DEVELOPING CRITICAL SENSE IN GRAPHING[1]

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In current social contexts there are various situations in which people participate in graphing activities. The school has an important role in the teaching of graphing knowledge to citizens. Several researchers have stressed critical sense as an important aspect of the data handling process. This paper reports on a pilot study exploring some tasks in which primary school teachers might approach graphing, using critical sense as an important element. Analysis of the results suggests factors, which may be significant in the design of such tasks.

1 Graphing

As a data handling activity, graphing might be conceptualised as a process by which people can establish relationships between data, and infer information through the construction and interpretation of graphs. If we isolate a graph as a construct, we can enumerate certain elements. For example, Friel, Curcio & Bright (2001) state that graphs share four similar structural components: framework (e.g. axes, scales, grids, references markings), specifiers (visual dimensions used to represent data), labels and background (any colouring, grid, and pictures over which the graph may be superimposed). However, familiarity with these components is not sufficient to ensure understanding of a particular graph: the composition of a graph might not be understood because context may be a key factor in understanding the elements of a graph.

The activity of graphing might be developed in various contexts. Gal (2002) states that data handling activities, including those related to graphing, may happen in two main kinds of contexts: 'enquiry' and 'reading'. In *enquiry* contexts people engage in empirical investigation of actual data. The individuals act as 'data producers' or 'data analysers' and usually have to interpret their own data and results and report their findings and conclusions, e.g. researchers, statisticians, students. The *reading* contexts emerge in everyday situations in which people see and interpret graphs (watching TV, reading newspaper, looking advertisements while shopping, visiting internet sites etc). In those *reading* situations, people encounter the much-heralded "information-laden" environments (Gal, 2002).

In the media, graphs are used to illustrate journalistic arguments and they might be used to emphasize and/or disguise aspects of data (Meira, 1997; Ainley, 2001). However, the specific knowledge of graphing is not the only factor that supports the interpretation of media graphs. Even specialists, who utilise graphs daily as professional tools, might develop interpretations related to the content of graphs, which do not consider important aspects (Monteiro, 2002).

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According to Gal (2002), *reading* contexts of print media graphs demands a certain level of 'statistical literacy' in which readers can interpret, critically evaluate, and comment on statistical information, arguments, and messages. In the activity of interpreting media graphs, adults mobilise diverse skills and types of knowledge (e.g. literacy skills, statistical and mathematical knowledge, beliefs and critical sense).

Several authors have stressed the importance of *critical sense* as part of 'statistical literacy' (Gal, 2002). This means the ability to look behind the data and deeply analyse information and its interrelations rather than simply accepting the initial impression given by the graph. This is a graphing skill related to the role of citizens in society (e.g. Adler, forthcoming; Evans & Rappaport, 1998; Watson, in press).

Let us give an example. We might produce a technically accurate graph. However, that graph might present unrealistic or incoherent data. Then, it is fundamental that the producer or the reader can analyse the graph by articulation between the graphical representations and the content presented by the graph and their previous knowledge. In this case, *critical sense* forms a link between the different aspects involved in graphing situation. The example described is the actual situation for many citizens who need to be able to look critically at statistics presented by different sources, such as governmental statistics about unemployment, inflation, poverty etc (Evans & Rappaport, 1998), where the line between 'mere description' and 'suggestion' may be very fuzzy (Konold & Pollatsek, in press).

1.1 Graphing in School contexts

Several countries have included the teaching of graphing skills as a curriculum topic in primary schools (e.g. England and Wales by National Curriculum, 1989; and Brazil by Parâmetros Curriculares Nacionais, 1997). This official inclusion in national curricula is recognition of another important context in which graphing activities are developed: school contexts.

Despite official inclusion, in some countries the teaching of graphing has been slow to develop. This issue is very complex because is associated with several factors, among those we can highlight the role of teacher.

