THEMATIC WORKING GROUP 3

BUILDING STRUCTURES IN MATHEMATICAL KNOWLEDGE

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Remark*. Pearla Nesher, a co-leader of TG3 was not able to be present at the conference in Bellaria, so Graham Littler was asked to take on this responsibility. However, Pearla suggested the structuring of papers of TG3, which we use below. During the conference Dvora Peretz was a great help to the group leaders, and we would like to express our thanks to her for it.

Sixteen papers were submitted to TG3. One was rejected since it did not match the theme of the conference. Two papers, sent by D. de Vries & I. Arnon and B. Sternberg, were not presented at the conference since authors were unable to come to Bellaria. We hope the papers will be presented at another conference in the future.

Therefore thirteen papers were presented during TG3 group meetings in Bellaria. All of them, except one, passed the final reviewing process. The authors of the rejected paper did not use the possibility to give a two-page presentation of their contribution.

The structuring of the papers in the group was suggested by Pearla Nesher and approved by group members. The thirteen accepted papers were divided into the following four sections : S – Structure, C – Concept, P – Processes, and R – Rest. This introduction to the Group's work follows this structure. Each of the sections will be presented separately. First, all the papers in each section are listed, then follows a short overview of each paper.

Since all participants of TG3 had access to all the contributions in advance, only short presentations were made and the majority of the time was given to discussions, which were rich and valuable and helped the authors to improve their papers.

Our attempt to find the common features of all or nearly all the contributions to TG3 resulted in the following list:

1) Creating mathematical structure plays an important and fundamental role in the mental mathematical development of a student, from several points of view;

2) Research in this rich area must cover a lot of cognitive, pedagogical, psychological, sociological and physiological aspects including methodology, developmental models, cognitive mechanisms, communication, metacognition, …;
3) A constructivist approach to learning mathematics is regarded by the majority of researches as more effective than a transmissive one;
4) The quality of the research depends on the methodology used, therefore further development of this methodology is needed;
5) Each tool suitable for the explanation of the process of structuring is based on some phenomena and the list of phenomena used in a particular research characterised its methodological background.

STRUCTURE

S1 Christou C., Demetriou A., Pitta-Pantazi D.: The specialized structural systems and mathematical performance.

S2 Hejny M.: Understanding and structure.

S3 Jirotková D., Littler G.H.: Structures of mathematical thinking.

S4 Meissner H.: Construction mathematical concepts with calculators or computers.

The 'common denominator' of this section is the creating of a new piece of knowledge with respect to the existing mathematical structure. Stress is put on procedure in (S1), on development in (S2), on communication in (S3), and on usage of calculators or computers in (S4).

S1

The problem solved in S1 is indicated in the title of the paper. It concerns 'specialized structural systems (SSS)'. According to this theory, the human mind is organized into three levels. The first one involves a set of environment-oriented SSSs – the set of specialized abilities which enable a person to represent, mentally manipulate, and understand specific domains of reality and knowledge. Four of these SSSs are discussed: the qualitative-analytic, the causal-experimental, the spatial-imaginal, and the verbal-propositional.

S2

A longitudinal study of students' 'knowledge without understanding' terminated in finding that this 'cognitive illness' is the consequence of teaching approaches which do not respect the nature of knowledge acquisition. The author's model of creating new knowledge consists of six stages and the first two – motivation and creating a set of concrete experiences of the future knowledge – are underestimated in traditional teaching.

S3

An analysis of the social and cognitive aspects of communication between a bright student, Gina and a weak student, Ben, resulted in finding several general phenomena. Namely, that for the understanding of such communication it is necessary to consider the projection of one person's knowledge to another person's mind: how person A understands and interprets the speech of person B. In addition, if A and B are students, how their communication is understood by the experimenter. These interesting relations are analysed from social and cognitive aspects. Valuable results are obtained from the different mathematical culture of both students.

S4

Calculators and computers are usually linked to calculations and rarely to concept creation. To describe the knowledge construction in the calculator-computer environment, two terms derived from German are introduced: Darstellung (external representation) and Vorstellung (evoked concept image). Procedures, guess-and-test used in different mathematical areas (additive and multiplicative structures, percentage, functions,...) are analysed deeply. Tools used for the research can be easily adjusted for pedagogical application.

CONCEPT

C1 Ouvrier-Buffet C.: Definition – construction and concept – formation

C2 Stehlíková N.: Building a finite arithmetic structure: interpretation via abstraction in context.

