PRE-ALGEBRA COMBINATORIAL PROBLEMS AND ALGORITHMS IN PRIMARY SCHOOL MATHEMATICS.

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Primary school mathematics deals mostly with properties of individual numbers and operations over them (NCTM, 2000). It expects special didactical efforts to provide transition from specific number set to suitable algebraic expression and handling of sense of variables beyond generalized numbers (Arcavi, 1994). So, problem design for early algebra learning focuses on effective tools to construct explicit algebraic expression for model dynamic situations (e.g., Hershkovitz *et al*, 2002).

We propose an additional option of introducing algebraic thinking in primary school practice. A set of context-related situations has been constructed in order to stimulate students to climb from properties of specific natural number, *N*, to varieties of these properties with changing *N*. It deals with combinatorial-type problems of partitions of a given natural number under several restrictions, and it starts with the question: "Is it possible?... And in how many ways?". Only the lack of explicit analytical expression for the functional dependence enables students to perceive general features of functions.

To solve typical combinatorial problems, one is usually been required the skill of algorithmic reasoning to construct and apply counting algorithms (Stanley, 1999), that is evidently a feature of a well-developed mathematical thinking (Polya, 1965; Mason, 1988).

In general mathematical form, those problems concern partitions of a natural number, *N*, as a sum of natural numbers under certain restrictions (for example, a sum of the closest natural addends, a sum of equal or consecutive numbers, *etc.*).

The mathematical difficulty of these problems varies, but even initial, nearly trivial, situations have been developed up to the level of a primary school math teacher. It turns out, that almost all of these problems can be treated and solved without any formal algebra. Moreover, we usually try to stimulate mathematical trial as a way to derive a rule rather than application of algebraic technique.

All of the problems in the set are firmly interconnected, so each problem can arise naturally through the discussion of a previous one. The set is treated as an open mathematical situation (Silver, 1995) that invites multi-level and multi-directional inquiry. Elements of the proposed approach have already been realized in a framework of pre-service and in-service learning workshops for primary school math teachers. It is clear that all of the tasks can be easily transformed into suitable problems for primary school pupils, and the open character of those situations invites teachers to bring out this adjustment.

As an illustration, we present some key questions concerning one of the problems *"How many possibilities are there to distribute a given number of items, N, between* *two persons?"*. This problem comes from splitting a natural number into two "close" (equal or consequent) addends.

Related questions: "Is it a well-defined problem? How does a realistic context influence splitting possibilities and restrictions?" Several mathematical interpretations are allowed: should each subset be a non-empty one? Are the two ways of splitting, namely $a+b$ and $b+a$, supposed to be the same or different?

"How can you fulfill each next splitting, and what is the criterion to stop the process?"

"How does the number of possibilities change with an increase of the number, N? May the number of possibilities decrease / remain the same when N increasing?" The correct answers vary according to the chosen version of splitting restrictions.

From a didactical point of view, proposed approach demonstrates a general idea of constructing theory-of-numbers functions from numbers' properties. Furthermore, they provide fine examples of functions with various (including non-monotonic) behaviors.

The proposed scheme also has some psychological benefits:

- The original situation is a simple and a very natural one. Most problems can be interpreted and displayed in a non-formal perceptional form. They visualize basic number properties and relations by real manipulative procedures.
- The open character of the situation discussed stimulates students' activity to solve the problem and to pose the next one. Multi-directional development invites various tools and methods of mathematical reasoning for students as investigators.

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