

DIFFERENT PROJECTING METHODS IN TEACHING SPATIAL GEOMETRY

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Over the past fifty years the teaching of geometry mainly the teaching of spatial geometry has been pushed into the background. Recently several Dynamic Geometry Systems have been developed to help the teaching and learning of plane geometry. Only a few 3D computer programs exist and all of them use the well-known axonometric projection to describe solids. Movies, animated cartoons, computer games, and advertisements use a different projection. The following question has arisen: Would it be profitable to use this projection in education? To answer this question the author wrote two programs according to the two different projections and made a survey. In this paper we describe this survey and its result.

1 Introduction

One role of education is to improve the ability of solving real life problems. Spatial orientation for example is our everyday problem. Although orientation is problematic for a lot of people, three-dimensional space is the usual for us. For this reason show business offers 3D cartoons, 3D computer games.

In the seventies algebraic structures dominated in mathematical education and a more general approach was aimed for. Classical geometry appeared only as a special case. From the beginning of this tendency were those who spoke of geometry to develop visual abilities, for example (Gaulin 1985, p.64):

For years mathematics educators like Bishop, Clements, Mitchelmore, Tatha and others have been advocating that one major goal for teaching geometry that has been overlooked during the 'new math' wave and ought to be re-established is the development of students' spatial intuition, including their ability to visualize and to communicate spatial information by various means.

The survey by the French Ministry of Education shows that the fifteen-year-old students most repulsive subjects in mathematics were spatial geometry and statistics. Only ten percent of teachers taught spatial geometry. They said that they did not have enough time to teach it, but the real reason is that the students 'cannot see in 3D'. We mean this, as the students cannot picture a spatial situation of a teacher's blackboard figure.

A survey of teachers by IREM of Strasbourg (Bayart et al., 2000) demonstrates this fact. They asked their students to answer the following questions. If ABCDEFGH is a cube and M, N, P are point of edges CG, BC, and HG, respectively like on Drawing 1, then 1) point P is on line AM, or not 2) point P is on line MN, or not 3) line AM and NM are the same lines, or not and then point P isn't on neither line. The

most of the student answered that the points A, M, N, and P are collinear. These students treat the drawing as a representation of a 2D figure. (In this article we follow Parzysz's (1988) notation, where the *figure* is the geometrical object, which is described by the text defining it. Figures can be represented by 2D *drawings* by 3D *models*.)

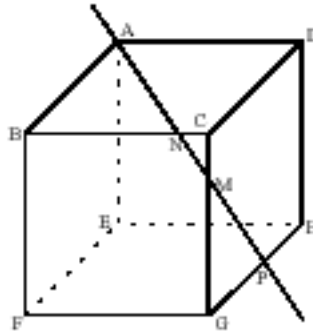


Figure 1

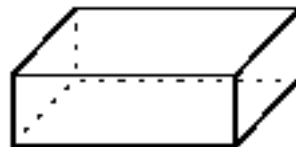
The recent reports on the teaching and learning of geometry recommend that the curriculum should be developed, they also recommend the teaching of methods especially with respect to 3D and the use of computer technology. As Jones and Fujita (2001) observed, while it is simplistic to just note that we live in a 3D world, there is a great need to be able to “develop the geometrical skills to represent 3D objects and to solve problems involving them”.

Our article is organized as follows. In section 2 we list the different representation methods. In section 3 we show our experiment, and our programs. In section 4, we discuss the results and finally, in section 5, we give our conclusion and some perspectives.

2 Different methods to represent solids in geometry

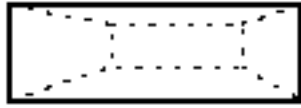
In geometry there are different methods to represent solids on a plane. Audibert (1990) gave a good survey about these methods. Here is a list of the most used:

- Axonometric projection.



Drawing 1

- Central projection with one and two centres, (Drawing 3 and Drawing 4, respectively).

