

SOLVING STRATEGIES IN STATISTICAL TASKS

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Abstract: The role of Statistics is becoming increasingly important in today's society. Collaborative work has shown to be one of the most adapted forms of facilitating knowledge appropriation and the mobilisation of competencies. In a school context we are aware of it. It is a fact that individuals construct explanations and solving strategies by themselves and also when interacting with others. The question is to understand how it works and how we can benefit from it in our classes. This is a research challenge. Responding to the challenge, we analysed the protocols of 136 dyads of a quasi-experimental study. One of the main results show five resolution strategies used by the dyads. In this paper we will discuss some of them.

1 INTRODUCTION

The past decades have brought new challenges for teachers: The educational agenda include contents as well as attitudes and values; new forms of understanding, teaching and learning; compulsory schooling was increased; pupils are more heterogeneous; new challenges brought by technologies and literacy has been considered an essential competency for everyone.

In Portugal, as in most western countries, Statistics started to be included in the set of contents that are taught in the compulsory Maths curricula in the 70s and 80s. Contributions to the search for solutions came not only from authors close to these two fields of knowledge, but also from Psychology, which has developed research that stresses the importance of social interactions in the classroom, namely the role of peer interactions in knowledge appropriation and in the mobilisation of competencies.

2 THEORETICAL FRAMEWORK

In present societies a great deal of numerical information is produced. Consequently, Statistics has become imperious for the challenge of transforming raw data into organised data in order to read and understand reality. Many of the decisions individuals are asked to take have important consequences. As modern societies regulate their citizens' lives through numerical indicators, they force them to appropriate knowledge that helps them understand their meaning and how these indicators come about. Later it forces them to mobilise that knowledge when they have to make decisions.

Having some statistical knowledge has become inevitable in order to practice a critical, reflexive and participant citizenship, since our decisions are based on critical data analyses, whether individually or collectively (Shaughnessy, 1992). Consequently, Statistics has become a part of Mathematics curricula in basic education, integrated in the form of a curricular unit (Gal & Garfield, 1997; Scheaffer, 2000). The process by which children and adults build attitudes and create values regarding many of the choices they will make in personal and social terms is

gradual (Batanero, 2000). They must therefore become critical and reflective towards statistical information, even when they use it inappropriately or in a naïve manner.

In order to reach these goals the emphasis of teaching should not be placed only on knowledge acquisition, but also on the mobilisation of competencies (Abrantes, Serrazina & Oliveira, 1999), that is, on knowing in action. In the case of Statistics, this implies knowing what is present in a statistical study, how to interpret it, how to pose questions, how to select a sample, or which measures are used according to a certain context and a given situation. The task proposed to pupils is what allows them to deal with more or less complex, more or less familiar situations. This is precisely how we can lead pupils to make decisions, run risks and have the opportunity to be meaningfully involved in a problem solving activity. When the task is unusual to the students in terms of either its contents or its working instructions, this facilitates an exchange of resolutions and reasonings and we find a great wealth of solving strategies.

However, tasks alone are not enough. Research has shown how learning situations in a collaborative atmosphere promote better performances in pupils when compared to traditional contexts where we often find individual working situations (Carvalho & César, 2000, 2001). Interaction between pupils gives them the opportunity to elaborate a common representation of the wider task, by exchanging ideas and strategies through the discussion of viewpoints and the sharing of explanations. Moreover, as van der Linden (2000) states, “peers do not learn because they are two, but because they perform some activities that trigger specific learning mechanisms” (p. 6).

In this paper, withdrawn from a larger investigation (Carvalho, 2001), we shall only consider the so-called unusual tasks – those that are unfamiliar to the students. Our main aim was to trigger a rich relational context that would encourage pupils’ mobilisation of cognitive and social competencies. Formed by the nature and instructions of the task, this context creates social interaction norms that account for the emergence of a certain cognitive functioning in pupils and, consequently, a solving strategy for the task.

3 METHOD

The data we are presenting belong to a larger quasi-experimental study (Carvalho, 2001) which compares the effects of collaborative work and unusual tasks in students from 7th grade.