The problem of how a student gets the understanding of a given concept is common to both papers. Both studies are based on the constructivistic approach to learning mathematics: a student, by solving problems constructs new pieces of knowledge. Differences are in the experimental approach and in the methodology used. While paper C1 consider a rich scale of views (from Aristotle to contemporary theories) of the definition of construction, paper C2 is directed to one very new theory.

C1

The research is based on the following experiment: a set of examples and a set of counter-examples of some concept is given to a student and his/her task is to explain the idea of the concept and construct its definition. The chosen term *tree* (from graph theory) gives the opportunity to construct several definitions. The thinking processes of solvers (students from grade 10 to university) are analyzed and classified and a general view of the situation is described. This approach to a new concept brings a good understanding of the concept to the student.

C2

The base for the study is one episode of a long-term process of investigating a non traditional arithmetic structure by university students. The episode concerns one concept (called the *inverse reduction*). This long term process has already been analyzed in a different framework and the results of this analysis were reconsidered from the point of view of the theory of abstraction in context (Dreyfus, Hershkowitz, Schwarz). The theory provided a new framework, which enabled the author to look at her data from a different viewpoint and to describe phenomena which had been vague and implicit. In this study the theory of abstraction in context was applied, for the first time, to the analyses of a long-term investigative work of several students. It was the basis for the construction of a generalised model of the development of the above concept. In the article, the importance of the *need for the construction* and the *need for the new structure* (e.g. inner motivation of a student) is identified as a crucial part of the investigative process.

PROCESSES

P1 Marcos P.,R., Lago O.,Hernámdez L., Jiménez L., Guerrero S.: Secondary pupils' understanding of story division problems with remainders: the influence of the semantic structure and situation of the problems.

P2 Peretz D., Gorodetsky M., Eisenberg T.: A closer look into cognitive abstraction processes in the learning of abstract mathematics.

P3 Pittalis M., Christou C., Papageorgiu E.: Student's ability in solving proportional problems.

P4 Rodd M: Emotion, intention and action for vital mathematical embodiment.

P1

Authors of this paper were asked to present two page version of this contribution. This version was not sent to the group leader.

P2

The goals of the paper cover two aspects: content and methodology. As far as content is concerned, the process of grasping and understanding the abstract concept of *relation* (a subset of the Cartesian product of two sets) is analysed. In the work on methodology, three aspects are emphasised: 1) the broad experimental context, 2) the methods used for monitoring and observing the students' processes of abstraction and 3) the mode of representation chosen for the presentation of experimental data.

P3

The SOLO taxonomy, comprising five levels of cognitive performance (prestructural, uni-structural, multi-structural, relational, and abstrac), is applied to students' processes dealing with the idea of proportionality. On the basis of experimental research, a model indicating the levels of students' understanding of simple ratio and proportional situations is described, and illustrated.

P4

The notion of the internalisation of mathematical structure, investigated previously by Stehlíková and Jirotková, is developed to include the roles of emotion and intention in developing a personal internalisation of mathematics. 'Inner mathematical structure' is related to the emotions, intentions and actions of the mathematics student; emotion and mathematical action are linked through the notion of intention. An application of this theme is applied to students learning with computer algebra systems and, briefly in concluding, to teaching mathematics

REST

R1 Ferri R.,B.: Mathematical thinking styles – an empirical study.

R2 Panaoura A., Philippou G., Christou C.: Young pupils´ metacognitive ability in mathematics.

R3 Singer, M.: How does learning occur – an hypothesis.

These three papers in the last section are linked to the theme of TG3 indirectly: the first one deals with cognitive styles, the second with metacognition and the third with learning processes.

R1

Starting from the ideas of Klein and Burton the author identified four types of mathematical thinking styles: analytic, visual, conceptual and mixed. Experimental data gained from 12 students, are analysed by means of ‘grounded theory’.

R2

The first part of the research presents the difficult problem of measuring metacognition. As the discussion of this contribution showed, the methodology of the research is not yet fully established and the main tool used in the research should be reconsidered. However the topic is interesting and very real.

R3

On the basis of recent findings in neuro-physiology, a learning strategy for primary mathematics is suggested. Three types of training are described and illustrated on the development of arithmetical thinking. This new approach has been applied in schools and the results are promising.

List of contributions

List of Thematic Groups