SWEDISH UNIVERSITY ENTRANTS' EXPERIENCES ABOUT AND ATTITUDES TOWARDS PROOFS AND PROVING

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Abstract

About 100 university entrants responded to questions about proofs and proving. The part of the questionnaire, which deals with students' attitudes, experiences and feelings, as well as their actual proving abilities, was analysed. Almost 50 percent of the students stated that they had had teachers who often proved statements but only about 30 percent of them said that they had had the possibility to exercise proving both orally and in writing. Students' attitudes to proofs and the learning of proofs were very positive. However, caution must be exercised when interpreting these figures because of the dichotomous nature of the statements. We must also expect the students to be influenced by the mathematics setting in which the questions were posed. The present survey is the first step of a deeper study to be developed concerning students' culture of proof in school and in basic university courses in Sweden.

1. Introduction

The role of proof in the Swedish schools has presumably diminished since the Swedish curriculum reform in 1969 (Skolöverstyrelsen, 1969), a development, which is similar to that of many other countries (e.g. Hanna, 1995; Niss, 2001; Waring 2001).

In the seventies, university teachers in Sweden noticed that many students in the basic courses were not capable to handle tasks, which dealt with proofs and mathematical theories. Teachers solved the problem by moving a part of such tasks to the upper courses (Boman, 1979). There still exists a gap between mathematics as it is taught in upper secondary school and university mathematics. A lot of students have difficulties in passing the examinations in the basic university courses.

Proofs are an essential part of mathematics and that is why you cannot ignore such activities in mathematics education (Hanna, 1995). Students need a lot of practice in order to develop their understanding of proofs and their abilities to prove statements. The aim of my thesis, of which this paper is a partial outcome, is to describe and characterise the culture of proof in Swedish upper secondary schools and undergraduate university courses in mathematics. I also want to examine if there is continuity between school mathematics and university mathematics concerning proof. Finally, I want to put this analysis in its historical context. This paper deals with university entrants' declared experiences, feelings and attitudes concerning proofs and learning of proofs when they start to study mathematics. Their actual proving abilities are also analysed. Proof in this study does not only refer to students' different ways of verifying statements but also to proofs in all their forms and levels that students meet in the context of school mathematics.

2. Methodology

The methodology of the global study is under consideration. Since different parts of the study vary in character the approaches to the research questions will vary as well. Both quantitative and qualitative methods will be used depending on the specific questions. Quantitative analysis of results gained from survey studies will be supplemented by qualitative analysis of textbooks and of observations of learning situations.

Since the aim of this first part of the study was to get some background information about students' attitudes, feelings and actual proving abilities, a questionnaire was created in order to gain an overview of the students' attitudes towards and experiences of proofs as these were expressed in responses to questions posed in the questionnaire. Thus, the aim of the present study is to give an overall view of the students' attitudes towards proofs, an overview which will be supplemented by in-depth studies carried out within the global project.

3. Research questions

The curriculum changes were reactions against the dominating role of proof and theory and the way proofs were previously taught, which did not make room for discussions (Niss, 2001). But what is the situation in the Swedish mathematics classroom of today? In this paper the focus is on the following questions.

- What are students' attitudes towards proofs and the learning of proofs when they start to study mathematics at the university?
- What are students' declared experiences of the treatment of proof in upper secondary school?
- What capacity do students have to prove some elementary statements?
- Is there any correlation between students' declared experiences and their proving abilities?
- Is there any correlation between students' declared experiences and their feelings?

4. Methods

A questionnaire consisting of two open questions, three multiple-choice questions, 28 dichotomous statements and two proving tasks was designed and distributed to the students at the beginning of their first term at the University of Stockholm in August 2002. One hundred students responded to the questionnaire. In the beginning of the questionnaire we asked the students to describe what they considered was characteristic of proof. After this open question they were asked to choose the proof which best corresponded to their picture of a correct proof among five different alternatives (Hoyles, 1997). The purpose of these questions was, on one hand to help the students enter the context of proving and on the other hand to give us information on their views concerning proofs in order to be sure we were in the same context as the students. These responses are not analysed in this paper.