The teachers play a fundamental role in teaching and learning situations, because the development of 'graphing skills' is not a spontaneous action. They should legitimise students' directions of enquiry, redirect their attention, encourage certain initiatives and discourage others; provoke meaning negotiation, maintain proper articulation of activities and conceptual matters (Nemirovsky & Tierney, 2001; diSessa et al., 1991; Ben-Zvi & Arcavi, 2001).

Therefore, the teacher has an important role in the construction of a teaching context for graphing which should be meaningful and purposeful for participants (Ainley, 2000). Consequently, the teacher needs to guide the pedagogical setting to situations in which statistically relevant aspects are discussed, such as questions related to the critical analysis of data or the necessity for the generation of new and useful information (Ainley, 2001; McClain & Cobb, 2001).

However, these ideal aspects of graphing teaching are not yet a reality in most school contexts. Ainley (2000) underlines that in conventional classroom settings the teaching of graphing emphasises the teaching of several sub-skills by a succession of tasks, such as scaling, drawing axes and plotting points.

Several studies investigated the interpretation of graphs as pedagogical issue. For example, Curcio (1987) assessed fourth and seventh grade students' interpretations of traditional "school" graphs (sic). From the analyses of the answers of students, three types of interpretations were identified. These might be related to different ways in which teachers design data analysis tasks, based on certain types of questions (Curcio and Artzt, 1996; Friel, Bright and Curcio, 1997):

- *Reading the data involves* "lifting" information to answer explicit questions for which the obvious answer is right there in the graph.

- *Reading between the data* involves interpolating and finding relationships in the data presented in a graph. This includes making comparisons (e.g. greater than, greatest, tallest, smallest, etc) as well s applying operations (e.g. addition, subtraction, multiplication, division) to data.

- *Reading beyond the data* involves extrapolating, predicting, or inferring from the representation to answer implicit questions.

Based on these three types of comprehension of graphs, Friel, Bright and Curcio (1997) introduce the idea of graph sense as a range of behaviours, such as: read, describe, interpret, analyse and extrapolate data from graphs. According to these authors, graph sense develops gradually as result of designing graphical displays of data, exploring their use in a variety of contexts, and relating them in ways that are not limited on graph construction or on simple data extraction as the purpose for reading graphs").

Friel, Curcio and Bright (2001) enumerate two reasons for using graphs: analysis and communication. Graphs used for analysis purposes are predominantly tools for detection of important or unusual features in the data. On the other hand, graphs used for communication are defined as pictures intended to convey information about numbers and relationships among numbers. The use of the kinds of "school graphs" which are used within Friel et al's study (for example displaying information about "the number of letters in students' names" or "how many raisins in various boxes) have limited purpose, in terms of analysing or communicating information which relates to interesting problems.

In these specific examples, the term *looking beyond the data* does not imply a need to look critically at the data. In others words, we might look beyond the data (extrapolating, predicting, or inferring from the representation) without being prompted to question the main idea presented in the graph. Outside the school context, the ability to look critically at the graph and the data presented, which we call *critical sense* is something more than *graph sense*, as defined above.

In particular, "because graphs are pervasive in our society and are found in such media as magazines, newspapers, and television, individuals must use graphs to make sense of information structured by and communicated from external sources" (Friel, Curcio and Bright, p.133, 2001). Therefore, the pedagogical approaches with media graphs might contribute for the development of other different skills, including those linked with the development of critical sense in graphing.

1.2 Articulation between outside school and school knowledge/use of graphs

Watson (in press) argues that teachers are enthusiastic about using newspaper articles containing significant mathematical content as pedagogic resources. Unusual and sometimes misleading graphical presentations, which occur in print media, might be excellent examples to motivate and challenge students.

Nevertheless, Adler (forthcoming) argues that the resources in and for school mathematics are drawn from both academic and everyday mathematical practices. Mathematical activity in school is by necessity neither an everyday activity nor the activity of the mathematician. Therefore, the utilization of resources from out-of-school practices produces an important challenge for teachers, because the recontextualisation processes are complicated and sometimes contradictory.

