ARGUMENTATION STRUCTURES IN CLASSROOM PROVING SITUATIONS

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Abstract

This paper focuses on argumentation structures of proving situations in class. Examples of different types of proving discourses which were observed in a comparative study of French and German lessons on the Pythagorean Theorem are presented. These illustrate two kinds of argumentation structures. Describing these discourses shows that there is a need to develop a theoretical framework for describing global argumentation structures of proving processes.

Introduction

Rav reminds us that mathematical proofs involve sequences of claims where the passage from one claim to another is generally not formal.

> "A proof in mainstream mathematics is set forth as a sequence of claims, where the passage from one claim to another is based on drawing consequences on the basis of meanings or through accepted symbol manipulation, not by citing rules of predicate logic" (Rav 1999, p. 13).

This point seems to be particularly relevant in proving processes in learning-teaching contexts. When proving is not formal deductive reasoning it is not evident that passages from one claim to another can be described purely as the recycling of conclusions of one claim as data for the next. This raises the question: What kinds of passages from one claim to another are to be found in informal proving discourses? How do these passages and the whole argumentation structure intertwine? In what ways are these passages negotiated in class? This paper focuses on the question of how the argumentation structure of a proving discourse in class and passages within this discourse can be described.

Argumentation structures

Research on proof and proving has focused on different types of reasoning and argumentations in proving processes of students (Reid 2002, Balacheff 1988). In other research Toulmin's model of arguments (1958) turns out to be a powerful tool to characterise different types of arguments, including formal and informal arguments in class (Krummheuer 1995, Knipping 2002). Pedemonte uses the model to characterise abductive and deductive types of arguments in proving processes of students and analyses the cognitive unity or break in those processes (Pedemonte

2002). In a comparative study on proving processes in French and German lessons on the Pythagorean Theorem, carried out by the author of this article, the functional analysis of arguments exposed in the Toulmin model turned out to be equally fruitful. The analyses of statements in terms of their function within an argument, i.e. as data (D), conclusions (C), warrants (W) and backings (B), helped particularly to single out distinct arguments in proving discourses in ordinary classroom situations.

On the other hand the structure of these arguments as a whole cannot be described in terms of the model. The analyses of proving discourses presented in this paper use the Toulmin model in order to separate different argumentation streams (i.e. steps of argumentation as described in the Toulmin model) as a basis for further analysis of the global argumentation structure. Proving discourses observed in class appeared to have complex and distinct argumentation structures. Argumentation streams (AS) were entwined in more complex ways than a linear chain of argumentation steps. In comparing the different classroom proving discourses that were observed, two types of argumentation structures were singled out. In the following these structures will be discussed and illustrated by two examples. The argumentation structure of the first discourse is metaphorically described as source like, while the second type of structure is characterised as reservoir like.

Source-structure

In proving discourses with a source like argumentation structure, arguments and ideas arise from a variety of origins, like water welling up from many springs. This is illustrated by the following proving discourse from a German lesson on the Pythagorean Theorem. Conjectures and different arguments are discussed in public. False conjectures are eventually disproved, but they are valued as fruitful in the meantime. More than one justification of a statement is appreciated and encouraged by the teacher's open or vague questions. The diversity of justifications characterizes an argumentation structure with parallel streams and diverse lines (see figure 1). Not only the target conclusion, but intermediate statements are justified in various ways. The teacher encourages the students to formulate conjectures which are examined together in class. Students' conjectures are appreciated even when they become publicly contested and refuted. The source like argumentation structure of this lesson is shown in figure 1.