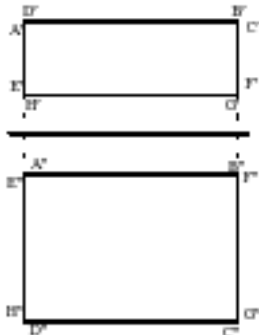


Drawing 2

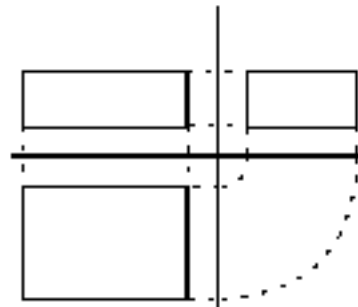


Drawing 3

- Methods of descriptive geometry. In the first case we have two orthogonal projections one is vertical the other is horizontal (Drawing 5). In the other case (Drawing 6), a third projection is added. Each of them are orthogonal to each-other.



Drawing 4



Drawing 5

The axonometric projection is the most used representation method. Almost every textbook uses this method. This is a parallel projection, and hence the parallel lines remain parallel, which is not true for example with central projection. This is the most common representation method, and this method can be applied most easily because its algorithm is the simplest.

With central projections the sizes are deformed, so it is problematic to make the drawing. The two-point central projection is spectacular, the students like this method, but if the solid is oblique then they cannot draw it.

The methods of descriptive geometry are used almost exclusively in technical education, and they are really challenging for students. Despite this, they like this method and they like to find out the original solid.

We asked seventeen students (training to become teachers of mathematics) at the University of Debrecen (Hungary) what was their opinion about these methods. Eleven/eight said that the central projection/methods of descriptive geometry are spectacular, but all of them recommended only the axonometric projection for teaching. We note that Rommevaux (1991) and Chevalier (1989) showed that it is dangerous to teach only the axonometric notation. They said that the axonometric projection is a plane algorithm and that students cogitate axonometric figures as plane figures.

In teaching spatial geometry, it is necessary to use models because only these models represent solids in 3D, yet, they only give ocular demonstrations for some strange facts. We can ask students to produce models. It is possible to make models within half an hour from paper; it takes a bit more time to make models from transparencies, but in this case we can add visible diagonals and we can mark plane sections. There exist small plastic pieces that can assemble into polyhedrons. There are solids (for example the sphere) that we cannot construct from paper. Hence, there are models from wood or plastic. Often these models are decomposable, so we can examine the plane sections for example at conic.

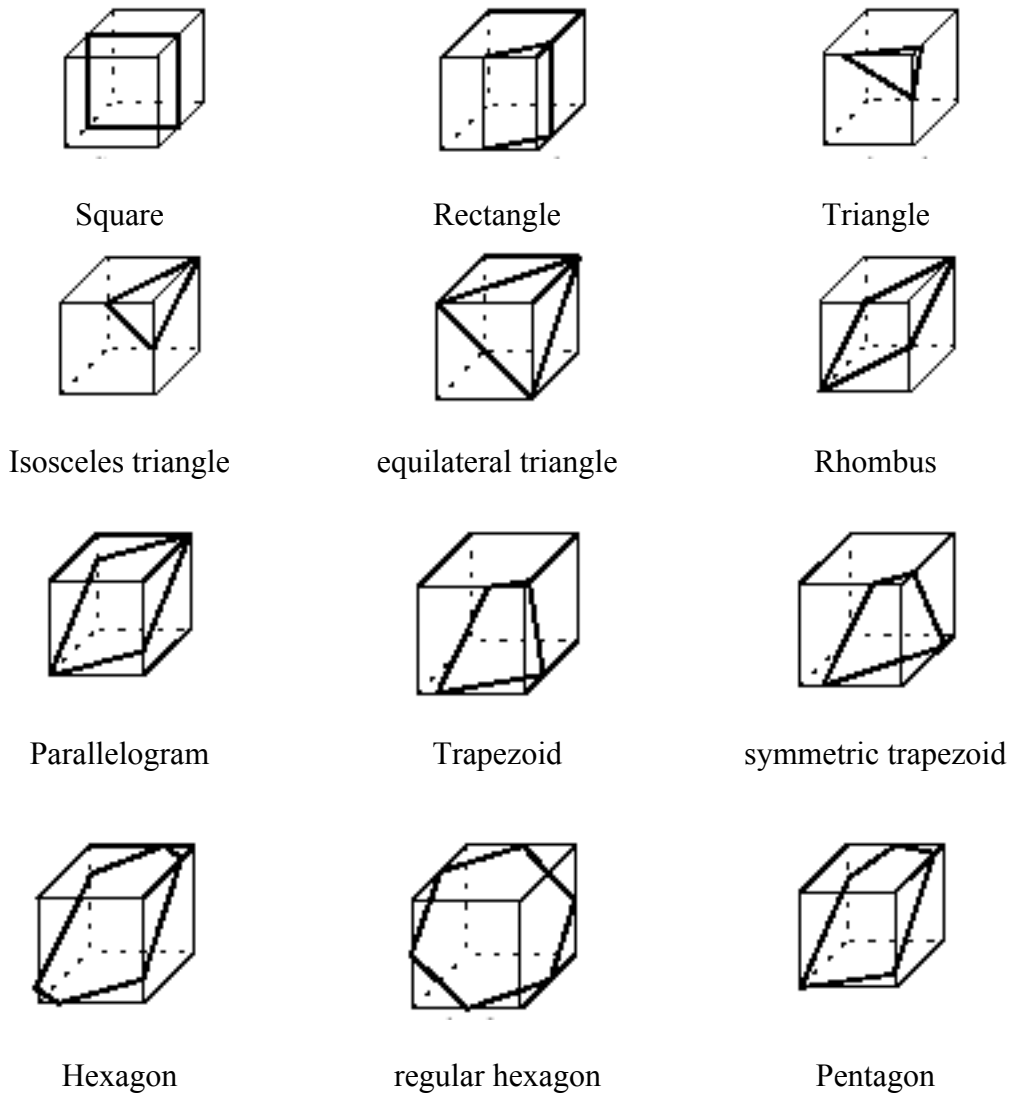
Although the screens are two dimensional and computers can produce only drawings, if we take time into account and use continuously changing pictures, it can help to giving the impression of a three dimensional vision. For example with CAD programs we can go around a planned houses. It is expensive to use models but at the beginning of spatial geometry, when students learn the fundamentals, we must use them. Moreover we need to return to the models if students stumble over difficulty (Parzysz, 1989).

