TEACHING AND LEARNING PLANE GEOMETRY IN PRIMARY SCHOOL: ACQUISITION OF A FIRST GEOMETRICAL THINKING

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Abstract

This paper presents an implantation of a teaching whose aim is to give Primary School students (8-10 years old) a first geometrical thinking about basic geometric objects and relations between objects. We first carry out an analysis of knowledge presented in the Primary School Official Curriculum. Then we present the working hypothesis upon which our"engineering" is based: changing simultaneously the size of the working space and the instrumentation is a factor of improvement for the transition from sensible space to geometric space and for the comprehension of geometric objects and their relationships. We further present an example of a sequence named "Square" designed and implemented in 2001-2002.

Teaching and learning plane geometry in French Primary School pose many problems.

Content of curriculum is not very developed and structured in the presentation of geometric objects and relations between these objects. Work in different spaces and sets of instruments are mentioned without any reason for their use. Different types of control, different registers to be employed are quickly evoked.

Nevertheless many misconceptions stay, even in adult population (Rolet 1996) about the first pieces of knowledge of geometric objects and relations. We note particular difficulties in reading a drawing, linking or differentiating spatial properties and geometric properties, and modifying a first reading. Difficulties also appear in giving a too important role to measurement, and not using several types of control.

Like other researchers, we think it is possible to address these difficulties early in Primary School. Several solutions can be found in Duval (1995), Berthelot and Salin (1992), or Hoyles and Noss (1996). We have tried to tackle the obstacles by joining these theoretical frameworks to build our own engineering, based on the principle of giving the students the same task in different instrumented spaces.

So, after a presentation of French primary school curriculum, we shall describe the theoretical framework for the design of sequences for 8-9 years old students. We present an example of a sequence and some elements of the associated analysis.

1. Analysis of "the knowledge to be taught"

1.1 Official Curriculum content

We describe here the main points concerning the plane geometry, contained in the Official Curriculum for Primary School in France.

One of the main purpose is to get the students familiar with some plane figures and to have them progressively passing from a geometry where objects and their properties are controlled by perception to a geometry where they are controlled by explicitation of properties and call for instruments. There is also mention of different contexts for problems of description and construction such as ordinary space, a sheet of paper and a computer screen.

In the knowledge content, we can find relations and geometric properties like collinearity, perpendicularity, parallelism, lengths equality, axial symmetry, midpoint of a segment. We also find know-how related to the use of different instruments and different techniques in problems of recognition, reproduction, construction, description and decomposition.

1.2 Our re-reading of the Official Curriculum

The Official Curriculum knowledge can be structured in concepts. We use the theory of Vergnaud (1990), who considers a concept as a group of three sets:

The set of situations which give sense to the concept (the reference),

the set of operative invariants including "concept-in-act" and "theorem-in-act"(the signified), and

the set of linguistic and non-linguistic forms for representing the concept, its properties, situations and treatment procedures.

We added a characteristic developed in our thesis (Rolet 1996) pertaining to the control. We distinguished several kinds of control. The "simple-perceptive control" works on spatial and/or spatial-geometrical properties of the drawing and uses eyesight as an instrument for construction and validation; it is associated to a purpose of getting a good likeness drawing. The "instrumented-perceptive control" works on spatial and/or spatial-geometrical properties of the drawing, and uses eyesight as an instrument for construction and validation but also several sets of other instruments; it is associated to a purpose of getting a drawing with certain properties. The sets of instruments have various forms ranging from ropes to software commands, passing through a classic set composed of non graduated ruler and compass.

Theoretical control is not used in Primary School.

We can distinguish four classes for the concepts explicitly or implicitly present in the

Curriculum:

- The *basic objects* like points, segments, lines, angles. They are not explicitly defined but only used in the different problems.
- The *constructed objects* like midpoint, circle, special quadrilaterals or triangles.
- The *basic relations* like length equality (isometry) and perpendicularity.
- The *constructed relations* like parallelism and symmetry.