The relationship between 'everyday knowledge' and 'school knowledge' is a complex issue because in the 'school context' the practices are neither *enquiry* activities (as mathematicians' or statisticians' actions), nor ordinary *reader* activities (as citizens' reading print media). In addition, the school graphing activities are not simply a continuation of solving mathematical problems outside school (Ainley, 2000; Gal, 2002; Adler, forthcoming).

2 Exploring critical sense

2.1 Description

In this paper, we report on a pilot study, which explored the approaches to graphing shown by some student teachers, using critical sense as an important element. The study is based on interview tasks, and focuses on how effective these are for supporting the development of critical sense. It will inform a larger study into ways in which primary school teachers can be helped to develop critical sense in their own use of graphing, and awareness of the importance of critical sense in their teaching of graphing.

2.2 Method

The 10 student teachers that took part in the study were from the second year of an undergraduate course and were following different specialisms (mathematics, art, science and English). However, they had all taken a curriculum methods course in primary school mathematics, which included a section on data handling. All the students were female. Each student was interviewed twice, once individually, and then with another student taking the same specialism. In the paired interviews,

students were asked to work on a computer-based task. However, because of lack of space, we focus here only on the individual interviews.

2.3 Individual Interview

In the first interview, each student was asked about their familiarity with media graphs in a *reader* context and their familiarity with computers. In this interview, the participants were also given two tasks based on print media graphs (See Figures 1 and 2).

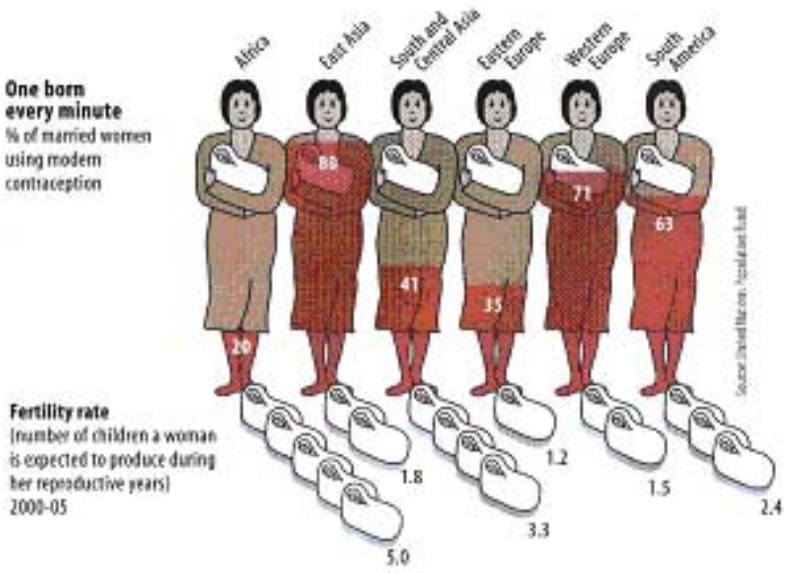

Figure 1: graph reprinted from The World in 2002, The Economist, 2001, p.132.

The second task used two graphs related to road accidents (see Figure 2). Initially, the same question was asked: '*if you could talk to the person who produced this graph, are there any questions you would like to ask?*' The students were then asked to consider the possibility of combining data from both graphs to produce one graph. In addition, they were required to think about how realistic the targets displayed in the both graphs were.