3 Experiments

Our experiments were made with fourteen/fifteen-year-old students of three classes at Fazekas Mihály Secondary School in Debrecen (Hungary). The students have mathematics lessons three times a week. We arranged the students from each class into three groups. The members of the first group got a transparent model. The members of second and third group used our computer programs. We asked the students to find as many different shapes (plane sections) of a cube as they can. On the next day we asked them to draw the plane sections, and how they can be obtained.

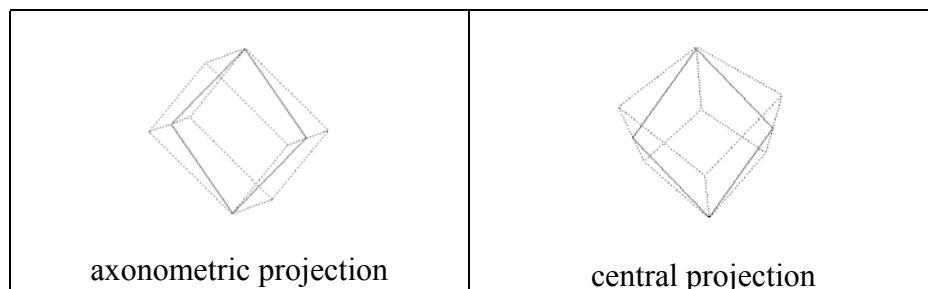
3.1 Cube's plane sections

All the solutions to our problem are represented below:



3.2 Our computer programs

We had no right to use Geospace or similar programs, so we wrote two small programs, which draw the cube's plane sections. We wrote these programs only for our survey, so we had to take into account the capacity of the computers available in the secondary school of our experiment. For this reason, we wrote DOS programs in Pascal.



Drawing 6

Both programs can be used in the same way; they only differ regarding the projection methods. At the beginning the centre of the cube is at the origin and always remain

on the z -axes. We can swivel the cube around the x , y and z -axes with keys R/F, E/D and W/S, respectively. With keys T and G we can pull up and lower the cube. The programs always draw the intersection of the cube with plane xy . We can make the cube's wire-frame disappear (or bring it back) by pressing key A. The programs use different colours to draw the intersection and the wire-frame. In the pictures of this article we are using solid and dotted lines. The plane xy is the fixed section plane. This plane xy remains parallel with the screen, hence we obtain undistorted sections. Of course we could have written the programs with a fixed cube and a moving plane, but in this case, we would have obtained ill-shaped sections. We chose the fixed section plane to simplify the recognition of the shapes. Drawing 7 shows two screen-captures from the programs.