For example let us consider Isometry. It is deemed as a basic relation. The main situations (with or without measurement) which give sense to this concept in Primary Schoolare recognition or construction of different objects: two segments with the same lengths (directly or indirectly), a segment *n* times as long as a given segment, and the midpoint of a segment. If numbers are not required, two types of operative invariants can be described. On the one hand we find the use of superposition in case of direct comparison, the use of a go-between length in case of indirect comparison or transfer and the folding. On the other hand we find transitivity of equivalence relation (to have the same length) and order relation (to be shorter than). If numbers are used, we must add operative invariants relative to the use of numbers and an instrument (ruler). Linguistic as well as non linguistic forms are not yet stabilized in Primary School.

We know that excessive and premature use of length measurement becomes an obstacle for the understanding of the concept of length (Rolet 1996). The "instrumented-perceptive control" with the graduated ruler prevents students from using other kinds of control. For example, to know if segments have the same length, students first measure them with their graduated rulers, without trying to estimate or to calculate this length.

Let us now consider the example of the concept of Perpendicularity. It is a second basic relation. The main situations giving sense to this concept in Primary School are the recognition or construction of a certain number of basic geometrical objects or relationships: a right angle in a plane figure, a right sector, a quarter of a turn (ninety-degree turn), vertical and horizontal lines, perpendicular lines in other positions, the shortest way from a point to a line and a perpendicular bisector or more generally an axis of symmetry. The operative invariants are, on the one hand, the use of "simple-perceptive control" or the use of "instrumented-perceptive control" for recognitions and constructions and, on the other hand, some convictions about existence and uniqueness of perpendicular line and about theorems on perpendicularity and parallelism. In Primary School, the used linguistic forms are "right angle" and "perpendicular lines", the figural symbol (on drawings) is rarely used.

2. Theoretical framework

First, we make general choices for the students' tasks, for the role given to writing, and for the teacher's role. The tasks must be interesting for students and must allow them a sufficient time for research thus we ask students to achieve non-obvious constructions with time allotted to them for conducting a research as a group. The conversion between the figural register and the textual register is an essential condition for comprehension (Duval 1995), so we ask the students to describe their figures and to write the instructions for their construction. Finally, we consider the teacher's role which is to organize situations but also to support students' learning, so we insist upon institutionalization (official recognitions of knowledge by students and students' learning by the teacher), and individual help and evaluation.

Beyond these general choices, we put forward the following particular hypothesis:

Changing the size of the space and changing the instrumentation can promote a better passage from sensible space to geometric space and a better comprehension of geometric objects and relations between geometric objects.

2.1 Different "instrumented spaces"

Brousseau (1983) specified two kinds of space: "meso-space" and "micro-space". The meso-space is a space defined with respect to the subject, including the space in which he can readily move and observe objects. The measure of these objects ranges between 0.5 and 50 times the person's size. Objects cannot be apprehended in one sight and a simple-perceptive control is difficult, even impossible. For example, a meso-space may be a playground, the floor of a gaming room or, more generally, the floor of an empty large room. Henceforth, we shall speak of a floor.

The micro-space is a space of interactions tied to manipulation of small objects. Objects can be seen in one sight and a simple-perceptive control is very easy. For example, a micro-space may be a sheet of paper or a computer screen.

Instruments play a great role in recognition and construction problems. They are not passive (Noss & Hoyles 1996) and, like these authors, we assume that abstraction comes from working in several contextualized worlds. We also agree with Laborde & Capponi (1994) when they say that the micro-world offered by the software of dynamic geometry named Cabri-geometry can be one of these worlds.

Thus we consider several instrumented spaces, defined both by their size and the instruments used. For example, we have:

The meso-space of the floor with ropes and plumb line.

The micro-space of a sheet of paper with strings.

The micro-space of a sheet of paper with a non graduated ruler, a compass and various templates figuring perpendicularity.

The micro-space of a computer screen using some commands of the dynamic geometry software Cabri-geometry.