Questions from some earlier studies were used in the present study. Almeida (2000) used a questionnaire consisting of 16 statements concerning students' declared perceptions of proof for each of which the students had to select one of five responses: strongly agree, agree, no opinion, disagree, and strongly disagree. Eleven of these statements were applied to the dichotomous part of the questionnaire. In this paper two of them, namely "I can't see the point of doing proofs: all the results in the course have already been proved by famous mathematicians." and "If a result in mathematics is obviously true then there is no point in proving it." were analysed besides three other statements concerning students' attitudes. In addition to the eight dichotomous statements concerning students' attitudes and declared experiences one of the multiple-choice questions about students' feelings, was analysed. In the first part of the analysis all students with foreign examinations were excluded.

The proving tasks used in the present study have previously been used by Recio and Godino (2001) and were adapted to be used in this study. The students' answers to proving tasks were classified into five categories.

1. The student gives no answer or the answer is very deficient.
2. The student checks the proposition with examples or draws a geometrical picture.
3. The student justifies the validity of the proposition by using informal reasoning (without algebraic symbols), sometimes combined with examples and visual representations.
4. The student uses algebraic symbols, but fails in operating with them.
5. The student gives a proof, which includes an appropriate symbolisation.

The categories differ in some aspects from those of Recio and Godino. Category 2 consists of their categories 2 and 3. Category 4 of R&G is separated to categories 3 and 4 in this study. This is because very few Swedish students had checked the propositions with examples. In this study there is no clear hierarchy between these categories. However, between the second and the third category there is a qualitative change in students' responses because of the attempts of using more general arguments as well as between the third and the fourth category because of the usage of algebraic symbols. After piloting the questionnaire we had to change the geometry question somewhat and insert a picture of adjacent angles with algebraic symbols. Very few students grasped the question without such a picture.

In order to analyse the correlation between students' declared experiences and their proving abilities the responses were divided into four groups depending on how the students responded to the following statements: "*I have had a possibility to exercise* *proving both orally and in writing in school.*" and "*My teacher in upper secondary school often proved statements to us.*". Group 1 consists of the students who disagreed with both of the statements. Group 2 consists of the students who disagreed with the first statement and agreed with the second statement. Group 3 consists of the students who agreed with the first statement and disagreed with the second one. Finally Group 4 consists of the students who agreed with both of the statements.

In order to analyse the correlation between the students' declared experiences and their feelings the students' responses were classified into three categories. The negative responses consist of alternatives *b) nervous, d) dull, e) insecure* and some of their own descriptions, like "anxious". The positive responses consist of the alternatives *a) curious, c) eager* and some of their own descriptions, like "This will be easy". The mixed group consists of those who had chosen both kinds of alternatives.

In this part of the analysis the students with experiences of mathematics after upper secondary school were excluded and the students with foreign examination were included.

5. A review of relevant research and theory

The role of proof in mathematics education has been studied in several countries during the last twenty years. (e.g. Hanna, 2000; Niss, 2001).

In Sweden such studies are largely lacking. Tomas Bergqvist's doctoral thesis To Explore and Verify in Mathematics (2001) is an exception. In the thesis he deals with the issue of how some pupils at the Swedish upper secondary school verify their solutions. In his analysis Bergqvist uses Balacheff's levels of proving. One observation he makes is that the students both wanted and were capable of using 'higher level reasoning' when verifying their solutions.

Almeida (2000) made a quantitative study of British mathematics undergraduates and described their declared perceptions of proof. He also studied a sub sample of these students and analysed their actual proof perceptions and proof practices and compared these with their declared perceptions. He found some differences between them. In the present study the focus is on students' attitudes towards proofs and the learning of proofs.

In a number of studies researchers have set out to classify pupils' levels of reasoning and proofs (e.g. Balacheff, 1988; Harel & Sowder, 1998). Some researchers, however, question the existence of a universal hierarchy of students' ability of proving (e.g. Hoyles, 1997). Hoyles (1997) argues that hierarchies of this kind (e.g. concrete/abstract or formal/informal) are largely artefacts of methodology. There are huge variations between different countries concerning teaching of proof. It is important to study feelings, teaching and school and home environment in order to find other than purely cognitive reasons to why students' responses may differ (Hoyles, 1997). It is also important to "ensure that the goals for including proof in the curriculum and how these are operationalised are clarified and taken account."

