

ALGEBRA IN UPPER SECONDARY SCHOOL

A STUDY OF TEACHERS' TEACHING AND STUDENT LEARNING

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Alarming data from Swedish universities of technology about an increasingly worsening situation concerning the students' mathematical skills were reported in articles in the years 1997 and 1999 (e.g., Höskoleverket, 2002; Johansson, 1998). There are certainly various reasons for students' problems with mathematics, but one that is not insignificant seems to be their difficulties with algebra. The current discussions in Sweden about the students' difficulties in mathematics are forming the background for the present study.

Many researchers have been concerned with the move of the students from arithmetic to algebra and in particular the cognitive gap that exists between the two (e.g. Bednarz, Radford, Janvier & Lepage, 1992). Researchers have understandably focused on particular aspects of algebra, such as the models used when solving word problems (e.g., MacGregor & Stacey, 1993, 1998), the understanding of the equal sign (e.g., Kieran, 1981), the translation from tabular form to symbolic form (e.g., Ryan & Williams, 1998), the solution of linear equations (e.g., Linchevski & Herscovics, 1994; MacGregor & Stacey, 1995), or functions and graphs (e.g., Herscovics, 1989). In Sweden, there are few studies available about students' algebraic thinking and reasoning in upper secondary school (e.g., Ekenstam & Greger, 1987; Persson & Wennström, 2000; Olteanu, 2000, 2001; Olteanu, Grevholm & Ottosson, 2003). There is a need for scientific studies of students' knowledge in algebra related to the intended curriculum. The present research project is focused on finding out ways in which learning can be improved, on how better learning outcomes can be achieved in algebra in upper secondary school in Sweden (related to the intended curriculum) and on helping teachers to develop relevant teaching practices.

The general aim of the present research project has two different foci, united in the overarching question of the shaping of an object for teaching and learning:

1. To study how students' algebraic thinking develops, how they move from one way of understanding to another, and how their knowledge, skills, and ways of relating to the content change.
2. How teachers think about their teaching of algebra, to what extent and in which ways they are aware of students' possible problems in understanding the content, and how the teachers give explanations and clarify students' problems in understanding.

Theoretical points of departure are found in Marton and Booth's (1997) and Sfard's (2002) conceptions of learning, Pong and Morris' (2002) concept of the "enacted object of learning," that is, the object of learning that students can possibly experience in the classroom, Hattie's (1999) "feedback" definition and Brekke's et al. (2000) notion of a diagnostic way of teaching. The teachers' understanding of teaching of algebra, as well

as their understanding of algebra as a content for teaching, both influence their ways of teaching it and the students' ways of coping with it. Although the teachers follow the same curriculum, the way they shape the content is based on their own understanding of the subject as a whole. The teachers are shaping algebra as an "object of teaching" (Carlgren & Marton, 2000), and the object thus formed is what the students direct their awareness to. The object of teaching formed may vary between different teachers, which results in variation in what the students are afforded in terms of learning possibilities.

During the spring 2002 a pilot study was carried out in order to test a preliminary design for a coming main study. Students and teachers were selected from three different upper secondary school programs. A total of 111 students took a diagnostic test (Step 1). The test was given after the students had studied the algebra part of Mathematics B. On the basis of the test results, 10 students were selected (in co-operation with the teachers) for an individual session, a post-test, and an interview (Steps 2-4), whereas 10 other students constituted a control group, only taking the post-test. Four teachers took part. Video sections and interviews were transcribed and analysed. The results of the pilot study were presented at the CERME 3 conference (Olteanu, Grevholm & Ottosson, 2003).

The results of the pilot study show that there were important differences between the intended object of learning and the enacted object of learning. Since the teaching act has an important effect on how students experience the object of learning (Marton & Booth, 1997), it is important that teachers become aware of the fact that there often is a gap between what they teach, and the resulting "enacted" object of learning (Pong & Morris, 2002) as experienced by students. The post-test reveals that the teachers' instructions do not result in much change of students' thinking. The feedback (Hattie, 1999) was neglected by teachers, and the teachers were not able to intervene in more varied and productive ways, they do not use complementary material in the instructions, for example, pictures, figures, representations, and concrete materials. The teachers and students have worked together with algebra for a substantial period of time and should know each other's ways of thinking well. Still the results indicate that teachers and students are not used to communicate deeply about the students' actions in problem solving and ways of reasoning. The individual sessions most often revolved around some stereotyped appellations and symbols instead of solving the cognitive conflict through discussions and reflections, and helping the students to overcome their errors and misconceptions (Benett et al., 1984).

The communication between student and teacher was in six cases out of nine developing into a monologue, where the teacher explains and writes on paper, and the student listens. The feedback from the teacher did not meet the needs of the student (Hattie, 1999).

The enacted object of learning and the object of learning (Pong & Morris, 2002) is what appears in the classroom. It is constituted jointly by the teacher and the students and

mediated by discourse (Sfard, 2002). From this pilot study, it was not possible to arrive at any firm conclusions about how algebra in mathematics course B was constituted. To develop a more solid basis for understanding the nature of the discourse in the classroom and how this discourse may influence the enacted object of learning, the research design will now be further developed for a new collection of data (with new teachers and students).

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