2.2 Sensible space, geometric space and issues of modelization

Ideas of Berthelot and Salin (1992, 2000) were the basis we dealt with to consider the modelling issues. These authors said that one of the functions of geometry is to model the sensible space. Sensible space is a context for actions with approximate results. This modelling is essential for further work in theoretical geometry. The modelling is spatial-geometric if one must use pieces of geometric knowledge to solve a problem taking place into a sensible space with result in the same space.

The solution must go beyond the given problem, must be communicated to others and must lay upon an explicit model. The authors have the students practising this modelling on the basis of different sized spaces.

We see two ways for using issues of modelling in different sensible instrumented spaces. The first way consists in solving the same problem in two (or more) instrumented spaces. For example the students must firstly construct a square on the floor with ropes, then on a sheet of paper with classical instruments and finally on a computer screen with software commands. The second way consists in giving a problem in a meso-space, transferring it into a micro-space to solve it, and coming back to the meso-space for the application and the validation. For example, to construct on the floor a “path” with parallel edges, the students can look for a solution in a micro-space with strings.

2.3 Best comprehension of objects or relationships between objects

We will have a clue of a better comprehension of objects and relationships between objects if students know how to recognize, describe and construct them:

in spaces having different sizes,

with different instruments,

in different semiotic registers.

This point still has to be subject of further practical and theoretical work. In next section we present the “Square” sequence and we give elements of analysis.

3. The “Square” sequence

3.1 Presentation and methodology for collecting data

This sequence was designed by the researcher, presented to the teacher and discussed with him. The teacher alone managed it in his class (25 students), the researcher staying in the back of the room. For the sessions on the floor and in the computer’s room, the group of students was split into two halves. The other sessions took place in the ordinary class-room with the whole group. The sequence lasted ten one-hour sessions.

One task was given in all cases:

“Construct a square with given small equipment or instruments”, and 4 different instrumented spaces were proposed:

Space A: the floor with ropes (plus strong Scotch tapes and markers).

Space B: a large sheet of paper with irregular torn edges and strings (plus Scotch tapes and pens).

Space C: a sheet of paper with non graduated rulers, string, a compass, small paper strips, a template of perpendicular lines on a transparency, and a set-square.

Space D: a computer screen using limited commands of "Cabri-geometry" software: Point, Segment, Line, Circle, Perpendicular line, Label, Hide/show, Colour, Thick. Of course we presented instrumented-perceptive control by dragging free points.

In the "classic"instrumented space (space C) the teacher asked groups to exchange messages with instructions for construction.

In a final synthesis, a comparison has been made between the different construction procedures in the four instrumented spaces.

To support our next analysis, we have at our disposal:

- Written evaluations of initial knowledge of students.

- Video tapes of lessons: the camera was fixed and hold by the researcher, pointing at the teacher in collective times and pointing at a group of students in research times.

- Written messages and files of the students.

- Written evaluations after the sequence.

3.2 Analysis

The construction of a square, a well-known figure, was the occasion to work with objects like points, segments, lines and circles and with the two basic relations of isometry and perpendicularity. We give the principal analysis of student's comprehension of these concepts.

<u>Points, segments, lines and circles</u>

In the first and second instrumented space, students had difficulties in distinguishing the piece of rope (string) and the segment marked on the rope (string). They were ill at ease to superpose segments: there was a mix-up between lengths of used pieces of ropes, lengths of segments marked on the ropes and even lengths of pieces between end of segment and end of the piece of rope. In other words they took the piece of rope not like a part of a line but like a segment. They often hid the ends of segments and the vertices of the square under the scotch tapes put to tighten the ropes.

No unusual problems were encountered during the construction of the objects in space C. In space D we were surprised by the fact that all students kept lines to determine their squares and didn't feel it necessary to define segments and to hide lines. Segments were in this case seen as part of lines. Cabri-geometry was also a good instrumented space to give a real status to concepts like points: "visible" vs defined,"draggable" (or independent) vs fixed (or dependent). But understanding of these concepts still presents many difficulties for all students.

