CHILDREN'S ARGUMENTS IN DISCUSSION OF A "DIFFICULT" RATIO PROBLEM: THE ROLE OF A PICTORIAL REPRESENTATION

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In this paper we complement our previous work on ways of facilitating children's performance on ratio and proportion tasks (Misailidou and Williams, 2002). The method used involved the formation of small discussion groups of children that provided different responses to diagnostic test items. Each group was involved in a researcher-guided discussion on the item and the children's arguments were recorded and analysed. Tools that would provoke more productive arguments were used where appropriate. As an example, we present here the case of a group for an item called "Paint". A pictorial representation of the problem was used as a tool to facilitate argumentation. The analysis suggests that such tools can enhance children's problem solving capabilities and provoke learning through argument.

Introduction

Previous research identified common errors and misconceptions that hinder pupils' performance on ratio and proportion tasks (see Tourniaire and Pulos (1985) for a review of relevant literature).

We believe that children's errors and misconceptions can be the starting point for an effective, diagnostically designed, teaching of ratio. Williams and Ryan (2001) demonstrated that argument in small group discussion between conflicting positions can provoke cognitive conflict and that tools and representations can sometimes be critical. Pesci (1998) on the other hand, demonstrated the effectiveness of a discussion-based approach to the construction of proportional reasoning in a classroom setting.

With this paper we explore how selected pupils' groups argue in different multiplicative contexts with the aid of appropriate tools and in which contexts their proportional reasoning skills can be enhanced.

Methodology

Firstly, we developed an instrument using missing value ratio items. Some of the items have been adopted with slight modifications of those used in previous research studies and others have been created based on findings of that research. All the problems were selected having as criterion their "diagnostic value": their potential to provoke a variety of responses from the pupils, including errors stemming from misconceptions already identified in the literature.

Two versions of this instrument were constructed. (Both of these versions can be seen in full on the web at http://www.education.man.ac.uk/lta/cm) The first version ("W Test") contains all the items presented as mere written statements. The second version ("P Test") contains the same items supplemented by "models" thought to be of service to children's proportional reasoning. These models involve pictures, tables and double number lines.

For each item of the test, all the pupils' answers, correct and erroneous, accompanied, where possible, by the corresponding strategies were recorded. Then the results were subjected to a Rasch analysis using the program Quest, which allowed us to scale the most common errors for each item with its difficulty in the W and P form. Thus, since the forms were equated through common items, we were able to identify items for which there were significant differences in responses in the two forms. For example, there was a highly significant difference in difficulty between the two forms of the item Paint (see below): the pictorial version was much easier.

Then, from selected classes of this sample, we formed discussion groups to work on the diagnostically most challenging items of the test. The pupils were chosen on the basis that they had provided a range of responses on the relevant item. Each group was involved in a researcher-guided discussion on the item and the children's arguments were recorded and analysed. Tools that would provoke more productive arguments were used were appropriate.

We worked following a general methodology that was used by Ryan and Williams (2001) and which was described in our previous work (Misailidou and Williams, 2002). Briefly, the starting point is an unresolved or not trivially resolvable problem, which can be the source of discussion. Children that provided a range of responses to the problem form small groups and are set the task of persuading each other by clear explanation and reasonable argument of their answer. The researcher, adopting the teacher's role, can establish rules for the children's argument in order to facilitate participation in discussion. Moreover, she tries to ensure that the arguments for the errors are clearly voiced and that potentially productive tools are introduced. Finally, children are asked whether and why they have changed their mind and how they would summarise what they have learnt.

We present one case of an item called "Paint". The plain version ("W") of the item was adopted from Tourniaire (1986) and is the following:

Sue and Jenny want to paint together.

They want to use each exactly the same colour.

Sue uses 3 cans of yellow paint and 6 cans of red paint.

Jenny uses 7 cans of yellow paint.

How much red paint does Jenny need?

It was decided that the most appropriate model for this item was a pictorial representation adapted from the textbook series SMP 11-16 (1983).