3.3 Design of the experiment

We chose the cube because this solid is well known to everybody in everyday life and textbooks, for example (Antibi et al., 2000), (Hajnal et Némethy, 1988), (Hajnal et Némethy, 1989), (Gábor Endréné, 1996), contains a lot of examples about cubes. But none of our students were familiar with the plane sections of a cube, they had not met such examples before, hence their previous knowledge could not distort our results. We were interested in which plane sections they could find and if they could draw how they can be obtained.

The students who used computers learnt the usage of programs easily and rapidly. They found a lot of sections, even some that do not exist, for example right triangles.

We would have liked to give to other students material they could slice, for example plasticine, but we could not organize this. Further there was an opportunity for the students to spend significant time playing with this material so we gave them a transparent model. All the students had 45 minutes to find the plane sections. The next day they got the same time to draw the sections, and how they could be obtained.

4. Results

	Square	rectangle	isosceles triangle	equilateral triangle	rhombus	parallelogram	Trapezoid	symmetric trapezoid	Hexagon	regular hexagon	pentagon	right triangle	right trapezoid
Without any program	32	20	2	25	0	1	0	2	1	4	0	0	0
Axonometric projection	31 (6+9)	24 (4)	9 (1)	13 (5)	9	10	1	13 (2)	26	22 (2)	31	13	12
Central projection	30 (5+7)	23 (3)	8	13 (3)	10	9	2	12 (1)	26	23 (1)	31	12	12

The table above summarises the results of the experiments. The students who used transparent models drew only a few sections but all of them drew how these sections could be obtained. Usually, they drew only the most obvious sections: square, rectangle and equilateral triangle. The students who used computers drew many more sections, but usually, not how they could be obtained. In the last two rows, the numbers in brackets show how many students gave the position of the plane section in the cube. Some students used the computer to draw erroneous figures, we didn't count them, but we counted the cases when they found a non-existent section.

Many students answered that a right triangle could be a plane section of a cube. The section represented on Drawing 8. is very similar to a right triangle, but it is easy to prove that if a plane section of a cube is a triangle, then it cannot have a right angle. The students needed to control the program's answer, but almost all of them rashly accepted what they believed to see on the computer screen. In this case, the program did not state that some sections were right triangles; it just drew an almost right triangle. The students needed to analyse the result, but they eliminated it.



Drawing 7

The results with each program are quite similar, there seems to be neither advantage nor disadvantage in using central projection instead of axonometric projection. However, if one pays a specific attention to the numbers in brackets, one can see a slight difference: the students who used the axonometric projection have better results than the students who used the central projection. From the group, which used the central projection program, two students drew cubes with central projection. We presume that they used their visual memory. The others used the familiar axonometric projection. They reproduced the program's answer in a different projecting method. We can state that they really learnt what they saw. On the other hand, in the group using the axonometric projection program, we cannot evaluate the rate of students who really learnt the sections to students who drew from memory, because they all used the axonometric projection in their drawings. We had nine and seven students who inscribed only the square in the cube. They just drew the other shapes, or they drew wrongly, e.g. drew a hexagon on one face of a cube.

5 Conclusion and perspectives

We made a survey about plane sections. Some students used computer programs, others used models. The students who used a computer found more sections. A lot of them learnt that a cube has a pentagon plane section, but they couldn't show the

section plane. Therefore it is not enough to use computers. If we want to improve the students' visual orientation, then they need to use models as well. Computers are helpful, that is a fact modern teachers cannot deny. However, the best solution seems to be a harmonious integration of computer within more traditional methods, like using models.

According to our experiment the central projection has no advantage over axonometric projection, hence we do not need to teach central projection *instead of* the familiar axonometric projection. But we not need to forget the central projection. According to Bishop (1980) and Ben-Chaim (1988) we can improve the spatial ability with training. The *conversion* between different projections could be such a tool.

As the experiment shows, the biggest problem for students is the conversion from one projection method to on other. Indeed, it is not enough to copy drawings, students need to interpret what they see.

Hence, we plan setting up experiments, where students would need to converse different sections into other types of projections, and vice versa.

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