This study was conducted over a period of two school years (1996/1997 and 1997/1998) in two schools in the Lisbon area. During the first year the total sample consisted of 315 subjects (172 girls and 143 boys). Pupils were aged between 11 and 15 at the start of the school year (average age: 12,3; standard deviation: 0,9). The following year, we had 218 subjects – 112 girls and 106 boys, aged between 11 and 15 (average age: 12,8; standard deviation: 0,7).

We used three types of tools: (a) a psychological evaluation test, applied by the researcher at the beginning and end of the school year; (b) two usual Statistics tasks (considered exercises), one pre-test and the other post-test, conducted by a different Maths teacher from the pupils'; (c) three unusual Statistics tasks that the experimental group pupils had to solve during the collaborative work and in the presence of the researcher. Subjects' performance regarding the first two tools allowed us to distribute them in two groups and form the dyads that would work collaboratively, constituting the experimental group. Two of the unusual tasks were conducted in the classroom, so did not audiotape them nor we transcribed the dyads' interaction while working. Therefore, our analysis of these two tasks was exclusively based on the answer sheets of the dyads forming the experimental group. The third unusual task was carried out in another room individually for each pair of pupils' and we recorded their interactions, which were transcribed later on. Besides this recording we analysed the pupils' answer sheets. The study was repeated the following year.

3.1 Analysis of the unusual Statistical tasks

In unusual tasks, pupils cannot find clear guidelines suggesting how to solve them, since the question itself is worded in such a way as to allow various solving possibilities. This is the opposite of what happens in usual tasks, where is explicit what the pupil should do. Therefore, pupils should start by identifying the essential features of the question, in order to mobilise the mental tools most suitable in that situation. Next, they must define a sequence of action, choosing a general strategy. This implies being able to attribute a meaning to the task, identify what they know and find out what is missing to get a solution. These steps, that form the pupils' solutions, produce the process of co-elaboration. While seeking to attribute a sense and a meaning to the unusual task they face, little by little each element of the dyad negotiates a plan to solve it. In this paper we will just analyse some parts of the three unusual statistical tasks.

One of the tasks (Example 1) asked pupils to compare two distributions that were identical except for one value that differed. In another one (Example 2), pupils had to interpret the statistical meaning of five workers' salaries in a firm. The distribution is deliberately asymmetrical, so that pupils are confronted with the values' heterogeneity, thus creating the opportunity to discuss different points of view. Besides, the fact that this question's data referred to workers' incomes introduces a favourable context for pupils' answers to conflict between their statistical knowledge and their social knowledge concerning the economic, social and political problematic related to work. In order to solve this part of the task, the dyad must confront several types of knowledge, thus gathering the necessary information to help solve the task. In the last one (Example 4), the pupils have to find the fifth missing number in order to reach a given average.

4 RESULTS

A strategy is a cognitive activity by which we can process, organise, save or remember knowledge in order to do something. A strategy gives us a possible way to solve a task and a task can have more than one way to be solved. In this paper we will only analyse those found in other investigations (Carvalho, 2001; César, 1994; Gattuso & Mary, 1995).

4.1 Arithmetic strategy

According to César (1994,) in an arithmetic strategy the student uses one of the four basics computations. This was the most frequent strategy in our subjects, since it underlies in most of the knowledge the pupils used when they had to calculate various statistical parameters. Sometimes subjects add supplementary information, resulting from the task demand to justify themselves, which implies discussing and reflecting upon their solutions and questioning the parameters they have chosen. In the following example, we find that the dyad simply calculates the different algorithms of the parameters that are requested (mode, median and average), without comparing them. In this case, the pupils only draw on instrumental knowledge (Skemp, 1978) - do the computations - without resorting to other knowledge that was essential for a more elaborated solution.

