# FAIRNESS IN A SPATIAL COMPUTER ENVIRONMENT

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*This paper focuses on how children (5_ -8 year-old) construct a fair sample space in computer-based game. The paper describes children's constructions of symmetric and asymmetric fairness, the two categories that children employed for the construction of fairness in a computer spatial environment. It illustrates what kinds of intuitions about chance and randomness children bring from their experiences and it refers to their expressions of fairness, while they are involved in a computer-based game. The game offered children the opportunity to manipulate sample space and distribution in order to achieve fairness in their game.*

## 1 INTRODUCTION

The literature on randomness is replete with references to children's and adults' incompetence in dealing with judgements of chance. For example, Kahneman et al. (1982) set out a series of heuristics that, because of their inherent bias and their ubiquity, have been termed by other authors as "misconceptions". Our assumption is that ways can be found for children to express randomness and chance that cannot be predicted by misconceptions. These intuitions can also used for building more formal probabilistic conceptions. Fischbein (1975) claims that intuitions are knowledge from experience that controls an action. This study adopts Fischbein's definition and builds also on diSessa`s (1988) work, which argues that intuitions consist of a number of fragments rather than one or even any small number of integrated structures one might call 'theories'.

In a study of intuitions and fairness, Pratt and Noss (1998) showed how older students than the children of the present study, made sense of dice-based situations. They illustrate how existing intuitions about fairness, often based on actual outcomes, are co-ordinated with new meanings and they derive from interacting with a computational microworld. They also concluded that some of the meanings that their subjects developed were insufficient as ways of making sense of randomness, because this naturalistic meaning-making encouraged a data-oriented view of the world, where Kahneman and Tversky's misconceived heuristics can flourish. These naïve theories of stochastics are abstracted from experience with dice and other kinds of random generators, like cards, during informal games playing. It is here that technology has some real potential, as has been shown, for example, by Wilensky (1997), Pratt (2000), Konold and Pollatsek (2002) who demonstrated how the most obvious traps described by Tversky and Kahneman can, under the appropriate conditions, be circumvented.

This paper will describe how children constructed fairness in a computer-based game that had at its core a spatial metaphor of probability. It will illustrate children's expressions for the construction of a fair environment to explore randomness, which fall into two categories: symmetric and asymmetric fairness.

## 2 SOFTWARE AND METHODOLOGY

The first author guided by the second author designed a game to afford children the opportunity to talk and think about probability in the context of quasi-concrete manipulations.

