

STUDENTS' CONCEPTION OF A POINT: A COMPARISON OF PHYLOGENESIS AND ONTOGENESIS

Magdalena Prokopová

Department of Mathematics and Mathematics Education, Faculty of Education,
Charles University in Prague

Department of Mathematics, Faculty of Education, University of JEP, Usti n.L.,
prokopova@pf.ujep.cz

A point (and a straight line) are fundamental objects of Euclidian geometry, which is taught in basic and grammar schools. Philosophers meditated on the nature of a point a long time before Euclid (from 6th century BC). But it was Euclid (about 325 – 265 BC) who delimited the concept of a point by means of the definition in the First book of his *Elements* (Stocheia). Phylogenesis of a point is marked out by names such as Viete, Kepler, Leibniz, Newton, Bolzano or Cantor.

Students meet the concept of a point before they can formulate a definition or put it into their geometrical structures. Analysis of interviews with 16 students from the age of 12 to 18 years show that there is a strong parallel between the ontogenetic and phylogenetic aspects of the conception of a point.

We can identify the following phenomena of description of a point in the phylogenetical and ontogenetical development.

Phenomena of phylogenesis and ontogenesis

- a gradual movement from pictorial form of representation of a point toward abstraction
- a transition from a real object to an ideal object
- determination of a position
- size and position
- a unit of a space
- smallness of a point
- an indivisibility of a point
- an endpoint of a segment

Phenomena used only in phylogenesis a materiality of a point

Phenomena used only in ontogenesis

- a shape of a point

At first, geometrical theories view a point as a shape. The emphasis laid on the materiality of a point in the phylogenesis corresponds to the priority attached to the tactile sense. Students tend to prefer the visual sense.

Many students are able to think about a point only by way of a picture. It corresponds to the first phase of phylogenesis of planimetry. There are present notions both of a real object and an ideal object. Students use them both. Sometimes these ideas are contrasted to one another. The students are gradually able to “leave” the physical world and think in the ideal geometrical world. Only a few students admit the indivisibility of a point. This is possible only if they stop thinking about a point as a drawn object on a paper. Children do not understand a line or a segment as a set of points which is compactly ordered. This corresponds to Democritus’ geometrical atomism. They emphasize that it is “only one place” like a unit of space.