ARE MY STUDENTS ACTUALLY DOING MATHEMATICS?

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By comparing two lessons on inequalities, we show that learning mathematics is not only learning definitions and theorems. Some other knowledge that is not openly part of the curriculum, is necessary if we want our students to improve their abilities in algebra. Special devices can be developed to reach this aim.

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I. Introduction

Many researchers have been working on the problem of transition from arithmetic to algebra and on early algebraic thinking, focusing on ways to prevent the difficulties of the students (see references). We choose to work on persistent errors and developed a theoretical framework that permitted us to imagine some teaching devices (Sackur & Maurel 2000), in order to help students overcome their difficulties. Confronting one of these with the script of another lesson[1], we'll try to fit to one of CERME 3 guidelines: "What theoretical frameworks could be used for designing instruction aiming at promoting student understanding? What curriculum innovations could be suggested?" The two lessons, which look very much the same at first, appear to have very different effects on the students. We'll try to understand this phenomena.

Our theoretical frame distinguishes between three different aspects of knowledge necessary to work in mathematics, (see Panizza & Drouhard in Algebra group). The "ordinary" mathematical knowledge: theorems, definitions, axioms constitutes the First Order knowledge: if $a<b$ then $ax<bx$ is a first order (incorrect) knowledge, The Second Order knowledge includes other knowledge such as the rules of the mathematical game: the uniqueness of the solution is a second order knowledge.

II. The two lessons

In both classes the students are aged 15-16.Seresine lesson lasts two hours, CESAME lasts one hour and a half. In both classes there is some time devoted to personal work. After this time, in Seresine, the students gather together and students go to the blackboard, one after the other, to explain their way of solving. If they make a mistake or if they have some difficulty explaining, someone else replaces them. In

[1] We will refer to this lesson as Seresine, ours being CESAME.

CESAME the personal work is followed by some work in small groups of four students, where they have to come to a common solution, which they will later present to the whole group.

The problems

Seresine problem is: solve algebraically the inequality: for a in [-29;58], $4a^2-1 < (2a-1)(7a+2)$

The factorisation $4a^2-1-(2a-1)(7a+2) = (2a-1)(-5a-1)$ followed by some computing brings two different sets of solutions: $S_1 = [-29;-1/5[\cup]1/2;58]$ and $S_2 =]-1/5;1/2[$.

CESAME problem is: solve the inequality: $3/x>x+2$. All methods are valid. In the same way, two sets of solutions are obtained, either algebraically or graphically. In both Seresine and CESAME classes, the following error leads to an incorrect solution:

if $a<b$ then $ax<bx$,

Analysis

The purpose of Seresine lesson is to have the students use knowledge they possess to solve inequalities and to control their work.

The purpose of CESAME lesson is to have the students correct their incorrect knowledge about inequalities and learn at the same time some “other” knowledge that is necessary to do mathematics.

What happens in both lessons is exactly what the teachers expected. In Seresine, the students are very active and very co-operative as far as mathematics is concerned. During this period of time, they have the responsibility of what is going on in the classroom and they cope with it very well. The teacher has nothing to say. When they come to the two different sets of solutions, the teacher leaves them no time to react. He is “*exasperated*” by the fact that they seem to admit that there could be two different sets of solution. He doesn’t leave this problem and its solution to the students. Clearly the knowledge about the uniqueness of the solution is not an aim for Seresine’s teacher. The students are supposed to have it, and nobody seems in charge of teaching it.

In CESAME, the students have time to confront their solutions, and each of them is eager to defend his/hers. When they are in the small groups, they experience the fact that only the correct rule about inequalities can lead to the unique correct set of solutions. Then, at the end of the lesson, the teacher makes clear to the whole class the different results obtained: the correct rule, the knowledge about the uniqueness of the set of solutions and the fact that the correct rule is necessary if one wants to solve correctly an inequality.

There are some other interesting differences between the two lessons that we can only cite quickly here. In CESAME, the use of graphs of the functions leads to the correct solution and the students are very confident in it. The role of others and the timing (specially the work in small groups) give to each of he students the responsibility of the correctness of the solution and also gives them means to work on it that are not purely mathematical.

Conclusion:

Teachers have to know about the “other knowledge”, which is necessary to learn and to do mathematics, This kind of knowledge, which can also help in correcting some resistant errors, is very often implicit in mathematics classes. It has to be openly taught, but cannot be taught in the same way as definitions or theorems. Particular devices are needed and we have tried to show that our CESAME lesson is helpful.

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