USE OF SLOPE AND Y-INTERCEPT IN PREDICTION AND DESCRIPTION, AS SEEN FROM STUDENTS´ PERSPECTIVE

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Abstract

In this paper I analyse, from a semiotic perspective, the student's useof the prediction of certain changes necessary to make two straight lines equal in their characteristics, and the ways they explain why two straight lines are different. Both tasks required to use the figural perception of slope and Y-intercept. In the cases of prediction and explanation, many students set out early knowledge about the graphic expressions and limit themselves to the use of the coordinate axis, the marks of the scale or the quadrants as references to describe changes in the straight lines which imply physical movement.

Resumen

En este trabajo usamos una perspectiva semiótica para analizar las respuestas de estudiantes de bachillerato en tareas de aprehensión figural. Solicitamos a los estudiantes que sugirieran los cambios necesarios para que dos líneas rectas sean iguales y que den una explicación de por qué dos rectas son distintas. La solución de las tareas requiere de la aprehensión figural de la pendiente y la ordenada al origen. Encontramos que, en situaciones especiales, muchos de los estudiantes hacen a un lado sus conocimientos previos sobre la recta y prefieren usar signos como los ejes coordenados, las marcas de la escala o los cuadrantes para hacer las comparaciones y hacen alusión a movimientos de tipo físico.

Keys words: Visualization, straight lines, drawing graphs

Introduction

This work is based on the visual aspects of the straight line, and the elements discussed are the semiotic signs, their meanings, and their relationships.

The visual information in the graph is crucial to comprehension. Some researchers, such as Friel, Curcio and Bright (2001, p. 132), claim that "by graph comprehension we mean graph readers' abilities to derive meaning from graphs created by others or by themselves".

Friel et al. (op cit., p. 130) say that there are three levels of graph comprehension: an elementary level focused on extracting data from a graph (namely, locating or translating); an intermediate level characterized by an interpolation of data and the

discovery of relationships between data as shown on a graph (namely, integrating or interpreting); and an advanced level that requires extrapolation from data and analysis of the implicit relationship in a graph (i.e., generating or predicting). But they do not consider explicitly in their graph-structure comprehension the importance of the semiotic meaning of the graph. Their classification does not deal with the meanings of graphs having to do with the pictorial or figural aspect of graphs, i.e., those semiotic signs related to spatial activities such as locating or translating figural elements which make sense even before we have determined a scale or established numerical and analytical relationships.

I have evidence of the important role that the figural or iconic elements play in graph comprehension. In the present paper I am reporting my observations of students' use of the shape of slope and the y-intercept as visual information in identifying and predicting, and will also comment on the ways they use them.

Theoretical considerations

From Bertin's (1968) perspective, the visual perception (in graphs) consists of the perception of three factors: the variation of shadows and the two dimensions on the plane, regardless of the time variable.

Duval (1996, p. 142) claims that "a figure is an organization of sharp contrast of the brightness. It emerges from a background through presence of traces or spots, governed by Gestalt laws and perceptual clues".

In a straight line there are three relevant signs or representational units (Duval, 2001): direction of the line (from left to right), y-intercept (position on the y-axis), and angle between the line and the x-axis (symmetrical partition, major angle, minor angle). In this paper I consider two visual variables: the shape of the slope as the slant of the straight line, and the y-intercept and the x-intercept as points of intersection of the two axes and the straight line.

From Duval's point of view, the function of a graphical representation is based on two figures: the ground-figure (axes, grid and orientation) and the form-figure (the graph). The relationship between both figures is completely defined by the Gestalt rule (Duval 1999, 2001), so the graph must do a clear distinction between them. In addition, we view the graphical representations (specially graphs) as one type of *figural concept* (Fischbein, 1993), i.e., occasionally, they are considered to be objects or concepts.

When we are dealing only with the figural aspects of the graph representation, it is necessary to distinguish between *drawing* and *figure*. (Hölzl, 1995; Parzysz, 1988; Laborde and Caponni, 1994; Robotti, 2001). The main goal is to show two different ways of thinking and using the visual representation (in particular geometric figures) and its related processes.

Shama and Dreyfus (1994, p. 45) consider that the term "visual" is used here in reference to the manner in which mathematical information is presented and

processed during a problem solution. In particular, I observed students whose visual strategies (Acuña, 2001) could be based on the graphical signs (figural) or on the natural language descriptions (linguistic) (Acuña, 2002) and I noted that students use some specific global strategies for comparing the slope and y-intercept. In particular:

S1. Students use visual movement, in general a translation or rotation, from one line to another. They can make a parallel translation in the case of the straight line position; they mentally move the first angle or line to the second figure, trying thus to determine whether or not it is the same as regards the shape of the slope and the position of the Y-intercept with respect to the x-axis.