The circle was approached in Cabri-geometry environment. "Circle" was the only command available to produce an isometric segment on the perpendicular line to the first segment. The support of students was very strong. The teacher used rigid sticks on the floor and rigid pencil on the computer screen to show the trajectory of the free stick end.

Isometry

In the first instrumented space, size must be an obstacle for measuring. The expected procedure to construct isometric segments was superposition of segments marked on ropes. About one third of the students tried to create a system to measure the first marked segment and report it upon a second rope with their shoes and theirmarkers: the teacher had to intervene to reject a too long procedure for a too imprecise result. All students obtained isometric segments by using the first given segment as a template: transitivity of isometry was not used.

The aim of space B was to carry out the transition from a meso-space to a micro-space, without changing the instrumentation. The superposition was not acquired: surreptitiously the rulers appeared.

In space C the variety of instrumentations was very rich. The students used them with a preference for paper strips. The use of compasses to report lengths was not obvious: the compass is an instrument to draw global circles.

This was a burning issue in "Cabri-geometry"space. All students first drew isometric segments using simple-perceptive control. Secondly they drew two horizontal or/and vertical segments which seemed perceptively isometric. In both cases, the dragging of points (often made by the teacher) showed a bad construction. The teacher managed a transfer from meso-space to micro-space, linking the trajectory of the free extremity of a stick on the floor with a part of a circle. The circle was finally used.

Perpendicularity

In space A, the problem of perpendicularity was tackled after the problem of isometry. It was more difficult because of the size and the position of the first askew segment taped by us on the floor. In theory, students couldn't use stereotypical positions and "simple-perceptive control" in constructing perpendicular lines. Nevertheless it was only after failing to construct the square with four isometric sides that they accepted to make an instrument (a template) to report right angles. The construction of a template was more difficult but also richer than expected: students were not aware of particular right angle done by a vertical line crossing an horizontal line; on the other hand they made very different templates with very long wooden sticks found in the room, before making a rope template with two sides and eventually building a “rope-set square” with three given sides. Students more often used their template to verify their perceptive right angles than to construct them.

In space B, students made a string-set square. They found its use very uncomfortable.

In space C, students had several ways to construct aninstrument: making a template with strips (and Scotch-tape) or with a sheet of paper twice folded in halves; using a given template of perpendicular lines on a transparency, or a set-square. The teacher asked them to perform several constructions. The students didn’t give preference to classical instruments. The use of transparent template, frequently used for verification, was actually very difficult for the construction because it needed an anticipation on the right angle to be constructed by prolongation of lines drawn outside the template. The teacher showed the students the superposition of all the templates.

In Cabri-geometry students first again drew their perpendicular segments with simple-perceptive control, using horizontal and vertical orientations. The function allowing the dragging of points lead them to use the previously introduced command“Perpendicular line”. Two things are worth noticing: right angles as well as perpendicular lines are recognised in all orientations; there is a real influence of the software command upon the student’s expression.

Textual register

The writing of the instructions for construction was a partial failure. Students’ difficulties came from several sources: They forgot geometric properties; In spite of the fact that different instruments were used, instructions were very contextualized; Without labelling points, the latest instructions were not clear; On the other hand, there were no superfluous properties.

4. Clues coming from reinvestment

Two months later we designed a new sequence in the same conditions that the first one. Students had to construct a rhombus with a diagonal twice longer than the other.

The students showed acquisition about midpoint and report of lengths: in the first three instrumented spaces, they constructed the lengths of the two diagonals and their midpoints without any difficulty. More particularly, in space D, the commands were manipulated in a better way, and students were able to use twice “Circle” to resolve the problem. But the problem ofthe right angle was not yet solved for all students who tended to use the instrument to verify rather than to construct. At last, the writing of instructions for construction was still very hard to do.

We noticed that, in Cabri-geometry, several constructions of the rhombus were found; the operative apprehension of the figure was better; the lexicon and the syntax of the menu were progressively employed.

A more documented analysis of this experiment still has to be done; it will take place next year, with the aid of video records and transcriptions.

We shall also design and implement sequences for parallelism and symmetry in the last year of Primary School.

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