(Hoyles, 1997, p. 7) In the present paper some attempts are made to relate students' different levels of reasoning to the students' feelings about proof and declared experiences of proof making.

Recio and Godino (2001) studied university entrants' mathematical proof schemes and related these schemes to the meaning of mathematical proof in different institutional contexts. One of their main conclusions was that it was difficult for the students to produce deductive mathematical proofs. In this study the students' proof schemes are related to the students' declared experiences of the treatment of proof in upper secondary school.

6. Proof in the Swedish curriculum

The Swedish curriculum does not clearly state the aims of introducing the students to proofs and proving activities. Only the main goals are stated. Local schools and teachers have the possibility of applying these goals in their own way. "The school in its teaching of mathematics should aim to ensure that pupils develop their ability to follow and reason mathematically, as well as present their thought orally and in writing." (Skolverket, 2002, p. 60) One of the Criteria for 'Pass' (lowest mark of a three-level grading scale: Pass, Pass with distinction, Pass with special distinction) for any of the five courses A-E is that "pupils differentiate between guesses and assumptions from given facts, as well as deductions and proof" (pp. 60-66). Furthermore one of the 'Criteria for Pass with special distinction' is that "pupils participate in mathematical discussions and provide mathematical proof, both orally and in writing" (pp. 60-66).

7. Analysis of the results

7.1. Students' attitudes

The results from the present study show that the students have positive attitudes towards proofs and the learning of proofs. If we compare the results with the results obtained by Almeida (2000) we find that the Swedish students tend to view proofs in a positive way. Almeida gave an 'ideal' response to each of the statements, i.e. the response a professional mathematician might have given. The most negative responses (about 2.5 mean with ideal response 5) in Almeida's survey concerned the students' attitudes, namely the two following statements. "I can't see the point of doing proofs: all the results in the course have already been proved by famous mathematicians." "If a result in mathematics is obviously true then there is no point in proving it."

In comparison, only 11 percent and 14 percent respectively of the Swedish students answered yes to these statements. Furthermore, as many as 90 percent of the students agreed with the statement "Proofs help me to understand mathematical connections." Most of the students wanted to learn more about proofs and would have liked to learn more about proofs in upper secondary school. These results can be interpreted in a way that supports the results of Bergqvist: that pupils in upper secondary school want to use 'higher level reasoning'.

However, we must be careful when interpreting these results. Firstly, the questions were of dichotomous nature. Secondly, the questionnaire was distributed to the students at a time when they were about to enter their studies at the mathematics department. We would expect them to be rather positive towards a subject, which they had freely chosen to study.

7.2. Students experiences and solving of the proving tasks

From the responses concerning the students' earlier experiences we notice that 29 percent agreed with the statement: "I have had opportunity to practice proving both orally and in writing in school." According to the students there are still many teachers who often prove statements to pupils because 48 percent agreed with the statement: "My teacher in upper secondary school often used to prove statements to us." Finally, 59 percent agreed with the statement: "I have met different kinds of proofs in school."

If we compare the results of the proving tasks with results from the study of Recio and Godino we find that using particular examples as explanatory arguments was not as popular among the Swedish students as among the Spanish students. When proving the geometry statement only three of the Swedish students used particular examples or visual representations compared to 37.3 percent of the Spanish students. One explanation of this result can be that the Swedish students were conscious of the fact that some examples were not valid as general arguments but unconscious of the means by which to prove the statements correctly. Another reason may be that the students lacked the practical acquaintance of conjecturing and verifying statements in school. Here we would need further investigations.