S2. Students consider some real or imaginary tokens, or apparent relationships, to determine the orientation of the graphic elements. For example, they use the quadrants where the straight line is located to make sense of the line position.

These strategies are neutral, and can produce either correct or incorrect results.

The main objectives of this research were to know if students that recognise the slope and y-interceptuse them in construction, identification and prediction tasks based only on visual considerations, to know how students use them, and, when students don't recognise them, to know what signs they take to do it.

Methodology

This research was conducted with tenth grade students (16 years old). We asked 135 students to do certain tasks between June and October 2001.

Previous to this study, the students had some knowledge, from other mathematics courses, about straight lines, namely, their point-by-point graph construction using an equation, and the slope definition. They had some skill in calculating line slopes and they could calculate slope and y-intercept from visual information on a graph; thus our starting point was the knowledge about slope and y-intercept they had been using in previous classes, and their own informal information about them by handling the graph.

In this research, I used a questionnaire whose items focused on the graphical aspects of slope and y-intercept without any kind of equation or analytical information. Students must do construction and identification tasks with straight lines; they have to suggest suitable changes on the lines to make them match other lines and also explain the differences between two straight lines. The items are supported only by visual information, and the indications were given in everyday (non-technical) language.

The answers in the construction section were expressed graphically, and I could easily see whether they were using the appropriate elements or not. In the explanation section, which required written explanations, answers were not judged to be "right" or "wrong", but merely "adequate" or "inadequate"; here, the main criterion was whether the students used words such as "slope", or "Y-intercept" or not, as well as how they described them.

The coordinate plane used in the questionnaire tasks had only four marks on the axes in almost all items, one on each axis, in order to take them for orientation in the plane, but not for regulation of the visual variable with a number scale.

The questionnaire

The questionnaire used in this research had four different tasks with some different items:

1. Students are asked to draw some constructions with restrictions over slope and y-intercept.

2. They are asked to identify of a straight line among several of them.

3. They are asked to say why two different lines were not the same.

4. They are asked to suggest what changes were necessary for two given lines becoming the same.

In this paper we take only some of the items to support this research about the figural usage of slope and y-intercept in prediction and description tasks, as seen from the students'perspective.

Items 1 to 6 in the questionnaire concerned the construction of straight lines under specific conditions of slope and y-intercept:

1. Draw three straight lines with different slopes.

3. Draw one straight line with the same slope and the same y-intercept as the given line, resulting in a line equal to the given.

4. Draw a straight line with the same slope and different y-intercept as the given line.

5. Draw a straight line with different slope and the same y-intercept as the given line.

6. Draw a straight line with different slope and different y-intercept from the given line.

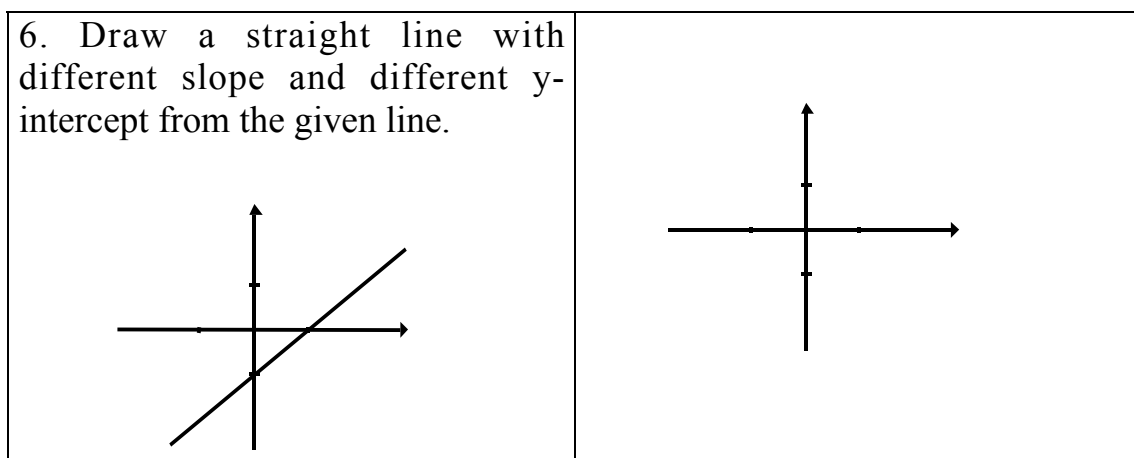The format of the items was as follows:



Figure 1. Format of item 6 in the questionnaire.