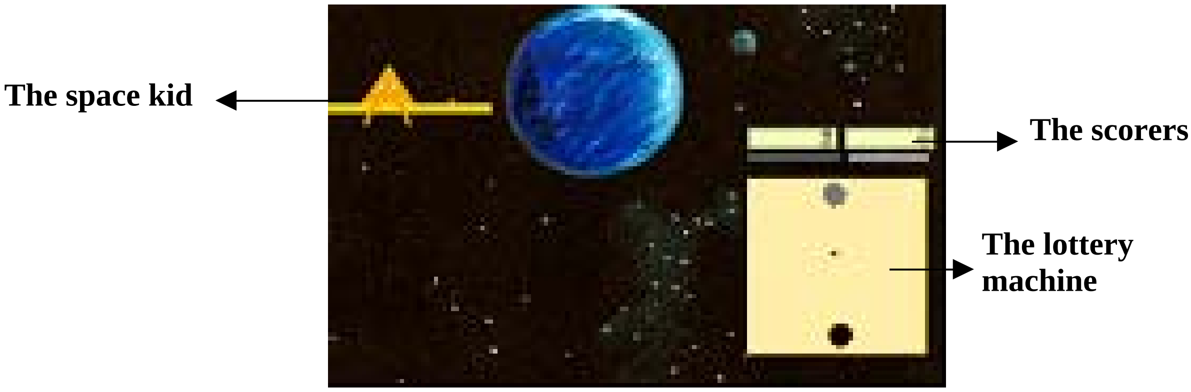


Figure 1: The outlook of the game

A space kid moves upwards and downwards on a yellow line (see Figure 1). A lottery machine, in which a small white ball bounces and collides continually with a set of blue and red balls, controls the movement of the space kid. Children could change and manipulate a number of aspects to construct their own lottery machine: the number, the size and/or the position of the balls. Also, children can create new objects associated with new rules. The game consists of two teams, the blue and the red one. Each collision of the white ball with a blue ball (blue balls appear in the figures as the light grey) adds one point to the 'blue score' and thus moves the space kid one step down the screen (blue is down). Conversely, each collision of the white ball with a re ball (red balls appear in the figures as the dark grey) adds one point to the 'red score' and thus moves the space kid one step up the screen (red is up). Thus, the space kid's vertical location constantly indexes the number of blue and red collisions. Whilst individual collisions can be seen as single trials in a stochastic experiment the totality of these movements gives an aggregated view of the long-term probability of the total events. In order to explore the connections children make between fairness and randomness, we began with a situation in which the children had to try to make the space kid move around a centre line. Within this situation, children tried to construct a 'fair sample space', or otherwise to 'balance' space kid's movement.

The results of the intervention are part of a broader study that adopted a strategy of iterative design, in which the computer-based game was developed iteratively alongside the gathering of evidence for children's use of the tool. The children were interviewed during their interaction with the computer. The role of the researcher was that of participant observer, interacting with the children in order to probe the reasons behind their answers and actions. In the final iteration, 23 children, aged between 5.5 and 8 years, participated. The children each worked with the software individually for duration of between 2 and 3 hours.

## 3 CONSTRUCTIONS OF FAIRNESS

The interviews showed that fairness is a major characteristic expressed by children in their games. As the game consists of two teams, the blue and the red one, children wanted to create a lottery machine where these two teams would get equal points.

Children used many symmetry strategies to construct a game, where two colours had the same probability as a collision event. They first placed the one colour in a symmetrical position to the other by indicating the symmetrical axis with the white ball. Characteristic, students carefully positioned each ball. Mathew (a 6 year-old boy) described his construction:

Mathew: The trick is to put them separately, near the white ball and get the same points, near to each other in the middle. Oh… It gets the red more times…. It needs another 4 points to be equal… Oops… Come on…come on….again…equal! (see Figure 2)

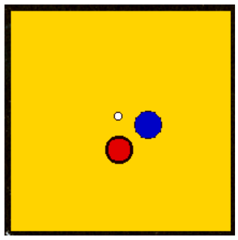

Figure 2

Mathew found that the 'trick' of fairness was to place the two balls in the middle and he put the white ball in a way to show the symmetrical axis between them.

In another strategy children grouped red and blue symmetrically across from each other, thinking of the balls in terms of two separate aggregates. They were concerned about the positioning of a group of the balls, seeing the behaviour of lottery machine as a long term one. Rachel illustrated her symmetrical idea:

Researcher: How did you think about that?

Rachel: I don't know…let me start the game to find out (Figure 3).

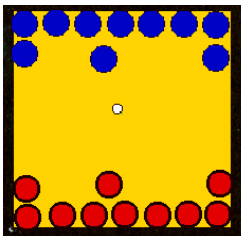

Figure 3

*She starts the game.*

Rachel: I put them like this to touch them easily… …

…Oh! Now, it touches the red more times. May be something went wrong. I will do something else….

She stops the game.

Rachel: I am thinking of something else. I have to copy some red balls…we will find out! (see Figure 4)

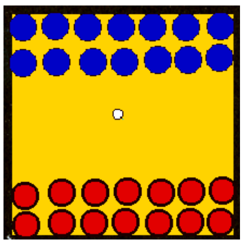

Figure 4

Rachel started by placing the balls symmetrically, but the gaps between the balls did not let this construction work properly on a short-term basis; something that she found in a spatial environment played a role. Continuity in the game gave her the opportunity to evaluate the adequacy of and modify her initial construction in a long-term basis. She finally decided to work with a more common symmetrical construction; she placed the two teams opposite each other, placing the white ball in the middle (see Figure 4). Also, Rachel's consequent increasing of the overall number of balls may be indicative of a burgeoning application of the law of large numbers. That is, the space kid's up-and-down motion is less erratic for larger numbers of balls.

