YOUNG CHILDREN MAKE SENSE OF TASKS ON FUNCTIONAL RELATIONS

Anna S. Steinweg

Department of Mathematics, University of Dortmund, Germany

e-mail: steinweg@math.uni-dortmund.de

German research in the field of algebra still focuses very much on higher grades. Functional relations and other themes related to pre- or early algebra are barely mentioned in the primary school syllabus. There is no data proving the results of foreign researcher (cf. e.g. Warren, 2001) for German children. This is a chance to be the first confronting children with tasks on early algebra. At least a kind of spontaneous reaction can be observed, because the influence by school mathematics is negligible.

The focus of the presented study is on *the very first beginning* of a process leading to thinking about mathematical functions symbolically. Regarding to Tall's, et al., (2000) sophistication of development, this research is located between 'routine mathematics' and 'performing mathematics flexible'. Symbols and forms of representation (cf. Carraher, 2001) are not the key interest here but shifts in interpretation of the task by the children (cf. Yackel, 1997). Leaving behind the routine forms of solving tasks demands a qualitative shift in reading and understanding the task. One possibility to describe this shift is to associate the routine procedures with numerical, arithmetical ways and the more flexible ways with pre-algebraic ones. It is of special interest here if a supposed 'transition phase' can be detected in the analysis of children's interaction with tasks on functional relations, i.e. interaction while solving the tasks in an individual way or discussing the solution. This leads to two basic research questions.

Research Questions

Because functional relations are not mentioned in the German primary school curriculum the first question was to find out if the children can handle the offered tasks and can make sense of the presentation form. The second –and more important– question was: *How* do they make sense of it? (within their very own framing)?

Methodology and Sample

One part of recent studies on primary school children's understanding of number patterns (Steinweg, 2001a, 2001b) dealt with functional relations. 63 children (9-10-year-olds, after 4 years of formal schooling) of three different schools and different socio-ethic-backgrounds participated in the written test. 15 students (9-10-year-olds of three other schools; 5 children per school) were interviewed. 46 children (23 in each of the classes) of the same age participated in the school project.

Quantitative Results – Do They Handle the Tasks?

The quantitative results are encouraging. The children were able to cope with the demands of the task –even though they did not have any experiences with functional relations in mathematics lessons at all. The figures do not give any answer to the question *how* the children found access to the task and how they used the structures of the functional relation. The mathematical structure –presented in different forms– is in no way self-explicit. The children had to make up their own sense and their interpretations are embedded in their social backgrounds (cf. e.g. Voigt, 1995).

Qualitative Results – How Do They Make Sense of the Tasks?

The transcripts of the interviews were analysed to find out if the existence of a transition phase between arithmetical-numerical thinking and pre-algebraic thinking can be proved.

Discussion

Algebraic solutions and interpretations do not have to include the use of letters. One might solve a task algebraically without even thinking or knowing of letters as variables. Schliemann, et al., (2001) describe the working on particular numbers and computing numerical answers in contrast to working on sets of numbers and describing relations among variables.

Mathematical objects, like functional relation, as artefacts can be interpreted in different ways and with various depth. The interpretation once used for a solution can change while solving the same problem and varies from one individual to another. The task itself has to offer the chance to view the relation as a function algebraically. In order to support the students the task has to ask for indirect, systematic, or reverse strategies (cf. Steinbring, 1999). The representation has to be understood as such, and has to lead to the underlying mathematical structure.

The results give rise to a new research question: Is it possible to guide the students to see the functional relation as such by posing the right questions and presenting tasks, which offer algebraic solving strategies?

The children in this study were not using variables but their uttered thoughts indicate some mathematical activities, which do no longer belong to routine mathematics –for them. This shows that even in elementary mathematics ‘unspoken changes of perspectives’ (Malara & Iaderose, 1999) could be identified. These changes have to be made ‘speakable’ for the children and for the teachers. An awareness for the transitions, which are founded very early and are regarded by Dooren, et al., (2001) as ‘precursors for algebraic thinking’ (p. 359), is vital. The shown examples put the finger on the important pivotal where the change of perspective can be successfully made. This might be the only way to prevent the constitution of two different ‘mathematics’ as one can find in the reactions of secondary school children (Cerulli & Mariotti, 2001).

The learner should become aware of the limits of the routines. More emphasis has to be put on the flexible use of mathematics and on fostering the individual strategies, which pave the way for structural ways of thinking. The design of suitable, 'potentially algebraic' tasks that will enrich primary school mathematics will be the major challenge of further research.

References

cf. longer version of this paper:

http://www.mathematik.uni-dortmund.de/didaktik/_personelles/people/ass.htm

or contact the author via e-mail.