Items 1 to 6 served as an introduction to Items 14 and 16, where students were asked for explanations and predictions about the differences between two graphs; to solved the task, student needed to recognize the shape of the slope and the y-intercept, since to grasp this graphical shape we must consider the graphical composition i.e. the relative position among the graphical signs as axes and the straight line, it is necessary to know where it cut the axes, what is the slant and if you make a comparison you have to decide which slant is bigger or smaller than other, among others things.

The tasks where description and prediction are needed were Items 14 and 16:

14. Both graphs are different, explain why.
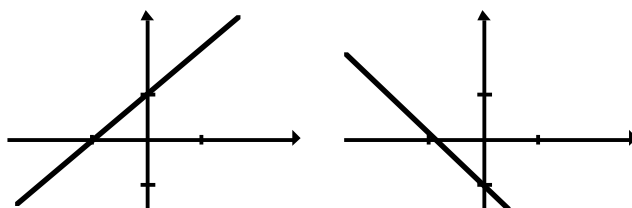
16. The following graphs are not equal, what would you change in Graph I in order to make both graphs match?

I   II
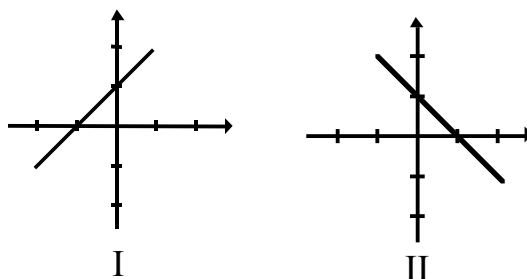
Results

The results of Item 1 reveal that 85.8% of the sample group know what a slope is and also when the slopes are graphically different; in Item 3 the results show that 79% of them can draw a straight line which is equal to another given line. In Item 4, two pieces of information are mixed, one is related to equal slope and the other to a different y-intercept; only 66.5% of the sample group could draw a correct line.

This result can be contrasted with Item 5, in which the instruction is equivalent to drawing a line with a different slope and the same y-intercept. Here, there were more correct answers than in the previous Item: 74%.

These results do not allow me to state whether one task is easier than the other, but certainly they are different. Finally, for Item 6, the frequency of correct answers is

75.5%, which is higher than the results of the two previous items, despite the fact that students have to make two changes.

In these construction tasks, we can see that most students, when they study a graph, can recognize differences in the slope and the y-intercept almost without any problem.
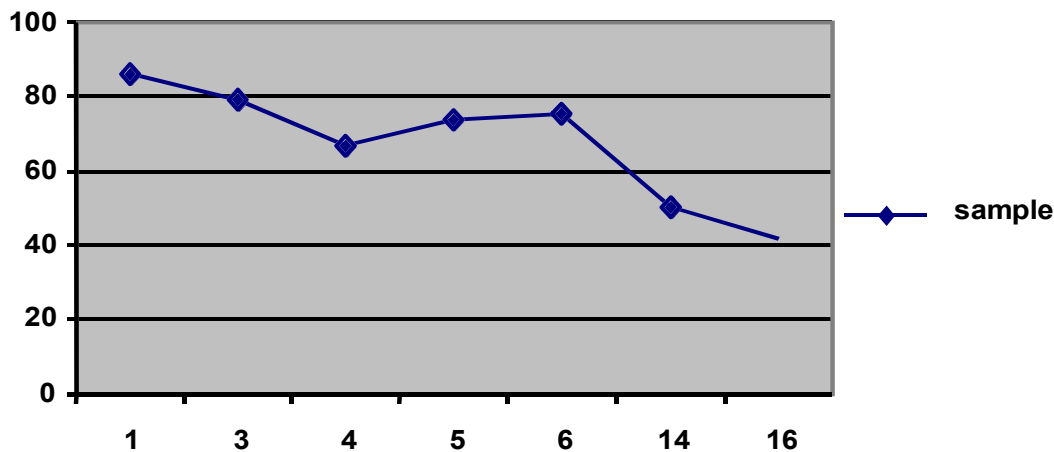

Figure 2. Frequency of correct and adequate answers of the sample group.

Items 14 and 16 ask for explanations and predictions about the differences between two graphs; to solve these tasks, students must recognize the shape of the slope and the y-intercept, but apparently, this is not enough.