Chris' (7 8/12 year-old boy) idea came from a combination of symmetrical teams and making patterns.

Researcher: What are you doing?
Chris: I am arranging one red, one blue, one red, one blue. I am building a wall! (see Figure 5)

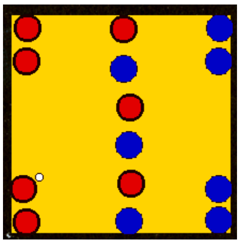
Figure 5

R: Ah! Why did you make a wall?
C: You will see!

Chris first made a lottery machine in which the two tams were spatially separated. He continued this construction by adding a 'wall' of alternately placed red and blue balls. He seemed to construct this wall as an attempt to build two separate sample spaces (see Figure 5). Chris also positioned his white ball in a 'whatever' location, indicating a moving form short-term to long-term behaviour of his lottery machine.

Another symmetrical strategy that children used for the construction of fairness is to make circles and trap the moving balls inside them. In Cathy's (7 6/12 year-old girl) case, a patterned idea was developed into a circle.

Cathy: Let me see…ok! I will put it here… But, will it (the white ball) move where I want? … I will put these two there. Ok! I will put the white ball in the middle to get every ball. It is like a cross! Let's try it out! (see Figure 6)

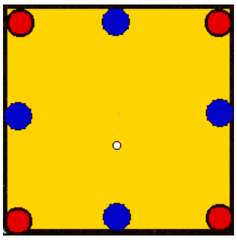
Figure 6

*She starts the game.*

Cathy: …Eh…Let me copy one to have a look…Ok! Another little ball. Move this a little bit. 1,2,3…Let's start it. (see Figure 7)

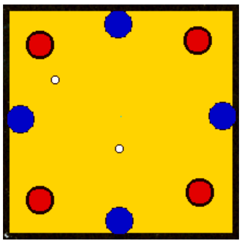
Figure 7

Cathy first decided to make a pattern, a 'cross' as she called it, and tried it out to see whether was working ok. As this didn't work in the short-term basis, she decided to bring the red balls closer, added another white ball, and developed her idea into a patterned circle. The idea of the circle was to make the white balls touch the coloured balls the same number of times. The circular arrangement of balls emerged in an attempt to place the balls equidistantly from the centre.

Children also used asymmetric fairness for the construction of a fair spatial computer environment. This characteristic was observed when children were not concerned about the arrangement of the balls. Fairness in this case was achieved by placing the balls around, by controlling their size and their number. Lucy (7 8/12 year-old girl), for example, based her decision on the movement of the white ball.

Lucy: It (the white ball) will go all the way round. Let's say the white ball will move right, left, in the middle, up, down, on the edges and it will touch all the balls. We have ten balls now… The score will get many points.

Researcher: Which score?

Lucy: The red or the blue one….They might get the same points. The balls are mixed up. (see Figure 8)

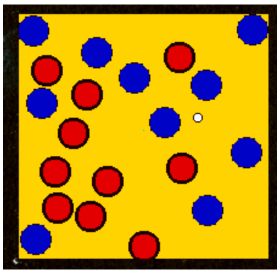
Figure 8

As Lucy couldn't find a pattern for the movement of the white ball, she took the same number and size of blue and white balls and mixed them around. In her game this construction didn't work for a fair result, because the white ball was blocked between the balls and it could not reach all the balls the same times. This led Lucy to make a further construction of mixed the balls, which included many duplicated white balls.