Example 1

2. Calculou-se a média, a mediana e a moda dos seguintes dados:
7, 4, 6, 34, 5, 8.
Se os dados forem alterados para: 7, 4, 6, 10, 5, 8 isso iria modificar as medidas calculadas? Porque?

$$\begin{array}{l} \bar{x} = \frac{7+4+6+34+5+8}{6} = \\ = 10,6 \\ \text{Mediana} = \frac{6+34}{2} = 20 \\ \text{moda} = h_{20} \text{ a } h_{21} \end{array} \quad \left| \quad \begin{array}{l} \bar{x} = \frac{7+4+6+10+5+8}{6} \\ = 7,0 \\ \frac{6+10}{2} = 8 \\ \text{moda} = h_{10} \text{ a } h_{11} \end{array} \right.$$

- [2. The average, median and mode were calculated for the following data: 7, 4, 6, 34, 5, 8.
If the data are changed to: 7, 4, 6, 10, 5, 8, would this change the calculated medians? Why?]

This example illustrates that the choice of an arithmetic strategy does not guarantee that the instrumental aspects are controlled (Skemp, 1978): the medians are calculated without previously ordering the data.

In the next example, one of the other unusual task, the dyad uses an arithmetic solving strategy to calculate the average and median, justifying its answer through the qualitative data they have reached, although the interpretation is not right.

Example 2

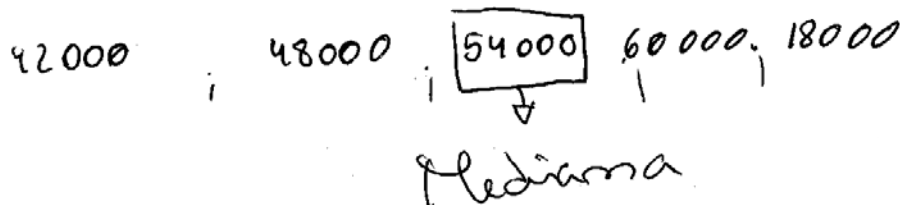
- 2.1 Achas que os cinco empregados estão de acordo quando se disser que a maioria dos empregados dessa empresa tem um salário igual à média? Porquê?

$$\frac{54000 + 42000 + 60000 + 48000 + 18000}{5} = 76800$$

Porque os trabalhadores ~~do~~ fazem o mesmo trabalho.

- [2.1 Do you think the five workers would agree if we told that almost of the firm's workers have salaries that is identical to the average? Why?]

[Because the workers do the same job.]



- 2.4 Escolhas a média ou a mediana para representar os salários dos empregados desta empresa. Porquê? A Média, porque a Mediana apenas mostra um salário enquanto que a Média é a soma de todos os ~~salários~~ e dividido pelo número de salários.

- [2.4 Would you choose the average or the median to represent the incomes of this firm's workers. Why?]

[The average, because the median just shows one income where as the average is the sum of all divided by the number of incomes.]

In Example 3 students doing the same task use an arithmetic solving strategy to calculate the average and median and are also capable of drawing conclusions from the obtained data.

Example 3

2.1 Achas que os cinco empregados estão de acordo quando se disser que a maioria dos empregados dessa empresa tem um salário igual à média? Porquê?

$$\bar{x} = \frac{54000 + 42000 + 60000 + 48000 + 18000}{5} = \frac{384000}{5} = 76800$$

$$\bar{x} = 76800$$

- Não, porque os empregados recebiam menos ou mais que a média.

[2.1 Do you think the five workers would agree if they were told that the most of the firm's workers have a salary that is identical to the average? Why?]

[- No, because the workers make more or less than the average.]

2.4 Escolhas a média ou a mediana para representar os salários dos empregados desta empresa. Porquê?

A mediana porque é o valor mais aproximado dos outros valores dos salários.

[2.4 Would you choose the average or the median to represent the incomes of this firm's workers? Why?]

[The median because it's the value closest to the other values of the incomes.]

A qualitative analysis of the interactions shows us that those who gave answers similar to the one illustrated in Example 3 often drew on their statistical knowledge and their social knowledge, one complementing the other, that is they use what Skemp (1978) calls relational knowledge. This is shown in the dialogue below, between these pupils while they were solving the task.

(...)

