# EXPLAINING vs UNDERSTANDING DYNAMICS: THE CASE OF ELEMENTARY ALGEBRAIC THINKING

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*As requested by the CERME Organizers, the original presentation has been reduced from 8 to 2 pages. In preparing such an abstract the structure of paragraphs has been preseved, while the actual teaching proposals (in part two) have been skipped. A full paper will be published elsewhere; the author is glad to mail a copy to interested Colleagues*

## Part One - (1.1) Foreword

Varieties of *articulated mathemathical thinking* ( **a.m.t.**) are today culturally accessible. However, as humanly "natural" as **a.m.t.** may be, it certainly appears as a not spontaneously developing strategic cluster: and standardized *mediation* actions (explaining, teaching, …) too often fail their job, ending into bilateral frustrations. Across countries, too many youngsters do not meet at school enough motivation to support understanding – enough understanding to support motivation. The endeavour of our teaching research is then: how to shape explaining & teaching inter-actions, in such a way to evocate (to induce, to drive) widely *resonant* understanding & learning, dynamic re-actions? By this paper we suggest some criteria to foster a successful development of **a.m.t**.. at primary school level, on the basis of about twenty years of systematic research (1) in normal classrooms: followed longitudinally (at times as long as five years), and transversely (vs natural language and sciences' areas). In this long-term work, insights from most recent neurological (2) and linguistic research converge to the ones from phenomenologically careful, field-based cognitive dynamics' analyses. Accordingly, efficient mediation patterns appear to emerge from: a) systematic, correlated realization and control of the intrinsic complexities of actual (children's and adults') *cognitive games;* b) handsome, dynamical, phenomenologically resonant *cognitive models*; c) open minded, "variational" attitudes vs *disciplinary structures*, whose compact coherence should be seen as a final goal of meaningful teaching and not as a blind instrument of efficient conditioning.

## (1.2) Backgrounds to cognitive modelling

A few, long lasting <philosophical cramps> still affect in many aspects our cognitive modelling, thus hampering explaining & teaching approaches. A typical one, is our misinterpreting our success in discretizing our *ways-to-look-at* complex realities: eventually attributing evidence for different aspects to intrinsic discontinuities (mistaking, in geometrical terms, projections vs partitions).

For example. Since Parmenides and Plato, up to Popper and Piaget, we contrast to each other as distinct "things" (not as different modes of a basic dynamics) *natural vs scientific thought.*

For example. Since old times we often contrast to each other *mind vs body* : keeping us away from realizing, and exploiting, the deep structural rooting of any kind of “abstract” thinking into the sophisticated “black-box” dynamics of our perception, in turn correlated to natural language.

For example. Since ever, we contrast *knowing (rational) vs feeling (emotional)* aspects within human (pupils’) attitudes and behaviours. However the “internal phenomenon” of understanding (as contrasted to misunderstanding, pseudounderstanding, conditioning, faking …) is always marked by an emotional dimension (likely to any “recognition” act), crucial to frame and stabilize one’s way across cognitive ambiguities and cognitive construction struggles.

For example. From Plato to Kant to Piaget to Cognitivism (with a few remarkable exceptions) one person’s thinking on a given subject at a given time has been seen as an univocally defined “entity” (mind’s state, in somebody’s terms). Researchers’ thinking-talking-acting in terms of “misconceptions” is no exception. However, any cognitive situation is actually shaped by some dominant whay-to-lok-at, temporarely and partially emerging by external and internal “couplings” from a population of virtually available ones: whose hardly controlled wholeness strongly interacts (as in figure-to-background perceptual dynamics) with foreground leading-patterns. (Remember <cognitive games> by Wittgenstein, Vigotskij’s <proximal development areas>, Aristoteles’ transitions from “potential” to “actual” - associated to peculiar “pleasure” feelings, and so on).

For example. Natural language and mathematics have been compared/contrasted to each other, looking for specific and/or common marks. However both language and mathematics mainly make explicit and stress by formalization an everpresent duality-interference process, linking syntactic metaphoric structures (spacetime, causality, correlation …) to all semantic contextual features (3).

For example. What “are” mathematical entities? Autonomous realities, gradually discovered and explored ? … human mental constructions, progressively structuring each other ? … Where the <unreasonable effectiveness of mathematics in shaping natural world> (Wigner) comes from? (4).

In all such cases, implicit hypotheses/answers often powerfully shape (distort) teaching attitudes.

**(1.3) About metaphorical thinking**

Very significantly Piaget stressed the role of *permanent object* constructs in founding human cognition. Careful observation of developing children allows to extend the pregnancy of this category to a few strictly correlated ones: *permanent phenomenon*, *permanent framing*, etc.

To understand teaching-learning interactions it is therefore crucial to realize at first the structure of perceptual-dynamics-as-a-model; and the parallel, essential role of language vs “possibilities’ spaces”, according to which perceptual findings can be variationally rearranged to interpret and to plan actual occurrences and behaviours. <Cognition is a variation-generating machine> (3).

Facing second order (third order …) correlations, i.e. the ones not directly supported by online sensory fluxes, cognition appears to choose a "conservative" metastrategy: thus relying on well tuned and vastly validated object/phenomenon, state/transformation, variables' relations/systems' interactions … "duplication" dynamics. As powerfully as implicitely, elements of referential syntax and semantics are directly transferred to metaphoric grounds, again marked by "naming": so, according to children's insight, <we use things we can't see to explain things we do see>. (This remark emerged in fourth grade). But the human "natural" ability to perceptually handle correlations in referential contexts by quick, automatic, temporary, interfering shifts of attention across sensory data, tends to miserable collapses as soon as the same flexible fuzziness has to be exploited on complex (though perception-isomorfic) metaphoric planes. As a matter of fact, "scientific" thinking becomes really and powerfully abstract only when it reaches the "safe" cognitive levels peculiar to perceptual dynamics: not by chance getting marked by "common" words. (Since ever, mathemathics has to do with "mathematical objects" of all kinds, and so on).

**Part two: two examples - (2.1) About numbers**

There are three "elementary" distinct *perceptual situations* where a (small-number) numerical feature can be most easily grasped. They are: a) recognition of a peculiarly invariant "discrete shape" - *numerosity - within a stable, observed state*; b) possible relations to be established between the recognized *numerosities belonging to two states, "contrasting" each other by diversity or by change;* c) actual *transition, by external "operation", from an initial to a final (different) state.*

**(2.2) About proportional thinking**

The process of realizing that <two numbers are one number>, in both additive and multiplicative structures, is strongly interfering in its evolution with the one of handling the correlations of four numbers, such as the ones featuring "translation" (a-b = c-d) and "proportion" (a:b = c:d): strongly rooted as they are in perception and experience since the very beginning of numerical competences.

(1) Guidoni, P.: 1985, *On natural thinking*, European Journal of Science Education, 7, 133/140

(2) Changeux, P.: 2002, *L'homme vérité*, Odile Jacob, Paris

(3) Cellucci, C.: 2002, *Matematica e filosofia,* Laterza, Bari

(4) Lakoff, G. and Nunez, R.: 2000, *Where mathematics comes from*, Basic Books, N.Y.