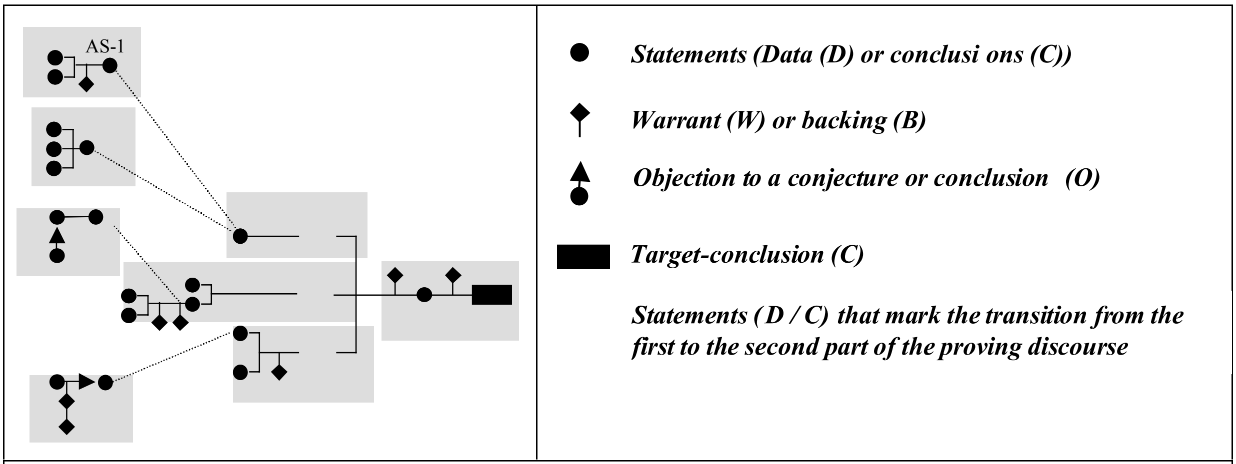


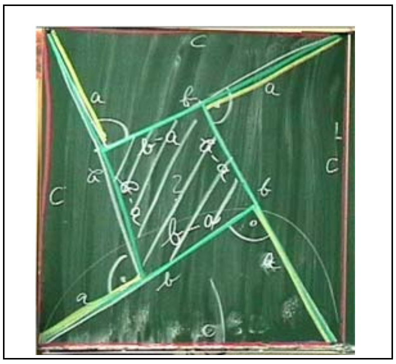
Figure 1 Source like argumentation structure

Eight different argumentation-streams (AS-1 to AS-8) make up the global argumentation structure of this proving discourse. In distinct streams arguments of various types are developed in parallel. These streams do not link statements and conclusions into a chain of arguments, but rather they nourish the global stream of argumentation as many springs nourish a stream.

The first four arguments in the proving discourse are of three different types. AS-1 relies on a pragmatic warrant. The correct drawing of the proof figure is claimed as a warrant for the fact that the inner quadrilateral is a square. Likewise it is argued in AS-2 that the outer shape is a square, based on the drawing. In the next argumentation-stream (AS-3) Maren suggests that the area of the inner square is b_. As is typical for the source like argumentation structure her conjecture is made an object of discussion by the teacher despite its falsity. The conjecture is visually refuted, which involves a different type of argument that can be described as visual-contemplative. In the proving discourse analysed here the teacher and the students develop another visual-contemplative argument which justifies that the area of the outer square is c_ (AS-4) and that the side of the inner square is b-a (AS-5). Pragmatic and visual-contemplative arguments, which are described in more detail in Knipping (2002), characterise the source like argumentation structure of proving discourse. Sascha's conjecture and its pragmatic refutation (AS-6), which is described in more detail below, fosters further informal arguments (AS-7). Finally the argumentation-streams join into a single stream. The discourse is closed by algebraic arguments which lead to the formula of the Pythagorean Theorem.

Conjectures such as AS-3 and AS-6, which are refuted, nonetheless nourish the global argumentation by introducing mathematical ideas which are important to the proof. The following episode, showing the negotiation and refutation of Sascha's conjecture illustrates this particular aspect of the source like structure of the discourse.

