ANALYZING ALGEBRAIC THINKING IN WRITTEN SOLUTIONS

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ABSTRACT

My doctoral research focuses on ways of algebraic thinking at the upper-secondary level. I have formulated a tentative model and used this model as a frame of reference. This model will be used to analyze the algebraic thinking apparent through a number of curricula, textbooks, and official tests. I present two levels of analysis: *a global level*, which distinguishes between different possible approaches to mathematical problems, and *a local level*, which classifies each step of a solution as a particular type of algebraic activity.

In my global analysis I distinguish between *synthetic* and *analytic thinking* respectively. It is possible to identify several types of solutions from ones using only synthetic thinking, through those using various mixtures of synthetic and analytic thinking, to purely analytic solutions. By describing solutions in terms of either synthetic or analytic approaches one gets a general picture of the approaches required for the solutions. The circumstances where no pictorial prompts whatsoever are given in a task would lead to an approach where one has to analyze the included conditions. The characteristics of the problem may change if a pictorial representation is present. Graphic representations and models characterize a synthetic approach. A visualization of a task can inspire various solutions. One can sketch the problem by hand or visualize the relationships with computer programs. There even exist "purely" synthetic solutions. By using Cabri one can create representations of a situation without axes. Visual representations can give rise to solutions involving various amounts of analytic approaches.

A local level analysis is needed to highlight more subtle differences in mathematical thinking. The model includes different components. I use the following abbreviations:

- ANA identifying and naming the variables of a situation (part of analytic thinking)
- MOD modeling or representing the relations between the variables of a situation
- DEP functional thinking or thinking in terms of dependent and independent variables
- CAL calculating using variables within an operational calculus
- STE structural thinking about expressions, i.e. thinking about formal expressions in terms of their structure and not in terms of a sequence of operations to do

STS structural thinking about structures, i.e. thinking about numbers and functions as elements of a structure (that is a set endowed with operations satisfying certain properties)

INT_1 interpreting the relations between the variables of the situation as an instance of application of a known formula

INT_2 interpreting results of formal calculations in terms of the meaning of the variables of the situation

Non-algebraic kinds of calculation appear in a solution. I use the following abbreviation for numerical computation:

ALG algorithmic thinking or following a procedure

ARI numerical calculations as opposed to formal calculations with variables

NOS using non-operational symbolism (shorthand, practical elements like, for example, arrows)

At the local level I code each line of the solution with respect to these aspects and summarize the frequencies and illustrate them in tables. By studying the frequencies of the different aspects, one gets an overall picture of the level and kind of the features of thinking.

To illustrate my approach, I analyzed two students' written solutions to an examination task. The students obtained different grades at an oral examination based on a discussion of their written solutions to this task. Student 1 got grade "Pass", and student 2 grade "Pass with distinction".

In an *A priori global analysis* of the task I have presented different possible solutions. It is obvious from the global analysis that the two students' approaches are similar. They use the given picture in the problem as a starting point for their synthetic solutions. They use the same theorem although the local analysis reveals that their thinking is quite different. Student 1 uses arrows to denote lengths, for example $D \rightarrow M = x$, and substitutes "algebraic" symbols by numbers. He expresses relationships by using numbers. In his conclusions the student stresses that the answer is a formula. Student 2 uses conventional algebraic symbolic representations. He identifies the variables in the problem and expresses the relations between them with a function. This also indicates that this student uses more algebraic thinking.

My tentative model grasps the differences in students' written solutions in qualitative and quantitative terms. I need to explore other kind of problems as well to find advantages and disadvantages of the model.