individuals' mathematics performance independently of their age.

CONCLUSIONS

In this paper, we have introduced a set of psychological factors that seem to influence mathematical understanding. These factors are based on sound psychological research and their exploration has potentially a lot to offer in mathematics education research. This study is only a small sample of what such an endeavor has to offer.

The results obtained in this study lead to a number of general conclusions, which may extend our knowledge in regard to the structure and development of mathematical knowledge and the influence that psychological factors such as the SSSs can exert on mathematical performance. These finding as a whole may have some serious implications for mathematics education. For example, much of the mathematics education research focuses on instructional practices that might contribute to learning or facilitate students to overcome their difficulties in understanding specific mathematical concepts. We do not oppose to such approaches or question their value. However, what seems to be implied by the results of this study is that improvement in mathematical performance does not necessarily need to be topic oriented. One may attempt to facilitate mathematical understanding by trying to develop the specialized structured system abilities which affect mathematical performance. It is conjectured that enhancement of these abilities may not only facilitate understanding of one mathematical concept but may result to improvement in a whole spectrum of mathematical topics.

With this study we have hardly scratched the surface of what we believe it is possible to explore by taking into account these factors. It seems warranted that future research in the field ought to focus even more on the SSS abilities as potential moderators of relationships affecting mathematical performance. Furthermore it would be useful to investigate the way in which mathematical performance may be developed or accelerated based on the development of separate specialized structural systems.

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