Sascha's Conjecture



Referring to the drawing on the blackboard (see figure 2) the teacher encourages the students to formulate a conjecture about the measure of the area of the four right angle triangles in the figure (98-100). Sascha guesses that the four triangles form a square (101-102).

Figure 2 Proof diagram from the German lesson

98 Teacher: Does anybody have an idea about the measure of these, we've noted this

Hat jemand 'ne Idee, was so die, wir haben das ja jetzt hier 'en

99 here a bit unsuitably. What measure do these right angle triangles

bißchen unzufriedenstellend aufgeschrieben. Was so die vier rechtwinkligen Dreiecke

100 perhaps have somehow? Sascha?

vielleicht zusammen irgendwie so für ein Flächenmaß haben? Sascha.

101 Sascha: I would say, they are a square. So, we can form

Die geben auch ein Quadrat, würd‘ ich sagen. Also man kann da draus auch ein

102 a square with them.

Quadrat formen.

The teacher expresses feelings of appreciation when the student comes up with his conjecture (103-106) and encourages the class to confirm or reject this conjecture (106-108). She hands four paper triangles to Sascha (109-114), representing the right angle triangles of the proof figure and asks him to verify his statement (114-118). But then she holds up two of the four paper triangles, showing that she interprets Sascha's conjecture as: 'Two of the right angle triangles form a square' (119). Overlaping the two triangles to make a square with two sides being the two shorter sides of the triangle, she shows that the longer sides extend beyond the square. Thus she argues pragmatically (120-122) that Sascha's conjecture, at least her interpretation of it which gets a public status, does not hold (121).

119 Teacher: So, this is a square? Is this a square? No, why not?

So, ist das ein Quadrat? Ist das ein Quadrat? Nein. Warum,

120 I've been folding and measuring. This side, which would appear twice in this square,

ich hab‘ nämlich gerade geknickt und gemessen, diese Seite, die hier ja in dem Quadrat

121 must fit twice in here, but all this is too much, it’s not a square.

zweimal vorkäme, müßte hier auch zweimal reinpassen, das bleibt aber leider soviel über, es ist kein Quadrat,

122 What a pity.

dumm gelaufen.

Some students seem not to be convinced that this argument is general and validate it privately using their own triangles (123). Again the teacher acknowledges Sascha’s effort (124-126), but she reinforces her argument against his conjecture. Once more she argues pragmatically, referring to the paper triangles, but this time she points out that if the triangles would have been more acute it would have been even more obvious that the two triangles do not form a square (126-129). By putting forward this backing for her warrant she tries to convince the students of her refutation of Sascha’s conjecture (see figure 3).

