DESIGNING TASKS FOR PURPOSEFUL ALGEBRA

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Introduction In this paper we report on the first part of the *Purposeful Algebraic Activity* project[1], in which we have been developing a sequence of tasks for the learning and teaching of algebra at ages 11-13, based on the use of spreadsheets. These tasks will be used within a longitudinal study of pupils' construction of meaning for algebra as they move from primary school and into the early part of secondary education. This project takes up the challenge set by Sutherland (1991) to create '*a school algebra culture in which pupils find a need for algebraic symbolism*' through the use of a framework of five design principles. The tasks are designed to provide opportunities for pupils to use algebraic notation and ideas in purposeful ways. The five design principles are:

Balance between different types of algebraic activity Kieran (1996) describes three kinds of activities within the scope of school algebra: *generational activities*, *transformational activities* and *global, meta-level activities.* The algebra curriculum in the early stages of secondary schooling is often characterised by an imbalance between these. *Generational activities* are often provided through tasks in which the use of algebraic notation to express a generalised relationship may appear (to pupils) to be an extra task imposed by the teacher, rather than an integral part of the activity. The kinds of problems which are set can often be solved by non-algebraic means, and so the opportunities for *global, meta-level activity* are limited. Attention tends to focus on *transformational activities*. Within our task design we have attempted to achieve a balance of all three sorts of activities within meaningful contexts.

The continuum from arithmetic to algebra Part of the body of research into the learning and teaching of algebra (for example, Herscovics and Linchevski, 1994) has identified a separation between arithmetic and algebra which many pupils fail to negotiate successfully. Other researchers suggest that the relationship between arithmetic and algebra may more usefully be seen as a continuum (for example Carraher, Schliemann & Brinzuela, 2001) and that new technologies may play an important role in supporting the development from arithmetic to algebraic thinking (Sutherland, 1993). In designing tasks we have aimed specifically to exploit pupils' fluency and familiarity with arithmetic structures in expressing generality through algebraic notation.

The spreadsheet as an algebraic environment An important feature of spreadsheets is that the use of the notation has an immediate purpose in producing some kind of result, and thus there is immediate feedback for the user. This contrasts with the use of algebraic symbolism to express relationships in more traditional

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pedagogic approaches, where the only feedback accessible to the pupil may be the teacher's approval. Spreadsheets use an algebra-like notation, in which the cell reference is used ambiguously to name both the physical location of a cell in a column and row, and the information that the cell may contain. The spreadsheet thus offers a strong visual image of the cell as a *number container* whose contents can be changed. However, the image offered by the spreadsheet is ambiguous in another powerful way: when a formula is entered in a column, it can be 'filled down' to operate not just on a single cell, but on a range of cells in a column. The cell reference can then be seen as both specific (the particular number I am going to enter in this cell) and general (all the values I may enter in this column).

Purpose Ainley and Pratt (2002) define a *purposeful* task as one which has a meaningful outcome for the learner, in terms of an actual or virtual product, the solution of an engaging problem, or an argument or justification for a point of view. We use the term *purpose* in a very specific way. The purpose of a task, as perceived by the learner, may be quite distinct from any objectives identified by the teacher, and does not depend on any connection to a 'real world' context. This may, of course, be true in a trivial sense: learners may construct the purpose of any task in ways other than those intended by the teacher. However we consider *purpose* is a distinct overarching element that needs to be considered separately in pedagogic task design.

Utility Alongside *purpose*, we also use Ainley and Pratt's (2002) second construct of *utility* as a design principle. Understanding the *utility* of a mathematical idea is defined as knowing how, when and why that idea is useful. Whilst engaged in a purposeful task, learners may learn to use a particular mathematical idea in ways that allow them to appreciate its utility by applying it in that purposeful context. We believe that opportunities to understand utility can be provided through purposeful tasks. In particular we aim to provide opportunities for understanding the utility of algebraic notation for generating data, finding the value of an unknown, showing structure, and explaining particular results or relationships.

Using the tasks in the classroom Our focus on the design of tasks does not imply that we are overlooking the crucial role played by teachers in developing pupils' understanding of algebra. Throughout the design process we have worked with a group of teachers from both primary and secondary schools. Their input and experience has helped to shape the overall design and presentation of the tasks. During 2002-03 the sequence of tasks will be used within the mathematics curriculum for first-year pupils in two secondary schools. We shall also begin detailed data collection which will allow us to evaluate the effectiveness of the tasks, and hence of the design principles on which we have built.

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