### INTERACTION IN COLLECTIVE WRITING PROCESSES AND EARLY MATHEMATICAL LEARNING

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### Abstract

The focus is put on the particular interrelation between collective writing processes and early mathematical learning. Introducing the framework of the two dimensions of orality and literacy and adapting it to mathematical education, empirically grounded contributions to the construction of theoretical ideas are developed. A comparative analysis based on the interpretation of two small-group episodes enables an appropriate understanding by interrelating this language approach and the reconstructed double interactivity of collective writing processes.

### 1. Introduction

Many studies of mathematical learning depict that "students show an inappropriate approach to the subject and its learning" [2, 347]. It has been argued that writing can contribute to the learning and meaning-making process and can provide a valuable means to facilitate the development of conceptual understanding [See e.g. 2; 10]. This assertion can be theoretically supported by BRUNER and VYGOTSKY. In order to describe the externalized form of thoughts and collective activities, their 'hard copy', BRUNER uses Meyerson's term "oeuvre" [3, 22]. The production of such identity-bestowing w125

orks is regarded as the main function of all collective activity. Oeuvres embody thoughts and intentions "in a form more accessible to reflective efforts" [3, 23]. This creation of a shared and negotiable way of thinking in a group may be the basis for intense reflection. BRUNER calls this demand for creating oeuvres "externalization tenet" [3, 22pp.]. The Vygotskian view on the characteristics of writing can be summarized as follows: "Written language forces the child to act more intellectually. It is forced to increase its awareness of the process of speaking" (Translated by Marei Fetzer [15, 228]). It remains an open question how a deep and personal understanding of a mathematical idea will take effect.

Studying the numerous approaches of writing in mathematics education reveals that research is limited to the products of students' writing. With the process itself being neglected there is no possibility to get hold of the aspects of learning involved in the act of writing. In this article novel theoretical ideas are developed to enable an appropriate understanding of the processes of collective writing in early mathematical learning situations.

### 2. Methodological context

Developing theoretical constructs of writing in mathematics education, this empirically grounded study is to be situated in the context of interpretative classroom research in the tradition of BAUERSFELD, VOIGT and KRUMMHEUER [12; 4; 11]. Accordingly, analysis is based on transcripts of particular video-taped episodes of discourse of the mathematics classroom. The systematic interpretation of those extracts serves as the empirical basis for the construction of novel elements of a theory [e. g. 11; 9]. In order to confirm the developed theoretical approaches and to valuate the generalization of the theoretical conclusions, selected data is compared to each other in comparative analyses.

### 3. The two-dimensional framework of orality and literacy

Language is often divided into the two categories of orality and literacy [See e. g. 1; 6; 7; 14]. In fact, these characterizations do not seem to meet reality appropriately. It is merely possible to tell between communicative forms that are either belonging to orality or to literacy. Hence a one-dimensional picture of spoken and written language is inappropriate. KOCH and OESTERREICHER developed a two-dimensional approach of orality and literacy referring to SÖLL [14; see also 7; 8]. They establish a distinction between medial and conceptual aspects. In order to distinguish between the two levels SÖLL introduces the terms "phonisch" and "graphisch" (phonic/graphic) when referring to medium and "mündlich" and "schriftlich" (oral/written) when concerning the conceptual level.

The medial dimension is according to KOCH and OESTERREICHER dichotomous. [13, 193; 7, 17] Something is carried out either in a written form and can be perceived with the eyes or it is spoken and can thus be heard. Concerning the conceptual side a wide range of communicative forms, graded in between the two poles "written" and "oral" of the conceptual continuum, can be realized. Independent on the medium, language may show aspects related closely to an either oral or written conception, it may be characterized by a rather informal style or dominated by a written design. Regarding phonic communicative forms talking to a good friend for example is to be located further towards the oral pole on the conceptual scale than a lecture. Referring to the graphic section of medium a newspaper article is closer to the written end of the conceptual continuum than the writing of a diary. KOCH and OESTER-REICHER introduce a number of parameters to scale the conceptual continuum and to enable the allocation of communicative forms within the poles. One of the parameters is e.g. the social relation to the opposite number, another refers to the grade of flexibility of the communicated topic [7].

The conceptual side of orality and literacy is derived from the continuum of immediacy and distance [7, 13]. "Conceptual orality is the language of immediacy, conceptual literacy is the language of distance." (Translated by Marei Fetzer [1, 70]).

Figure 1 depicts the two-dimensional framework of orality and literacy including some illustrating examples.



Fig. 1 The two dimensions of orality and literacy

#### 4. The interrelation between the two-dimensional framework of orality and literacy and writing processes in mathematics education

This two-dimensional framework appears to provide a helpful approach as a basis to describe interactional processes connected to writing in primary classes.

Adapting the aspects of writing processes in mathematics education to this language approach provides the following model:

When starting school, children are familiar with spoken language. Concerning the medial dimension they cling to the phonic mode. On the conceptual level their language is characterized by restriction to certain contexts and communicative immediacy. Accordingly children's language when starting school should be situated in the upper part on the left hand side of the graphic presentation of the two-dimensional framework. In contrast, elaborated mathematical language is characterized by items and symbols. They are a constituent part of mathematical language. Understanding must be possible in any given context. Being dominated by written forms, elaborated mathematical language is to be placed graphically speaking in the right part close to the bottom.