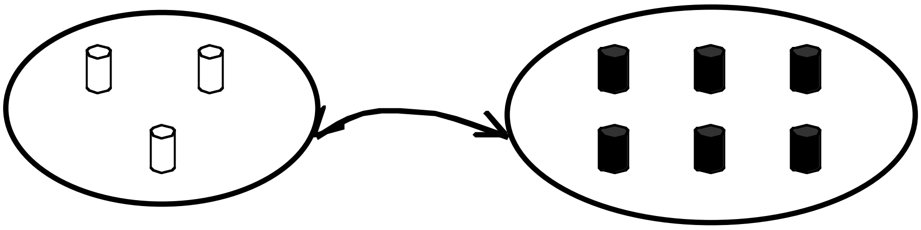
The "models" version of the item is the following:

Sue and Jenny want to paint together.

They want to use each exactly the same colour.

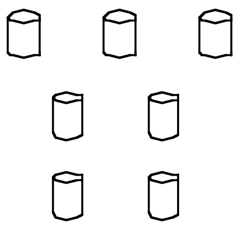
Sue uses 3 cans of yellow paint and 6 cans of red paint.

3 cans of yellow paint *6 cans of red paint*



Jenny uses 7 cans of yellow paint.

7 cans of yellow paint *How many cans of red paint?*



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How much red paint does Jenny need?

Lamon (1989) suggests that the addition of a pictorial representation to a problem may facilitate proportional reasoning something that we have also noticed at the pupils' test scripts. In a difficulty scale output from Quest where the difficulty estimate for each item is given in logits (starting from negative to positive logits for easier to more difficult items respectively) the W form of the paint item has a value of 1.62 (error 0.43) and its P form has a value of –0.86 (error 0.33)! (Roughly speaking, a third of the sample were able enough to do the easier version but not the harder one.)

Three 12 years old pupils, "Ruth", "Cia" and "Ahmed" were selected form the same class to form a "discussion group" because they had provided three different responses on the "non model" version of the item: "2", "10" and "14". The discussion, with the guidance of one of the authors, lasted approximately 50 minutes and was audio taped.

Presentation of the group discussion

Initially, the pupils recalled their response by consulting their test scripts. They were also invited to present an argument for their response to the group.

Ruth explained how she came up with the answer “2”: *“It’s 6 add 3 which equals 9…Then I did…add 2…add on the yellow paint… So that means they use the same amount…”* She used the “constant sum” strategy to answer the problem. She thought that the sum of Sue’s cans should be equal to the sum of Jenny’s cans: 3+6=9 therefore 7+2=9 and so the answer should be 2.

Cia explained her answer “10” as: “Because if Jenny uses 4 more cans of yellow paint than Sue does then I added…4 to 6 to get 10.” She used the “constant difference” or “additive” strategy to obtain her answer. This is a frequently used error strategy where “…the relationship within the ratios is computed by subtracting one term from another, and then the difference is applied to the second ratio.” (Tourniaire & Pulos 1985, p.186) In this particular problem, the answer 10 can be obtained either by thinking that 3+4=7 so 6+4=10 or by thinking that 3+3=6 and so 7+3=10.

Finally, Ahmed explained how he got the answer “14*”: “Jenny just uses more paint generally…if they are painting together they are probably painting the same thing…and since Sue uses 6 cans of yellow paint and Jenny uses 7 cans…I thought the ratio was 3 to 7. So I said…the 3 cans of yellow paint have been doubled. So Sue uses double the cans of yellow paint…but she uses her red…so I did the same with Jenny…since she uses 7 cans of yellow paint. I multiplied it by 2 and got 14 cans of red paint.”*

Ahmed connected the problem with the topic of “ratio” but the ratio 3:7 that he mentioned was not connected with the doubling that he used to find the answer.

After each pupil had presented their method, the researcher made sure that the other two had, in a sense, “understood it” by asking them to repeat it. When she realised that there were difficulties in understanding Cia’s method she distributed the sheet that is shown in Figure 1, as a tool that was supposed to facilitate discussion.

She stressed that pupils could use it as an aid to their explanations and they could write or draw on it anything they wanted. Cia drew the 10 cans on her sheet and with this additional visual help she presented a more elaborated explanation about her answer leading the other two to an understanding of her method:

Cia: *Jenny used 4 more cans of yellow paint than Sue did…*

Interviewer: *Cia said that Jenny used 4 more cans of yellow paint than Sue did. Can we see that on our paper? [and when the others answer yes] Please go on.*

Cia: *So if Jenny adds 4 to 6 then you get 10. Then it makes it like…because Jenny has 4 more of yellow than Sue…she will have 4 more of red…*

Then the researcher invited the pupils to consider all three answers and explanations and share with each other their opinions.