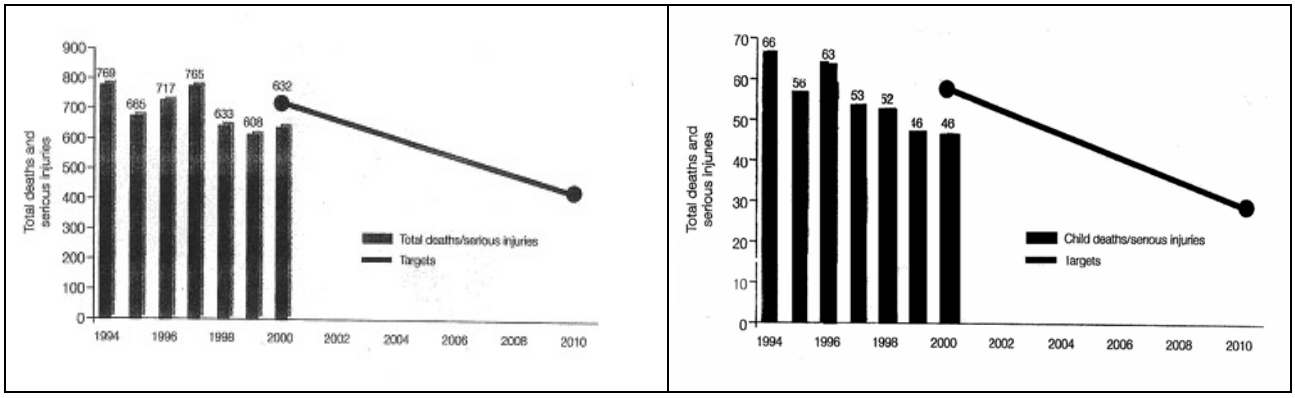

Figure 2: graphs reprinted from *Quality of life in Warwickshire*, September 2001, pp. 93-94.

We chose these graphs for three main reasons. Firstly, we anticipated that the topics associated with the graphs were related to the interests of the students, all

female student teachers living and studying near by Warwickshire. Secondly, these graphs have accessible levels of complexity of mathematical relationships and concepts. Basically, the graphs present absolute and rational numbers, and percentages. Thirdly, the way in which graphs show data might mislead the participants. In particular, we believe that the reading of the two settings of data associated with the pictures of women and babies on figure 1, and the line representative of the targets on figure 2 might be an aspect in which the participants might use their critical sense (or might not).

The graph about contraception and fertility rate (See Figure 1) was the first graph proposed for interpretation. Even though the graph was presented in the context of a magazine, we did not ask the participants to read the article. We use this procedure because the article did not explicitly mention the graph, and from the detailed examination of paragraphs of the article we could enumerate several sub-themes, which were not associated directly with the data related to the graph.

The questions related to the road accidents graphs tried to mobilise aspects of critical sense of the students more effectively. Therefore, initially they were invited to imagine that they could talk to the person who had produced the graph and invited to ask any questions about it, and secondly we asked more specific questions about aspects of data displayed on graphs.

2.4 Analysis of interviews

Half of the students said that they regularly read a newspaper and/or magazines aimed at a female audience. One student was a subscriber of a periodical.

2.4.1 Fertility graph

The students' responses to the fertility graph (Figure 1) were limited. Some students formulated interesting questions, which were mainly related to factors such as the source of data, and the methodology used to collect it. For example Anne (English):

> "Yeah – I'd like to know how they got the pictures in the first place. (...) I'd like to ask about, how they did the survey, the article around it. The actual survey itself. Did they test a small concentrated group? How did they make sure it wasn't varied? Can't see it from the graph (...)"

In general, students' comments regarding the 'fertility graph' focused on the appearance of the graph or technical aspects of production of data, rather than the subject matter itself. For example, they did not ask questions associated with the relationship between the use of contraception for women and rates of fertility in the regions shown by the graph, or about why only data on married women was included.

2.4.2 Road accident graph

The variability of questions here was greater than for the fertility graph. Many of the questions were connected with the conceptual aspects of data presented by the graph, rather than technical aspects. For example, Jackie (Mathematics):

"As I was saying what's serious injury, what's classed as serious injuries? And what age do they class as children? …"

In this task, students were also invited to compare and combine the data that came from both graphs. This necessitated that they (at least visually) manipulated data. The manually drawn graphs produced by the students were an important resource for them in beginning to establish relationships between the data. For example, Caroline (art) drew a line joining the tops of the bars on the first graph, and then added a line to represent the tops of the bars for the children's graph below this.

"I think I'd probably do line graphs more like these ones. Because… then it be like you would use that scale and then it would be easier to show that children are quite a lot less than... I'd have a line joining like these up to show… and then I'd have like the children's ones like down here (drawing on the copy) showing the difference between them… so that there would be a line for those. Probably have to use a different scale like that… And then the targets…then… I'd keep probably keep the same, but not obviously not join the lines… and then you could see the child's target down here. But by doing like that, you're saying that you could compare them more easily. Suppose that when they're on the same graph it would be easier to compare it."

