

WHAT DO THE STUDENTS NEED TO KNOW, IN ORDER TO BE ABLE TO ACTUALLY DO ALGEBRA?

THE THREE ORDERS OF KNOWLEDGE.

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Introduction

The general aim of our research within the CESAME group is to build a model of mathematical knowledge, focusing on the “rules of the game” of doing mathematics. These rules of the game could be an answer to the question: what does a student have to learn, beyond the mere textbook knowledge, to do mathematics? We propose to consider « *three orders of knowledge*», and we claim that this threefold model is quite relevant to the teaching and learning of algebra.

This model, expressed in terms of knowledge (the description of what the student *has to know*) and only in these terms, takes place in the general work of the group CESAME focusing on the conditions of teaching which can contribute (Drouhard et al., 1999) to the construction of this knowledge. To do that, we adopt an «epistemological» approach (knowledge-centred), a «phenomenological» approach (by considering the *actual experience* of the subject and the role of Others) and a «cognitive» approach. Following the work of Duval (1998) we consider that these three points of view (epistemological, phenomenological and cognitive) are necessary to understand the nature of the acquisition of this kind of knowledge, and that it is also necessary to be able to shift from one of these three points of view to another, which we do in our general research. Due to the limited size of this text, we will focus here just on the first, “epistemological”, point of view.

In general terms, we claim that, to actually do mathematics, one has to know the corresponding statements and definitions (*first order knowledge*) and the rules of the mathematical game (*second order knowledge*). But one has also to know that what one does *is* mathematics and not another thing (*third order knowledge*).

First Order Knowledge

We propose to consider that the ‘contents’ of the true mathematical statements (axioms, definitions, properties, theorems...) is first order knowledge. It concerns all which is explicitly considered as an objet of knowledge in textbooks, curriculum, classroom, etc.

Second Order Knowledge

In general terms, second order knowledge is what allows the mathematical discourse to function as it is supposed to function: for this reason we call it the rules of the mathematical game. It makes the definitions define as they are supposed to define, the theorems establish as they are supposed to establish, the writings mean (and denote) what they are supposed to mean (and denote), and the list is long.

At this stage of our research, we consider that there are (at least) two kinds of second order knowledge: one is semiotic (Duval, 1995, 1998) and the other is about validity and truth. The rules which control mathematical symbolism in its various «registers» (semiotic systems of representation (Duval, 1995, 2000)), i.e. the rules of use, interpretation and processing in symbolic systems, the rules of transformation of the mathematical semiosis, constitute second order semiotic knowledge. In the case of the symbolic system of *algebraic writings*, expressions are not mere strings of letters but instead have a denotation and students have to know it (second order) (Drouhard et al., 1994, Sackur, 1995). Students must know as well that the same letter indicates the same number in the context of a calculation, that two different letters are supposed to have different denotation, that the denotation of an expression does not change when they carry out a valid transformation and so on.

The rules which control mathematical reasoning, for instance that statements requires validation, or that a counterexample is sufficient for invalidating a general statement, the rules for demonstrating a valid statement, etc, constitute second order knowledge on validity and truth. In the case of algebra, students must learn that statements are related one to another by rules of inference, starting from a small number of basic statements (describing the properties of the structures). More generally, students must learn a that the result of a valid inference is true, and that it is *necessarily* true (Drouhard et al., 1999).

Third Order Knowledge.

During teaching situations students or professors can be led to utter some statements which are not just on first or second order knowledge. For example, a student can say to another: «in algebra, there is nothing to understand, you just have to learn»; a professor can return his copy to a student and say to him «no, this is not mathematics!» or on the contrary say to the students : “this *is* geometry”. These are rather general statements, not stating a particular mathematical result (i.e. a first order knowledge), nor a particular second order knowledge. However these particular statements have a function in the acquisition and teaching of mathematics. Third order knowledge is about conceptions on the nature of the first and second order knowledge. Maybe all third order knowledge, could be summarised as follows: mathematical activity consists in playing with (first order) knowledge according to the (second order) rules of the game.

Conclusion

We postulate that being able to do mathematics requires first, second and third order knowledge. A question arises then naturally: how are (didactic) teaching situations related to the order of the involved knowledge? (Panizza & Drouhard, 2003). Our present research aims at looking at how second and third order knowledge are (or are not) present in the middle school (whatever the explicit objectives of teaching would be), how they are learned, and how they are taught (in particular in algebra).

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