Writing processes are to be located in the graphic section. But where do they belong conceptually speaking? Is distance their dominant factor or are they characterized by immediacy? Discussing the communicative parameters (see above) one will conclude the dominant presence of immediacy. Besides, most of the parameters show the potential of variation and allow distance as well. The social relationship to the opposite number may serve as an example: When writing to a friend in the own class the addressee is very familiar. In the case of writing to pupils in a different class distance would be the dominant factor. Consequently writing processes can be located in the whole right part of the diagram. These processes seem to enable a development from conceptual immediacy to distance. The development might be interpreted as a movement from top to bottom within the figure.

Mathematical learning in the context of writing processes can accordingly be described as follows: As children get familiar with spoken as well as written mathematical language right from the start, a medium-related shift of communicative forms is provided initially. Besides, with the language of immediacy as the starting point a gradual development to a conceptually more distant language seems possible. Figure 2 visualizes the interrelations.



*Fig. 2* Writing processes in the context of the two-dimensional framework of orality and literacy

## 5. Analysis of an episode in the specific context of conceptual orality and literacy

The following episode is analysed on the basis of the two-dimensional approach of orality and literacy. In the extract two nine and ten year old girls are working on the task:

### Share 1000 dots among three children.

They are supposed to write about their problem-solving process.

First they distribute 999 dots to three piles representing the three children. Afterwards Martina starts to cut the last dot into little pieces. Up to this time the girls use the expression 'the little ones' for the tiny pieces that result from the division of the last remaining dot. This term emerged from the interaction and can be regarded as commonly shared. To save space only the summary of the transcript and the interpretation is given below.

In the analysed episode Sonja announces: "I'm going to write already." But instead of beginning to write, she asks: "Yes – but how do the little ones **count**?" As Martina does not react she insists: "Yes- how much **do** the little ones **count** – how m u c h ?" Martina answers: "A quarter- half of a quarter."

The term for or the value of the little pieces appears to be of central interest for Sonja in the context of the writing process. Concerning spoken language 'the little ones' seemed to be a satisfactory expression whereas in the context of written language it does not seem to be adequate. Written language appears to evoke the endeavour to increase precision in describing mathematical ideas or/and a deeper understanding of the mathematical subjects. Sonja's acting can thus be interpreted as striving for a 'context free understanding'<sup>1</sup> and an increasing degree of conventionalism of her individual mathematical language. Martina answers by paraphrasing what she did. Seen as a form of (story) telling she uses language of immediacy. In contrast, her choice of words gives reference to conceptually written forms. Accordingly it can be interpreted as an attempt to conventionalize. Martina appears to move on the edges of language of immediacy and conceptual literacy.

# 6. Empirically grounded contributions to the construction of a theory (Episode 1)

This episode can be interpreted as a qualitative development from language of immediacy towards conceptual literacy. Writing seems to be interrelated with the striving for increasing precision and conventionalism of the individual mathematical language. The attempt to describe mathematical ideas more precisely might be connected to a deeper understanding of a mathematical topic.

## 7. Analysis of a contrasting episode

The approach of the two dimensions of orality and literacy seems to be an appropriate means to describe certain processes of writing in mathematics education. The analysis of the second extract modifies the generated elements of the theory and makes them more sophisticated.

In the selected episode Rasputin and Marius (age 9 and 10) are working on the task:

## 524 How much is missing up to 1000?

Besides they are expected to communicate their mathematical activity in writing.

Marius starts thinking aloud as soon as the teacher leaves the boys alone. Trying to give reasons for his spontaneously worked out result 486, Marius mentions the number 76. As Rasputin interrupts his friend's attempt, Marius tries to work with the help of one-hundred-dot-tables. He does not succeed and demands Rasputin's assistance. This is where the episode selected for the comparison begins. Here only a summary is given.

Instead of providing help Rasputin suggests to write down their names first. Without any obvious hesitation Marius follows the proposal by additionally remarking that he is going to write down his name by himself. At this time of the interaction Rasputin brings the typeface into discussion: "Cursive writing or printing?" Marius reacts immediately: "Cursive hand writing – we are no babies any longer." (The children started their writing experiences in printing and changed later on to cursive hand writing.)

<sup>&</sup>lt;sup>1</sup> Members of an interacting group strive for a shared meaning and understanding of that certain situation in the given context. 'Context free understanding' refers to the possibility to understand an expression or action in any given context - independent on time and space.