Finally, students were asked whether the targets shown on the graphs were realistic. Different interpretations were given based on the same information. Generally, these considered contextual factors not indicated by the graphs, rather than focussing on the graphs themselves. For instance, Julia (science) referred to the continual rise in the amount of traffic, which was not taken into account by the graph:

R – Do you think that these targets are realistic?

J – Going on the data there, no. Because there is a slight rise… Here… the data stayed the same but … its a lot to achieve… I mean I'd like those deaths reduce like much… but I think it is a quite hopeful target … I don't think it's realistic, no.

R – Why?

J – …Because if I was just going on the data alone… But, personally having children the amount of education that they get on road safety … it just goes straight over their heads you know they still run across roads and with the increase in traffic, the increase in cars… I can't see the correlation of an increase in traffic and the reduction in road accidents, but… That's personally me…

Even though the students were following different specialisms, we could not notice differences at answers of participants related to this aspect.

2.4.3 Comparing the tasks

In general, the comments of most of the students (including Julia, who was herself a mother) were more limited during the interpretation of the fertility graph than in the task using the road accidents graph. Comments on the fertility graph also tended to be concerned with features of the graph itself whereas in commenting on the road accident graphs, students drew on a range of contextual information. It seemed that

during the road accident task, many students were more engaged in the data-handling situation, and their critical sense was activated more strongly. The bar graphs related to road accidents were closely linked with the daily lives of the students, particularly as it came from the region in which they study and/or live. The task was also the second in the interview, when students were more relaxed. However these arguments seems too simplistic to explain why the students demonstrated more critical sense in discussing these graphs.

2.5 Conclusions

We view the interviews in this study not simply as opportunities for data collection, but as situations in which learning and teaching happened. Analysis of the differences between the responses to the two tasks is therefore significant in exploring aspects of the interview tasks, which were important for the development of critical sense. We consider a number of factors.

2.5.1 The nature of the graphs used

The fertility graph is typical of many graphs presented in print media, in that it uses pictorial images related to the subject matter. It is essentially a combination of two bar graphs, showing levels of contraception and fertility rates, but the presentation tends to disguise the fact that two different data sets are being offered for comparison. In contrast, the road accident data are displayed on two separate bar graphs, with no decorative material. This invites comparison between the data sets, although the differences in scale necessitate some level of manipulation.

2.5.2 The questions asked

The initial question asked in both tasks ('*if you could talk to the person who produced this graph, are there any questions you would like to ask?*') was designed elicit critical comment, and also to legitimise such comment and questioning, in contrast to traditional pedagogic settings which are limited to closed reading of graphs.

However, in the road accident task, a further question required the students to make judgements about the reasonableness of the targets. This question seemed to be effective in activating critical sense.

2.5.3 Moving towards an enquiry context

In the fertility graph task students were only involved in reading the graph. The road accidents task required the students to do some simple manipulation in order to produce a graph to combine the two sets of data. This moved them from being simply readers of the graphs towards being more actively involved in data analysis and presentation. In Gal's (2002) terms, this is closer to an *enquiry* context than the *reader* context in which people would generally engage with media graphs. Results from this pilot study support our conjecture that enquiry contexts are more likely to develop the use of critical sense.

2.5.4 The relevance of data content

Although both graphs were chosen because we felt that the data content would be relevant to the participants (all female student teachers), the road accident data seems to have engaged their interest and concern to a far greater extent than the fertility data. The limited data available from this study means that we can do no more than speculate about the reasons for this. Engagement in purposeful activity plays an important role in deriving meaning from content and we conjecture that this will support the development of critical sense. The relationship between engagement with data content and the development of critical approaches will be an important aspect of investigation of further study. At present we simply note that it is not easy, even when considering this factor explicitly, to predict the relevance of data content for particular groups of students.

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