THE CONSTRUCTION OF MEANING OF ALGEBRAIC SYMBOLISM AT DIFFERENT SCHOOL LEVELS. AN EPISTEMOLOGICAL AND DIDACTICAL APPROACH.

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Epistemology and didactics are two different research fields, that not only treat different questions, but also have different methods. The aim of the present work is precisely to investigate how one could make them work together by the means of finding a common point, namely algebraic symbolism, through which each other could be strengthened. Throughout the next paragraphs, we will try to summarise the evolution of our work, by sketching the outline of the progression of its main questions.

Working with mathematical symbolism seems a good starting point from which to try to answer our initial question, that is, how didactics and epistemology can be linked together. First of all, the relation one has to symbolism not only plays a crucial role in learning process, but mastering symbolism is also unquestionably a sign of some mastery of mathematics. Studying the relations students have to symbolism is therefore of major importance in educational research. This importance has besides already been recognised by several authors, providing us with different studies. Secondly, mathematical symbolism has already been the object of epistemological research, furnishing us with sharp analysis about algebraic symbolism. Thus taking into account educational research about algebraic symbolism as well as epistemological works developed so far seems a good starting point from which to confront the two research fields.

A more specific question, in order to answer our prior questioning seems then to arise : what are the relationships between didactics and epistemology as regards the construction of mathematics symbolism?

We started this study by an analysis of several educational research, taking a look on the different approaches each author used to study algebraic symbolism. By doing so, we soon observed that a number of authors have used the history of algebra in their research and developed different uses of it. The first one, which essentially leads to modelling the different stages of the construction of mathematical concepts, is exemplified by Kieran's (1996) and Sfard's (1991, 1994) works and the second one, which helps the author design teaching sequences, is illustrated by Harper's (1987) and Radford's (1996) studies.

Getting further in our analysis, we noticed that history does not seem to be a sufficient tool for analysing the construction of meaning of algebraic symbolism. It does give us some clues for better understanding the different processes through

which algebraic symbols came to light, but it seems essential to go beyond detecting historical stages of construction of algebraic symbolism: it is of the highest importance to study, guided by history, the essence of the symbols.

Among the different educational studies we analysed, we observed that this kind of approach has been adopted by some educational researchers whose work are tinged whith semiotics such as Duval (1995), Drouhard (1992) and Arzarello (2001), who rely more specifically on Frege's distinction between *sense* and *denotation*.

However, Frege did not conceive, *a priori*, mathematics as being the unique application domain of his work, even though he used some mathematical examples to illustrate his talk. Following the philosophical current, it seemed important to us, in order to supplement our prior studies, to take a look at works whose aim is that of specifically analysing mathematical symbols. To do that, we took a look at epistemological studies and more precisely we relied on Serfati's thesis (1997) about the constitution of symbolic language.

In order to analyse the development of mathematics through the development of algebraic symbolism, Serfati identifies six main categories to describe what he calls the different "figures of representation" : the representation of the *requisite* , the representation of the *given*, the representation of elementary instructions, the representation of the entanglement of instructions, the representation of the adequation and the representation of compound concepts. The epistemological study developed by Serfati is not about simple notations of mathematical "concepts". It is also far from being a simple and flat historical description of algebraic symbols. In his work, Serfati is more interested in highlighting the interconnection between mathematical symbols and the evolution of mathematics concepts themselves.

From the epistemological study developed in our work, new questions arise. First of all, do we find a similar categorisation on educational research; that is, more than wondering if didactics research has already dealt with all the "figures", we are interested in knowing if such categorisation is suitable for educational research. For instance, can didactics study the unknown notion without dealing at the same time with the notion of variable? Secondly, regarding the "figures" that educational research has already dealt with, what conclusions have been drawn? Can we establish some links with what epistemology has revealed to us? Third of all, does educational research about mathematical symbolism developed so far present complementary points of view to those revealed by epistemological studies? More specifically, do they suggest another categorisation of the "figures" of representation?

In order to answer some of these questions, we have first collated educational research to the epistemological work mentioned above by comparing, for each "figure" defined by Serfati, the educational research related to it. This dual analysis has revealed, among others, many resonances between epistemology and didactics, and more particularly allowed us to highlight some students errors rarely mentioned in educational research. In a second time, the collation between the two research

fields has been used to tackle a crucial question developped in our work, that is: taking into account all the previous studies, both didactical and epistemological with regard to mathematical symbolism, what kind of experimental work could be set about in order to improve our knowledge on student's relations to symbolism?

In order to analyse the student's construction of mathematics symbolism, the experimental work has been elaborated for three levels: it was designed for students whose relation to symbolism has just started (grade 8), then to students already familiar with symbolism (grade 10) and it was finally meant for future elementary teachers whose relation to symbolism remains often problematic.

The various exercices have mainly been based on three epistemological ideas : the use of letters to express generality, the importance of substituting signs in a formula and the reading and writing (in natural language) an algebraic expression.

Nevertheless, epistemology has not only contributed to the design of the tasks: it has also played a crucial role when interpretating the students answers. Indeed, as regards the results collected so far, the epistemological lens has indicated, among others, germs of mastery of algebraic manipulation, which wouldn't have emerged through a sole didactical analysis.

The two and a half years of research carried out so far, supervised both by didactical and epistemological experts, produced many fruitful discussions. Talking about the same object (namely mathematical symbolism) from different points of view unquestionably enriched the prior questioning about the construction of mathematical symbolism.On the one hand, it was with great interest that epistemologists found, through educational studies, similar patterns of reasoning to those they've underlined in their research. On the other hand, the epistemological work provided a more systematic analysis of the nature of mathematical symbolism. It allowed us not only to refine our questioning about student's construction of symbolism but also to detect germs of mastery of algebraic symbolism manipulation, which would not have been perceived otherwise.

This double analysis will, we hope, allow us to formulate the answers to some of the questions that guided our work. That is, how can didactics and epistemology be linked together? How can the epistemological analysis, combined with the prior didactical works based on the history of mathematics, guide us through a better understanding of students' relationship with mathematics symbolism? And in a further step, what are the limits of this dual contribution? Could it be applied to other educational domains?

Another application of this dual analysis is being implemented to develop engineering designs allowing these students to overcome the identified difficulties, an issue which becomes crucial within the "early algebra" perspective.