Cia noted*: “I think Ahmed’s is the easiest to work out...2 times 3 is 6, 2 times 7 is 14 You just double it…Ruth’s was harder because if both use the same but they don’t use the same amount…they use the same amount but different amounts of colours…*

mine was all right… because if Jane's is 4 more then she uses 4 more of the other…of red paint and it will be the same."

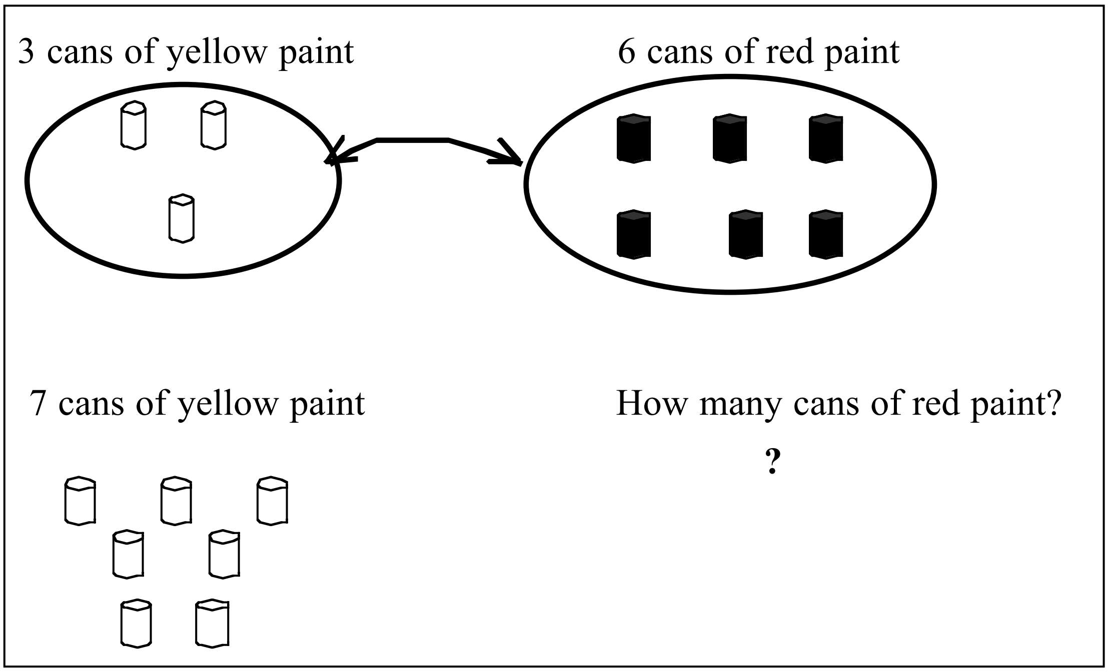

Figure 1: A tool for facilitating discussion

According to Ruth*: "Ahmed's is the easiest and the one that you can make sure that's all right… mine…I answered the question…but I don't think that the answer is right…about Cia's I don't know…because she said that if you get 4 more here you get 4 more there…but in the end Jenny gets more overall…Sue will only get 6 and 3, 9 and Jenny will have 17"*

Then due to Ruth's comment another issue for discussion was brought up:

Interviewer: *What does it matter if altogether these are more than these?*

Ruth: *Because…Sue does half the paint there…and she uses yellow first…[Sue] paints the yellow first.*

Interviewer: *Are they using the colours separately? Are they painting with the yellow first and then with the red?*

Ahmed: *They are painting together…they want to use exactly the same colour…it means like…probably that they are painting all the walls the same.*

Interviewer: *So what colour would be that?*

Ahmed: *Yellow and red put together…it should be orange.*

Cia and Ruth agreed with Ahmed and the researcher did not encourage the discussion on this point any more due to lack of time. But the wording of this specific problem provides opportunities for exploring different interpretations: not mixing paint and not using ratio and proportion to find the answer or mixing paint and using proportional reasoning. After these clarifications, Ruth concluded: ***"**So Cia's is the best in that way because it has got 3 more of that and 3 more of that so in the end it will all be the same colour."* But she could not explain why she preferred Cia's method to Ahmed's. Ahmed stressed that in his method: "I'm using mathematical…I am using proportion and ratio…." and Cia persisted in her additive method as well.

