WHY IS MODELLING NOT INCLUDED IN THE TEACHING OF ALGEBRA AT SECONDARY SCHOOL?[1]

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Teaching elementary algebra as generalised arithmetic

Mathematics at secondary school has its own particular features that differentiate it in many aspects from the works where they were firstly developed before being introduced at school through a complex process of *didactic transposition* (Chevallard 1985). When we deal with the problem of teaching elementary algebra, we are driven to study firstly what we, as researchers, understand *algebra* to be (Bolea *et al.* 1998). But we also need to know what the educational system itself understands *algebra* to be, that is, which kinds of mathematical activities are developed in the teaching and learning processes related to algebra. It is a common fact, established by numerous research projects (Sutherland *et al.* 2001), that the first learning of algebra takes place within arithmetic, an area of mathematics that is closer to the student and which acts as a reference point for the algebraic work thereafter. This prevailing and more or less explicit interpretation of algebra as a *generalised arithmetic* can be expressed in a fairly specific group of types of school mathematical tasks (Bolea *et al.* 2001a). It is mainly present in official curriculum guidelines and institutional instructions for teachers, as well as in the key textbooks and other kind of teaching materials. Bolea (2002) presents an experimental study about how Spanish teachers describe their algebraic teaching practices as well as their personal understanding about what school algebra is and what it should be. It confirms that introductory algebraic practices are based on arithmetical contents, that the understanding of algebraic symbols given in class are almost always presented in numerical terms, and that the abstraction level assigned to algebra is always higher than that of arithmetic or geometry.

The absence of the teaching of algebra as a modelling tool

Beside this interpretation of algebra as a generalised arithmetic, we can also see algebra as essentially a mathematical modelling activity. In this case, algebra can be considered not as a content of its own, but as a tool for modelling mathematical systems, what we called the *algebraisation process of mathematical organisations* (Bolea *et al.* 1998). From the findings of our empirical study, we can confirm that the interpretation of algebra as a modelling tool has a very weak presence in current Spanish secondary education. These findings allow us to corroborate the phenomenon of the "disalgebraisation" of the school curriculum.

Transpositive restrictions on school algebra

Didactic transposition theory establishes the existence of four different kinds of generic restrictions, which are imposed on the taught knowledge of any educational

[1] There exist a longer version of this abstract available from the authors (mbosch@fundemi.com).

system: (1) the need to adapt school mathematical activities to the institutional representation of the mathematical knowledge; (2) the need to evaluate the students activities; (3) the fact that all taught knowledge must appear as definitive and unquestionable; (4) and restrictions imposed by the *didactic time* in various senses, such as structuring of taught knowledge into an *ordered series of diferenciate subjects* or the need for "*fast learning*". If we consider these major kinds of constraints, it is easy to verify that they are much more compatible with the types of tasks which correspond to the view of algebra as a 'generalised arithmetic' rather than as a 'modelling tool': evaluation of algorithmic *vs* non algorithmic activities; sequential relations between mathematical topics; division of school mathematics into unrelated blocks of contents; instantaneous learning *vs* long term goals.

Is it possible to integrate algebraic modelling in secondary education?

Inevitably the question that arises now is if it will be possible, and didactically viable, within the current mathematics educational system, to design a mathematical curriculum that could incorporate elementary algebra as a modelling tool. At this stage, we cannot take for granted the 'viability' of a new kind of teaching proposals based upon the introduction of elementary algebra as a modelling tool. In other words, the possibility that modelling activities can exist in a generalised and stable manner in current mathematical educational systems cannot be guaranteed just by the quality of the teaching materials or by the cognitive characteristics of the students. Any curriculum proposal which aims to directly influence the didactic transposition of algebra should take into account the conditions which are imposed by the educational system itself. Moreover, because the algebraisation process ends up affecting all mathematical content blocks, it is foreseeable that any curricular proposal that attempts to integrate algebra as a modelling tool will lead us further than the specific area of the teaching of algebra, in order to encompass all of the mathematical secondary school curriculum.

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