In Item 14, there are two different lines, and students are asked to say how are they different. I was expecting the answers to include the words "slope" and "y-intercept", or perhaps a more precise description of them, yet the frequency was only 50.3% (the presence of these terms in the answer determined whether the answer was to be deemed "adequate" or not).

The frequency of correct answers to Item 16 was 41.4%. Most answers were related to changes in different elements of the graphs, and only 41.4% of the sample group considered the elements "slope" and "y-intercept". In their descriptions, we can see how most students used their own "Gestalt" ways of seeing the changes in a line. Many of them used allegoric means of movement as if the line were a concrete, physical line.

The assessment in Q14 and Q16 was based on placing the answers in four different classifications:

A1. They only use references to move graphic or geometric elements such as "side", "quadrants"or "position".

For example: *The position, because they do not have the same angle.*

A2. They refer to movements on the graph by means of changing the graph sign, for example: "to change the point", "to change the number",

"to change negative to positive", "to change the y-intercept", "to change the side". In all these answers, "to change" means "to move".

For example: *I will change its position when it cut the x-axes, instead to put the negative I will turn to the positive "x"* (sic).

A3. They used expressions similar to those of A2 but including the words "slope" and "y-intercept" in their answers.

For example: *Its slope must pass to other side to be positive.*

A4. They gave short and clear indications: "slope and y-intercept" or "slope".

 For example: *Slope and y-intercept.*

In answers A1, A2 and A3, students used slope and y-intercept as objects, that is, they moved the line by a reflection movement, first across the x-axis and then across the y-axis. Only in A4 they treated them as concepts using graphical meanings. Despite this, I accepted A3 and A4 as correct answers in tasks 14 and 16, due to the use of the right word. Below, we show the frequencies of the different types of answers for Q14 and Q16.
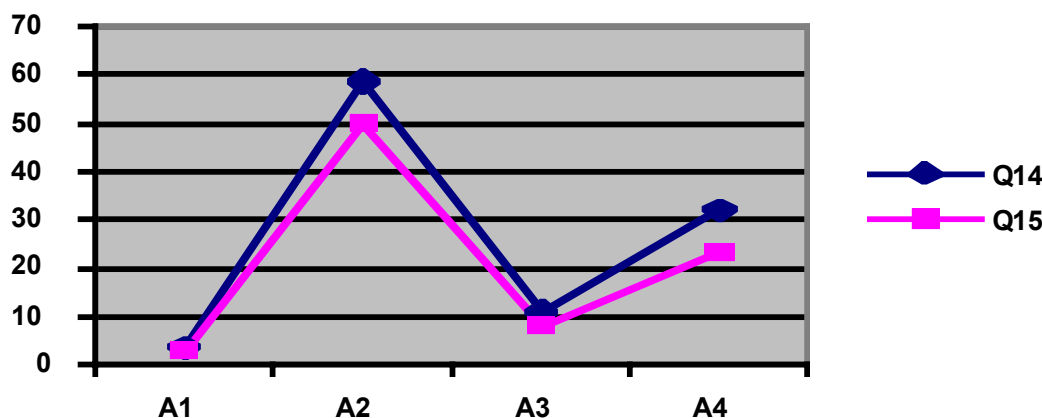
Q14 and Q16 were items which will give us an idea of the signs that students used as tokens (marks on the axis, axes, and quadrants) and their orientation (minus or plus signs, positive and negative zones of numbers) when they gave descriptive answers, and how they used them.

We can also see that the slope and y-intercept are not the most popular graphic signs for predicting or for suggesting a change which would make one line equal to another; students preferred to use their own orientation signs.

Discussion and Perspectives

Although most students seemed to make a successful identification of the shape of the slope and the y-intercept using a construction easily recognizable by its graphic shape, this identification seemed tobe different in other kinds of tasks, such as predicting or explaining.

I found that students viewed the points associated with the y-intercept and x-intercept as the same thing, and sometimes the figure-form and figure-ground are joined in a "triangle". In such cases, they moved them "sideways". In fact, they even seemed to be modifying the shape of the slope and the y-intercept.

In a similar vein, the usage of the graph signs (axis, marks on the axis, minus or plus signs associated with the orientation) by these students was similar to a physical movement rather than a spatial transformation.

Where strategies S1 and S2 were concerned, students had trouble expressing their ideas, this, with no doubt, because their instruction in school did not include visual recognition and prediction, specially because there is a double role of the figural shape as a figure and a drawing.

These findings point out towards the need of reviewing the in-class didactic treatment of the graph of slope and y-intercept. This is necessary because it is at the point when students learn how to draw graphs that they perceive spatial transformation as being associated with physical movement.

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