63. F.: *What do you think, would they be satisfied to know they make the same as the value of the average?*

64. H.: *Of course not! Their income's nowhere near...*

65. F.: *But in firms people can't all make the same...the bosses, the ones giving orders, must make more money. They're the ones who know what the others have to do...*

66. H.: *That's precisely why they won't agree... with the average...*

67. F.: *But they'll agree more with the value of the median... because it's closer to the values...*

68. H.: *Exactly, closer to the values of their incomes...*

69. F.: *So we'll put the one on top...*

(...)

79. H.: *You know what I think?*

80. F.: *What?*

81. H.: *That maybe being the average or the median depends on something?*

82. F.: *On what?*

83. H.: *Whether you're the boss or the worker.*

84. F.: *Sorry? I didn't get it.*

85. H.: *If you're the worker you want the median, but if you're the boss you'll want the average.*

86. F.: *That makes sense, as the median is less it's good for the worker who wants to make more...*

87. H.: *And as the boss wants to pay less than the average...*

88. F.: *Cool... but the problem refers to the workers so we're right.*

(...).

As we can see, in this case this pupil's argumentation refers to a relational type of knowledge (Skemp, 1978), namely when they say: *If you're the worker you want the median, but if you're the boss you'll want the average.* Relational approach allows for the construction of a scheme of a concept that can be updated whenever this is demanded by new situations, that is, knowledge that can be mobilised in face of new situations and which satisfy the current educational suggestions for the Statistics unit.

4.2 Algebraic strategy

According to César (1994), an “algebraic solving strategy assumes that subjects are able to transform the problem into an equation and to solve it” (p. 257). We find this kind of strategy in situations where pupils have to find out which value is missing when they have some data and the value of the average. The next example illustrates this strategy.

Example 4

$$\frac{15 + 25 + 50 + x}{4} = 25$$

$$15 + 25 + 50 + x = 25 \times 4$$

$$x = 100 - 15 - 25 - 50$$

$$x = 10$$

[The mean of four numbers is 25. Three of those numbers are 15, 25 and 50. Which is the missing number?]

The scarcity of this strategy among our pupils leaves us no choice but to agree with Gattuso and Mary (1995). These authors maintain that there are very few children in the age range of elementary schooling who choose and successfully apply this more abstract type of strategy when they can use a more concrete one, such as the arithmetic strategy. Actually being able to use this strategy would mean these pupils have reached a more advanced stage of their logic development and not just a consequence of the task. This may be a reason to explain that to the same task the students use more times a trial and error strategy, as we can see in the next example.

Example 5

(...)

14. A.: *First we must sum up $15 + 25 + 50$.*

15. C.: *$15 + 25 + 50$ is 90 [She uses the calculator]. If this were the mean of 90, 90 dividing by three would be 30.*

16. A.: *But we must have another number. It's the missing one.*

17. C.: *90 plus what?*

18. A.: *So, how do you calculate the mean? When you calculate the mean you sum up all this [She points to the values] and you divide by the same number.*

19. C.: *How many they are?*

20. A.: *4. So, what you should do is this, $15 + 25 + 50 +$ a missing number and divide by 4 to have 25.*

21. C.: *How much is the missing number?*

22. A.: *I don't know. We must find it adding number to the 90 and divide the total to 4.*

(...)

In this episode from a large interaction we can see that student's know how to calculate a mean. Statistical classroom practices usually stick to applying algorithms and procedures learn through repetition and routine and this fact associated to the logic development of the pupils may explain that these students solved this task with a trial and error strategy merely to apply an algorithm.

5 FINAL REMARKS

The analysis of the examples presented shows how the strategies pupils adopt depend on the statistical knowledge they already have and on mobilising it through the routines they are capable of use or through other information which, together with the task's context, help them attribute a meaning and thus to choose a solving plan. Consequently, the type of tasks proposed and their corresponding instructions play a crucial part and have great potential in terms of educational activities. However, an overview of statistical content in the most popular compulsory education mathematics textbooks in Portugal show that the tasks they propose are often just

variations of traditional exercises. According to Borasi (1986) if we want to change this, we should provide richer sources of tasks of various kinds and learn how to analyse the educational implications of the specific tasks we choose to use in our classes.

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