This extract evokes the impression of a multistage decision-process regarding the negotiation of meaning in the interaction. Apart from discussing aspects of time, social and subject matters, the need to take certain aspects of writing into consideration emerges. First the time and subject of writing are settled. The boys want to write their names - instantly. Apparently they agree on the academic task structure (ATS)<sup>2</sup>: They concur on the next task to be tackled. Afterwards the social participation structure (SPS)<sup>3</sup> is negotiated. However, they do not reach an explicit agreement on the question of who is supposed to carry out the writing. In contrast, aspects concerning the style or form of writing are discussed explicitly. Rasputin brings up the topic formulating the question: "Cursive writing or printing?" Taking into consideration how explicitly the writing matter is dealt with, one can interpret this topic as a new subject of negotiation. In addition to the well known interactive aspects of ATS and SPS thoughts about the way of presenting get increasingly relevant. Marius reacts immediately to Rasputin's question. He appears to include the potential reader: "Cursive hand writing - we are no babies any more." The second part might be understood as an argument or reason for the first part of the utterance. Considering the intended effect on the reader he seems to prefer cursive writing to printing. He connects the latter typeface with babies and thus generates an associative interrelation of both aspects. Accordingly printing might correlate with novices and beginners or with an unprofessional way of writing or problem-solving. In contrast, cursive hand writing might be associated with adults and elaborated writing skills.

With Marius reflecting on the effect on the potential reader his acting can be understood as the striving for conceptually written forms aiming towards conventionalism. Hence the "oeuvre" [see above] is supposed to evoke the impression of a skilled writer. Instead of being temporary or provisional the product of writing is intended to appear to be conventionalized. Conceptually oral forms are to be replaced by reflection and conceptual literacy.

# 8. Empirically grounded contributions to the construction of a theory (Episode 2)

Interaction in general as well as interaction with the writing partner is an oral act, thus phonic referring to medium. The interacting persons know each other, immediacy of time and place is given. Accordingly, the process of negotiating is character-

<sup>&</sup>lt;sup>2</sup> The academic task structure deals with the insight within a sequence of solution steps and their chronological order but is not identical with logical considerations about a sequence of solution steps according to the subject matter of mathematics. "The academic task structure (ATS) can be thought of as a patterned set of sequencing in the subject matter content of the lesson." [5, 156; see also 10, 16p.]

<sup>&</sup>lt;sup>3</sup> The social participation structure (SPS) involves the sequence of interactional moves and the chronological change of speakers. "The social participation structure can be thought of as a patterned set of constraints of the allocation of interactional rights and obligations of various members of the interacting group." [5, 156; 10, 16p.] ATS and SPS are mutually dependent on one another.

ized by communicative immediacy and is located close to the oral pole of conception. With the phonic interaction shifting to the discussion of graphic matters the potential reader has to be considered. Reflections on the way of presentation get necessary reaching beyond the edges of spoken language. Hence this transition can be interpreted as a medium-related shift. In this specific situation a second aspect of interactivity occurs. Consequently collective writing processes are to be seen as double interactive processes. On the one hand the communicating individuals have to negotiate meanings and adjust their interpretations and intentions face to face in order to reach agreement on the writing matters. On the other hand they are interacting with the potential reader who is not present at the moment of writing.

The interaction with the imagined addressee encourages aspects of planning, presentation and effect. Time and space are characterized by distance. Thus the preoccupation with ways of presenting a written product forces a gradual development towards language of distance and conceptually written forms. In fact, double interactivity in writing processes appears to be explainable in terms of the two-dimensional framework of orality and literacy. Figure 3 depicts the generated model and visualizes the interrelations.



*Fig. 3* Double interactivity of collective writing processes in the context of the two-dimensional approach of orality and literacy

## 9. Results of the comparative analysis

The analysis of the second episode confirms the developed theoretical ideas gained on the basis of the interpretation of the first extract. The results of the comparison are summarized below.

In the context of interactive writing processes a gradual qualitative development from the language of immediacy to communicative distance and conceptually rather written forms can be observed. This development can be interrelated to the reconstructed double interactivity of collective writing processes. Thus the shift to the application of a further elaborated and conventionalized mathematical language can be described more precisely:

The face to face interaction of the two authors allows a restriction to the language of immediacy. Shifting to the negotiating of writing matters, the interaction with the potential reader becomes an inherent component of reflection. As the addressee is not present but only imagined, communicative distance dominates the language. The interacting authors are forced to reflect on conventionalized forms, on aspects of conceptual literacy. In this way the development from the exclusive application of language of immediacy to written orientated forms emerges.

The interrelation of the concept of double interactivity and the framework of orality and literacy provides at the same time a valuable means to describe the genesis of a deeper and personal understanding of mathematic subjects:

The development of mathematical language within the conceptual continuum mirrors the endeavour to gain a deeper conceptual understanding and can be understood as the cause to achieve a higher intellectual level. Integrating the activity of interacting with an imagined reader might provide the potential of (mathematical) learning in the context of writing processes. The striving for increasing conventionalism and decontextualization, the efforts to describe specific ideas more precisely and the necessity to consider the potential reader creates a growing degree of complexity. Forced to apply language of distance the speed of interaction seems to slow down. This is where the reflective components of collective writing processes in mathematics education might be situated. Accordingly an empirically grounded correlation to the Vygotskian view on the effects of writing is indicated and can be mathematically adapted: 'Writing forces the development of a further elaborated mathematical language as well as a deeper understanding of mathematical ideas: Written language forces the child to act more intellectually.'

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