Lucy: I mixed them up…I put them just in a way to be equal. …I will copy some more white balls, here and here. Now, they might get all the balls the same times. (see Figure 9)

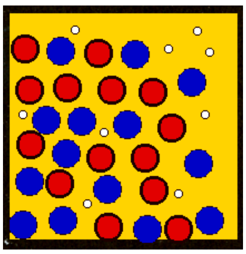
Figure 9

Lucy by building this construction tried to block the white balls in different places in order to get all the 'unreachable' balls. The construction worked over the long-term, to Lucy's delight and satisfaction. Lucy's ingenious response to the unexpected technological constraint further demonstrated her looser conception of the sample space.

George (6 8/12 year-old boy) made a more 'strict' mixture of the balls. He constructed inside his sample space a pyramid without worrying about the number of the balls of each colour.

George: …I have a very good idea. I will make… you will see in a while. I will make a shape… You will see. I will put this here… I am making a shape.
Researcher: Which shape?
G: Something that I forgot its name.
R: Ah! Is it like a pyramid?
G: Yes! I will do it like this. The blue will get the first point and first our space kid will move down and when it will touch the red it will move upwards. (see Figure 10)

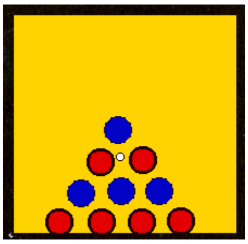

Figure 10

Although George placed more red balls in his construction, he placed them in such a way that some coloured balls would prevent the white ball from touching other balls. Thus, he expected to achieve fairness in his game.

Another construction that falls to the category of asymmetric fairness was Tom's (7 year-old boy). Tom did not have the same number of balls of each colour, but he attempted to compensate for the overall spatial imbalance by modifying the size of the balls.

Tom: I know. To put all these up… (see Figure 11)

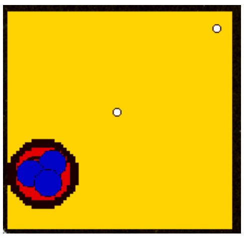

Figure 11

Researcher: How are you arranging them?
T: When it goes to the red to get the blue as well.

Tom's idea showed that he did not think of equality of the two colours in terms of the number of balls, but in terms of the space that each colour occupied inside the sample space. It is obvious that Tom's construction was influenced by the spatial characteristic of the tool environment and he was making a number-space connection.

# 4 DISCUSSION

The paper illustrates that children used one or more strategies to construct their fair environment. Many children tried symmetrical arrangements in their game, and a reason for that might be that their feeling of a fair environment is strongly associated with symmetry. Also, it is remarkable to note that some children's shift from symmetric to asymmetric constructions seemed to be informed by their interaction with the game, In particular, after visualising the arbitrary movement of the white ball in the lottery machine, these students seemed to care less for strict spatial symmetry. Their sense of fairness-as-symmetry seemed to expand so as to include stochastically symmetrical constructions.

The results show that children's thinking moved from finding and describing outcomes to *constructing* models of random behaviour. For the construction of fairness they used their intuition that symmetry represents a fair situation and they worked on that for the construction of their lottery machine. The continuous movement of the ball in the lottery machine gave them the opportunity to judge their ideas through the lens of the global outcomes. Thus, the manipulations and continuity of the game afforded these students a concrete instantiation of their intuitions, and thus an opportunity to 'debug' and develop these intuitions. Technology, in this study provides children with a way of talking and thinking about probability by manipulations. It affords the learner the opportunity to move smoothly between different meanings, derived from actions and language, and simultaneously to building new meanings, a kernel idea of Noss and Hoyles (1996).

The study shows that children have various cognitive resources for constructing a random fair environment. The spatial arrangement, the visualisation and the manipulations in the lottery machine allowed children to judge and build many symmetrical and asymmetrical constructions towards achieving a random yet fair result in their game. Based on this evidence we believe that cognitive resources for making sense of random mixture may be more likely to find a means of expression in continuous two-dimensional movement than in more conventional contexts that involve discrete number.

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