COGNITIVE FACTORS AFFECTING PROBLEM SOLVING AT THE PRE-ALGEBRAIC LEVEL

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The paper continues the authors' previous studies on algebraic solving strategies of the one comparison problem. The study focuses on arithmetic solutions of the same word problem, in an attempt to find out whether the choice of the arithmetic strategy is influenced by the same variables as in the algebraic case.

In our earlier studies (Nesher, Hershkovitz, and Novotná, to be published; Hershkovitz, Nesher, and Novotná, 2002; Hershkovitz, Nesher, and Novotná, 2001), algebraic solutions[1] of twelve separate formulations of the same comparison problem[2] were analyzed in terms of its linguistic surface structure, its underlying propositional structure, its underlying schemes, and the symbolic modeling selected empirically by the solvers in their attempt to solve. The purpose of the studies was to find factors that affect the way solvers construct their equations to solve a simple comparison problem. The work is based on cognitive theories related to solving word problems such as that of Kintsch distinguishing between the *text base* and the *situation model*, see e.g. (Kintsch 1986), and works emphasizing the *schemes* approach, see e.g. (Nesher, Greeno et al. 1982; Vergnaud 1998; Fischbein 1997). In (Hershkovitz, Nesher, and Novotná, 2001) the notion of a level of complexity was introduced and the research hypothesis was: Solvers of each problem choose the independent variable in an algebraic strategy that leads to a minimal level of complexity.

In the present study we are analyzing *arithmetic solutions* to 24 variations of the above mentioned word problem solved by students who did not as yet study algebra systematically. In continuation of our previous work, the current research question is: *Does the "complexity level" that we have found in algebraic strategies explain the arithmetical strategies as well?*

The set of 24 problems was grouped into 6 tests consisting of 4 problems. The tests were given to fifty six 12 to 13-year old students who have not as yet studied equations with one variable. They came from three different schools in Prague. The students were allowed a single 45-minute lesson period given to complete the task. We divided the students' solving strategies into two subgroups:

[1] After their mathematical background, solving strategies are divided here into *arithmetical* (numerical) where the solver does not use equations when solving the problem, and *algebraic* where one or more equations are used. Our previous experiments (Novotná, 2000) showed that solvers who successfully grasped solving equations prefer algebraic strategies. But in many cases, the arithmetic solving procedure can be more economical and comprehensible than the use of equations and of their systems.

[2] The basic situation of our study is: David has 22 marbles, Jirka 44 marbles and Peter 132 marbles.

a1. The student solves the problem on the level of relations among parts.

a2. The student takes an "average" value from the whole according to the number of participants and then balances (successfully or unsuccessfully) or does not balance this value according to conditions given in the assignment.

In the case a1, we concentrated on the value chosen as the "independent variable", i.e. as the reference (Nesher, P., Hershkovitz, S. and Novotná, J., 1999). Similarly (Hershkovitz, S., Nesher, P. and Novotná, J., 2001) to the algebraic solutions, the goal of our experiment was to find whether in the case of arithmetic solving strategies, the choice of the "independent variable" leads to the minimal complexity level.

In the experiment the relation between the complexity level and the choice of "independent variable" was not replicated. We identified other phenomena strongly influencing the choice of "independent variable", namely the manner of dealing with the assigned data. We found the following three main sub-strategies of a1:

a1-1: Finding the number of the unit part by logical reasoning

a1-2: Systematic trial and error

a1-3: Finding the number of parts by using graphic records of the assigned conditions

Conclusions and Discussion

In our experiment 55 solvers out of 56 used the same strategy for solving all four problems. In case of a1-2, a mixture of the choice of the "independent variable" occurred. In this case there we did not identify any system of choice by the solvers.

In our study we found *consistency of strategy* which we did not find in the algebraic study. Students mostly used the same strategy in all problems in the arithmetic strategies.

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