TABLE 1

Frequencies and percentages of types of answers

Category	Arithmetic problem		Geometry problem	
	Frequency	%	Frequency	%
1	39	44.8	50	57.5
2	10	11.5	3	3.5
3	8	9.2	2	2.3
4	7	8.0	1	1.1
5	23	26.4	31	35.6

7.3. Some correlations

A clear tendency is visible in these results (TABLE 2-3). The students who stated that they lacked experiences of both their own investigations and teachers' lectures of proofs in upper secondary school had very weak results. In this group 65 percent had not even tried to solve the tasks compared to 35 percent in Group 4 who stated that they had had a possibility to practice proving both orally and in writing and that their teachers had often proved statements to them. Furthermore, in Group 1 only 16 percent used the algebraic symbols. In comparison, more than half of the students in Group 4 solved the problem using algebraic symbols. The results of the geometry task were quite similar.

Considering the students' feelings when faced with the proving tasks we also see a tendency, although very slight (TABLE 4).

TABLE 2

Solving of the arithmetic problem in different groups

	Group 1 31		Group 2 23		Group 3 9		Group 4 23	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Category 1	20	65	11	48	2	22	8	35
Category 2	3	10	2	9	1	11	3	12
Category 3	3	10	4	17	1	11	1	4
Category 4	-	0	2	9	2	22	1	4
Category 5	5	16	4	17	3	33	11	48

TABLE 3

Solving of the geometry problem in different groups

	Group 1 31		Group 2 23		Group 3 10		Group 4 23	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Category 1	22	71	15	65	4	40	8	30
Category 2	-	0	2	9	-	0	1	4
Category 3	1	3	1	4	-	0	-	0
Category 4	1	3	-	0	-	0	-	0
Category 5	7	23	5	22	6	60	15	65

TABLE 4

Students' feelings towards proving in different groups

	Group 1		Group 2		Group 3		Group 4	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Negative	14	45	9	41	4	40	7	30
Mixed	7	23	2	9	1	10	2	9
Positive	10	32	10	45	5	50	14	61

8. Discussion

The results show that the students have positive attitudes towards proofs and the learning of proofs but have great difficulties to prove some elementary statements. There is certainly cause for concern when considering the students in Group 1. It has to be pointed out that the students, who begin to study mathematics at the university, often come from the natural science programme, where the focus has been put on advanced mathematics, chemistry and physics. Group 1 might even become bigger if students from other programmes had been given the opportunity to answer the questions. Another interesting question is if the correlations between feelings and abilities or declared experiences and abilities would have been the same if we had posed the questions to natural science students who did not choose to study mathematics at the tertiary level of education.

The main result is that the students who report they have been taught to prove statements at the secondary level of education also report of positive feelings towards proofs. This does not necessarily imply that they have been taught proof in the way they state. It is important to keep in mind that students may have different views and interpretations of the classroom activities than their teachers. In order to get a more varied picture of the actual teaching practices it is important to study local syllabuses, to interview teachers about their ways of introducing students to proofs and to analyse textbooks in search of different ways to present proof-making to students in upper secondary school.

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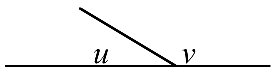
Appendice: The part of the questionnaire which is analysed in this paper

1. Proving tasks: Prove the following statements. A) The difference between the squares of every two consecutive natural number is always an odd number.

   B) The bisectors of any two adjacent angles form a right (90°) angle. (The angle bisector is the ray that splits the angle into two equal parts. *u* and *v* in the picture are adjacent angles.)



2. The question of students' feelings:

   When I get a task starting "Show that…" I most often feel

   a) Curious
   b) Nervous
   c) Eager
   d) Dull
   e) Insecure
   f) Some other way__________________________
   g) I have never got a task like that.

3. Write yes or no after the statements depending on if you agree or not.

   a) I see no meaning with proving; famous mathematicians have already proved all the results. ______
   b) I have had a possibility to practise proving both orally and in writing in school______
   c) I have met different kinds of proofs in school. ______
   d) If a result in mathematics is obviously true there is no point of proving it. ______
   e) I would like to have learned more about proofs in school. ______
   f) My teacher in upper secondary school often proved statements to us. ______
   g) Proof helps me to understand mathematical connections. ______
   h) I would like to learn more about mathematical proof. ______