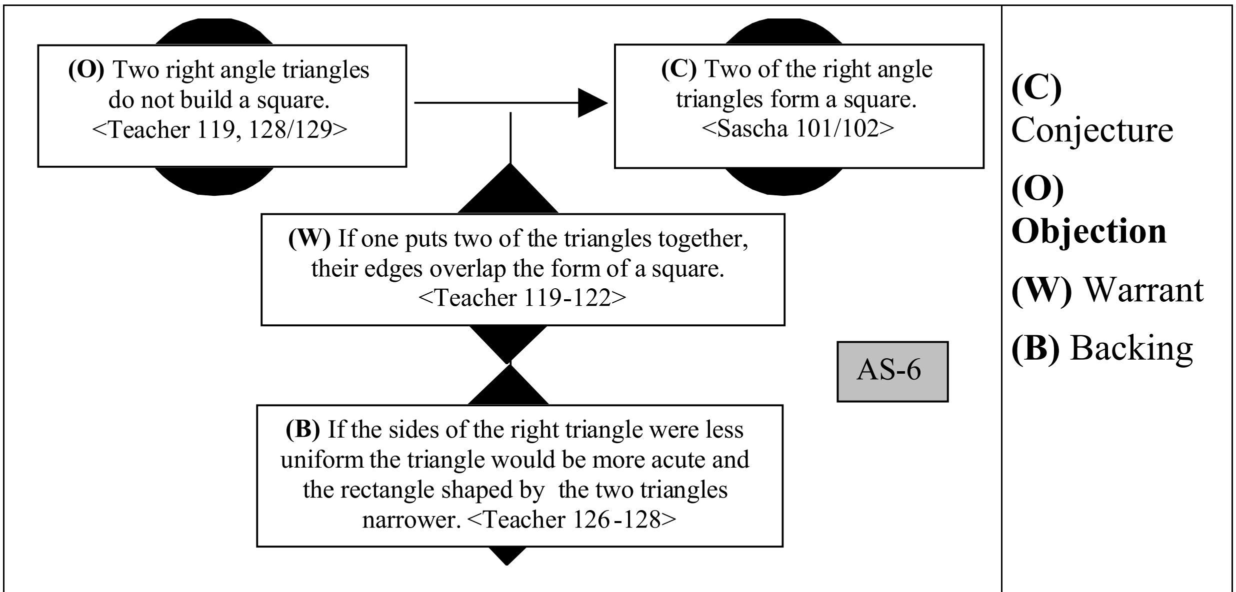


Figure 3 Refutation of Sascha’s conjecture (AS-6)

In the source like argumentation structure refuting a conjecture receives significant attention in the discourse, as is illustrated by the refutation of Sascha’s conjecture. In contrast, in the second type of argumentation structure, the reservoir structure described below, the focus is on deductive elements of the global argument so students’ false conjectures do not become a common issue of discourse.

In the following paragraphs the second type of argumentation structure is discussed through an example from the proving discourse on the Pythagorean Theorem in a French class.

Reservoir-Structure

Argumentations with a reservoir-like structure flow towards intermediate target conclusions that structure the whole argumentation into parts that are distinct and self-contained. The parts that make up the argumentation are like reservoirs that contain and purify water before allowing it to flow on to the next stage. What distinguishes the reservoir like structure from a simple chain of deductive arguments is that abduction allows for moving backwards in a logical structure and then moving forward in deductions again. This is illustrated in the first part of the discourse that is described in the following example. Deductions (see AS-1, AS-2 and AS-3 in figure 4), with the support of an abduction (AS-X), form a closed logical structure which I describe metaphorically as a reservoir.

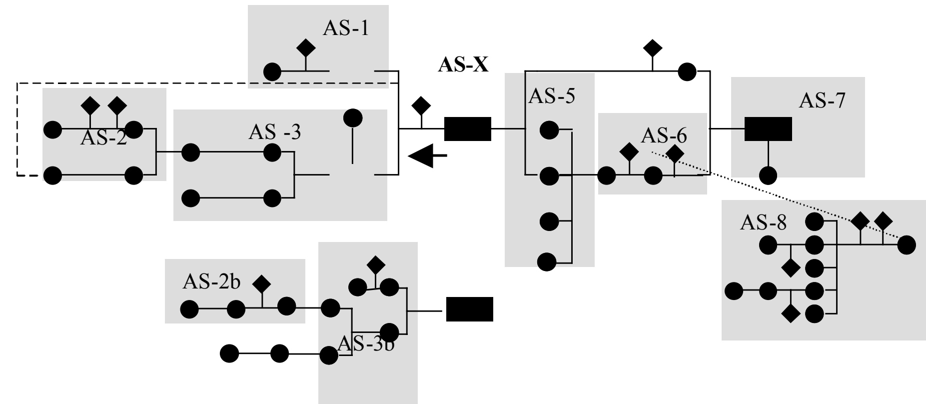
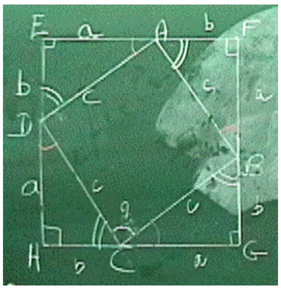


Figure 4 Argumentation structure of the French lesson

A closed structure can also be found in the second part of the discourse, formed by AS-5, AS-6 and AS-7. In contrast to the reservoir like structure in the first part, the argumentation in the second part only flows forwards. The proving discourse is in this part characterised by algebraic reasoning (AS-5 and AS-6), streaming towards the conclusion that a_+b_=c_ (AS-7). In this way this part shows similar features to the second part of the German lesson that has been discussed above. Before comparing these two discourses let us first have a closer look at the abduction as an essential passage in the reservoir like argumentation structure.