Since both of them could not agree to an answer they tried to elaborate more their justifications. Cia stated that Ahmed's 14 cans are more than the ones needed for the same shade and adopted Ruth's argument for another "proof" of her answer: "*7 add 3 is 10, 3 add 3 is 6…so it's got equal parts of red and yellow paint…"* Ahmed on the other hand introduced a new element to the conversation: ***"**I think…it's 3 to 6…it's double the amount of yellow paint…to produce a certain colour…you know like…if you add more yellow than red you'll get like a darker or a lighter colour…since they want to use the same colour it would be like…Sue is using 3 cans of yellow and they want to use like…the same…"*

The continuation of their disagreement generated more productive arguments. Ahmed in his attempt to convince the others came up with an idea for provoking "cognitive conflict" to them:

***Ahmed:** Since they want to use the same colour lighter or darker but exactly the same…I think…it means that double the amount of whatever colour…if yellow was to be used one [can] it would mean that red paint would be using two to make the same colour but it wouldn't be the same if you added.*

***Researcher:** [On the same time she is drawing the cans] So you are saying that if we had one…we would need two for the method that gives 14 whereas with this method, Ahmed, if we had one we would need how many cans of red? If we were going to use this method? [she shows the additive method]*

***Ruth:** It's 4…she adds 3.*

***Cia:** She should have 4 when the other is supposed to be one….*

***Interviewer:** So what do you think now? Do we have the same colour?*

***Cia:** I think it works either way…but I don't know what the answer is…so…*

***Interviewer:** Here it will be an orange but will it be a reddish orange or a yellowish orange?*

***Ahmed:** A reddish orange*

***Interviewer:** A very reddish orange. Yeah? Here?*

***Ruth:** Less reddish…*

Cia was not yet convinced so finally the researcher attempted a "recourse to zero (0)" strategy:

Interviewer: *[She is drawing on the same time] Let me do it the other way around. Here, if we have 3 cans of red paint with the method that gives us 14 how many cans of yellow paint do we have Ahmed?*

Ahmed: *One and a half.*

Interviewer: *So, here…with this method [she shows the additive method]… if we had three how many cans of yellow would we have? Cia?*

Cia: *If you got 6 and you take 3 you got 3 so now it would be…none.*

Interviewer: *What colour is this?*

Ahmed: *Red.*

Cia concluded that her "addition" method does not work all the time:

Cia: *No, it's not [the same colour]…that one [shows the method that gives 14] is better…that [shows the addition method] could work sometimes…*

At the end of the discussion Cia wrote down "14" as the correct answer and explained "by what" she had changed her mind:

Cia: *By a different method…by the pictures…*

We think that in this case the pictures supported (Ahmed's) arguments about the between measures relationship (the relationship between the yellow and red colour), and that this, supported by the researcher focussing on the extreme case "zero yellow paint", led to the necessary conflict for Cia, which seemed to be the cause of her change of mind.

Conclusion

In previous work, Williams and Ryan (2001) have shown how conflict groups have learnt through discussion, and have pointed out how the teacher can play a critical role in providing significant problems and ensuring that appropriate cultural tools are made available. This approach was shown to work, more specifically, in the context of children's multiplicative argumentation (Misailidou and Williams, 2002). The cultural device highlighted in that work was that of a sharing context and of a pictorial diagram selected to facilitate "grouping" as a predecessor of a multiplicative strategy.

Here, we implement the above approach in a more "difficult" conceptually multiplicative context: the "mixing of paint". The cultural tool that we propose is a context that provokes various interpretations on the "way the paint is used" combined with a pictorial representation that draws attention to the relationship between measures, which affords the crucial argument about (0,3) giving an unacceptable red colour instead of orange.

In conclusion, we find that an apparently successful "change of mind" brought about by cognitive conflict is related to the context (of paint mixing), the pictorial presentation (which draws attention to the necessary between measures relation), and the particular argument (that generalises the incorrect method to the absurd extreme.)

We suggest that in general a theory about the use of cultural tools will need to accommodate the particularities of all these elements, and this reinforces our belief that the pedagogical knowledge that is required is not only content-specific, but is local to the task at hand. This demands a great richness of expertise rather than general strategies, and encourages us to embed as much of this knowledge as possible in tools and artefacts for teachers to plan with.

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