In argumentations with a reservoir-like structure initial deductions lead to desired conclusions that demand further support by data. This need is made explicit by an abduction. Abductions allow reasoning backward from a desired conclusion to establish data that further deductions can be based on. Once these data are confirmed further deductions lead reliably to the desired conclusion. This characterises a self-contained argumentation-reservoir that flows forward towards and backwards from a target conclusion.

Abduction



In the first stream of the proving discourse it is deduced that the inner quadrilateral of the proving figure is a rhombus (AS-1, see figure 4 and 6). The reasoning can be characterised as deducing forward towards this conclusion. The deduction is also oriented towards the further target conclusion that the inner figure is a square. This target statement is the starting point of an abduction in the next step that flows backwards: ‘For ABCD to be a square, BCD must be 90°’ (AS-X).

Figure 5 Proving diagram from the French lesson

58 Teacher: Square. So, under what condition is a square, uh, is a rhombus a square?

Carré. Alors, à quelle condition un carré, eh un losange est-il un carré ?

59 Students: If it has, if it has a right angle.

S’il a, s’il a un angle droit

60 Teacher: If it has …?

S’il a… ?

61 Students: A right angle.

Un angle droit.

62 Teacher: A right angle, that’s enough. So, which angle do you choose? You calculate an angle. Ah, yes

Un angle droit, ça suffit. Alors quel angle tu vas prendre ? Tu calcules un angle. Ah, oui mais

63 but you have to prove that. The one you want. Which angle are looking to calculate? That one there?

il faut le démontrer. Celui que tu veux. Quel angle tu vas chercher à calculer ? Celui là . Non,

64 No, don’t mark it a right angle immediately, put a question mark. … So,

non, ne mets pas un angle droit tout de suite, tu mets un point d’interrogation … . Alors qu’est ce

65 what do you suggest? How did you calculate that? You told me that you’ve done that.

tu nous as propose ? Comment tu as fait ce calcul ? …Tu m’as dit que tu l’avais fait.

The backing of this abduction is the definition of a square. This backing leads to conjecturing of a premise: ‘ABCD has a right angle’ (62) of the target conclusion:

'ABCD is a square' (55-58). The students and the teacher make this premise specific: 'BCD is a right angle' (62-64). Therefore the abductive argumentation (schematised in figure 6) makes clear the premises of the target conclusion through reasoning and then matches this concretely with a visual representation, the proving figure on the blackboard (see figure 5).

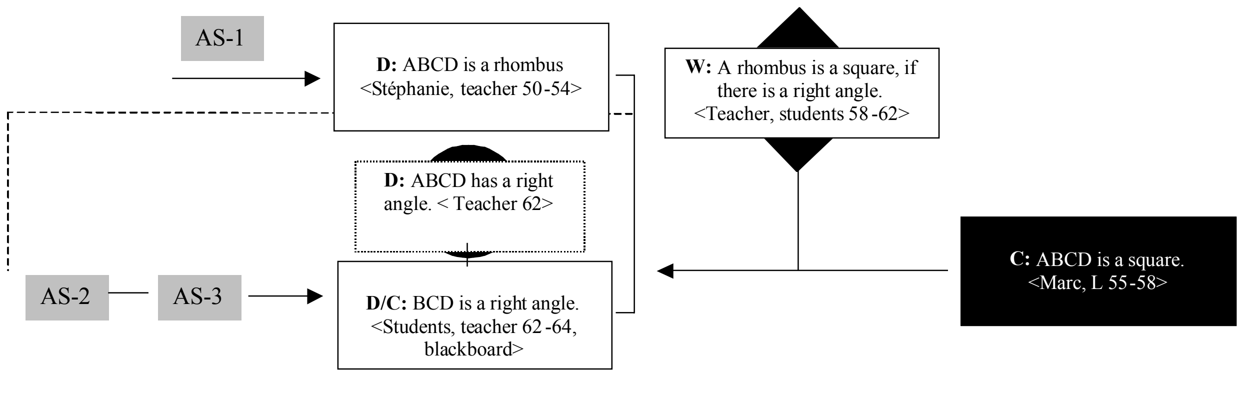


Figure 6 Abduction (AS-X) in the reservoir structure

The conjecture 'BCD is a right angle', supported by the drawing on the blackboard, becomes a target conclusion for further deductions. It is argued that the sum of the acute angles of the given right angle triangle are 90° (AS-2) and as the measure of the corresponding angles in the different triangles are the same the angle HCD and BCG are complementary. This flows to the desired conclusion that BCD is 90° (AS-3). From this the desired conclusion follows immediately.

This initial part of the discourse, which is at first discussed only orally in class, is afterwards fixed on the blackboard, accompanied by further oral comments and considerations (AS-2b and AS-3b, see figure 4). The deductive structure of this written argumentation is similar to the oral argumentation, however the abductive move that is essential to the overall structure is not expressed in the writing.

Conclusion

In this paper two different types of argumentation structures and passages that constitute these types are described. The examples given in this paper illustrate that proving discourses not only differ in the types of arguments used but as well in the global structure of the discourse and the passages from one argument to another. This underlines the need for further research on structures of argumentations. One possible question for further research concerns the role of abductions in argumentation structures. In the lessons described above, abductions were only found in the reservoir like structure. Still, it is an open question whether forms of abduction may also be found in proving discourses similar to that characterised here as source like.

The metaphorical descriptions of the proving discourses given in this paper allow for reflection on underlying functions of proving in class, revealing differences even though the proofs seem to be close from a mathematical viewpoint. In the source like proving discourse, where the function of proving is to get insight into why the theorem is true, many sources are explored. In contrast in the reservoir like discourse, where the function of proving is to justify that the theorem is true, conceptual relations are studied in a more focussed and enclosed way. Additional study of the relationship between functions of proving and argumentation structures would be an interesting point for future research. Here there is a need for a more developed theoretical framework based on empirical research.

References

Balacheff, N. (1988). Une Étude des Processus de Preuve en Mathématique chez les Élèves de Collège. Grenoble: Université Joseph Fourier.

Knipping, C. (2002) Beweisprozesse in der Unterrichtspraxis – Vergleichende Analysen von Mathematikunterricht in Deutschland und Frankreich. Unpublished Dissertation, Department of Education, University of Hamburg.

Krummheuer, G. (1995). The ethnography of argumentation. In: P. Cobb und H. Bauersfeld (Ed.), The Emergence of Mathematical Meaning: Interaction in Classroom Cultures. Hillsdale, NJ.: Lawrence Erlbaum Associates, 229-269.

Pedemonte, B. (2002). Relation between argumentation and proof in mathematics: cognitive unity or break? In: Proceedings of the 2nd Conference of the European Society for Research in Mathematics Education, 2, Marienbad, 2001.

Rav, Y. (1999). Why do we prove theorems? In: Philosophia Mathematica, 7(3), 5-41.

Reid, D. (2002). Conjectures and Refutations in Grade 5 Mathematics. Journal for Research in Mathematics Education, 33(1), 5-29.

Toulmin, S. E. (1958). The uses of argument. Cambridge: Cambridge University Press.