TOWARD A DIDACTIC PRACTICE BASED ON MATHEMATICS LABORATORY ACTIVITIES

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The introduction of computer technology in teaching has created the conditions for new teaching modalities of the constructive type. Nevertheless, the transformation of didactic practices is not guaranteed only by interaction with the tool. As a matter of fact, it requires the creation of the necessary conditions to the point where the pupils can, by means of the tools, conduct effective experiences with regard to the "mathematical objects" of teaching and then utilise them as a basis for the internalising of appropriate mathematical meanings. These conditions bring us to a particular didactic activity that, in this paper, will be called mathematics laboratory activity (MLA). At the centre of the analysis of this report there is the definition of MLA and the study of what characterises it.

Introduction

It is well known that the introduction of new tools into didactic practice can bring profound transformations to the way in which it is structured and consequently to the way in which the participants are then involved in the activities that characterise it.

This is obviously not specifically connected to the presence of ICT; many experiences conducted with didactic materials of other types have demonstrated that tools, in general, influence the teaching-learning process.

Many researches of didactic innovation based on the use of ICT in the field of mathematics education have produced a vast and rich repertoire of examples of activities which are profoundly different from that conducted in the didactic practice which we will call "traditional".

The introduction of computer technology in the schools has encouraged the large-scale diffusion of teaching practices not based on a transmissive model, but on a more constructive model. For transmissive model, we mean a type of teaching that has at its centre the "contents" to transmit and the method for their presentation to the students, while the constructive modalities of teaching focus the attention on the student with the aim to construct the experiential base necessary to allow him to come into contact with mathematics concepts. In this framework, the tools which are introduced into the didactical practice support and mediate the construction of the experiential base necessary for learning.

However, the results of many researches have highlighted the fact that the introduction of tools into the didactic practice does not improve by itself the teaching learning process.

It has been shown that such improvement, when realised, does not depend on the use of the tool itself, but on the transformation that the whole learning environment has to

undergo with the introduction of technology (Bottino & Chiappini, 2002). We underline that such transformation is not automatic and can obviously emerge in other situations in which tools not based on ICT are used.

In order to define the new type of activity that has to be developed by means of the mediation of the ICT and to distinguish it from the traditional practice, we will use the notion of the Mathematics Laboratory Activity (MLA) (Chiappini, 2002).

Research problem

In this paper we will use the notion of MLA to indicate a particular type of didactic practice mediated by the use of ICT, aimed to the construction of a rich and differentiated experiential mathematical base in the students that, we think, is necessary for the construction of appropriate meanings in mathematics.

This paper has to be considered as part of a more general research that has the aim to study the role of ICT in transforming the mathematical objects of teaching and in structuring and mediating the whole didactical practice, creating the conditions so that a mathematical knowledge domain, which is formal and abstract, can become a field of experience (Boero *et al*., 1985) for students and teachers.

The conditions for the transformation of mathematical knowledge domain into a field of experience can occur only in a didactical practice of laboratory, in which tools, embedding specific mathematical resource, are used.

We call MLA such a didactical practice and we are interested to study what characterize it and differentiate it from the ordinary didactical practice.

In this report we would like to outline the theme of research in its complexity and to focus the attention on some problematic aspects that we have observed during the experimentations conducted..

Theoretical framework

Research in the psychology of learning has highlighted that the approach to knowledge occurs fundamentally in two ways: the perceptive-motor one and the symbolic-reconstructive one. The first approach not only characterises the first phases of cognitive development, but is also involved in the most advanced learning processes, mathematics included (one just thinks about the role played by graphical type representations in mathematics or the role of mathematical machines for the development of specific ideas). It involves action and perception and produces learning based on doing, touching, moving and seeing. The second modality of knowledge is the symbolic-reconstructive one, which is formerly present at the beginning of the cognitive development of the child. It works on symbols (linguistic, mathematical, logical) and reconstructs in the mind "objects", meanings and their mental representations. It is a more sophisticated way to know. It requires awareness of the procedures and the appropriation of the symbols used and their meanings.

It is important to note that both of these approaches to knowledge are mediated by tools of different nature which carry out their mediational functions in ways which

are profoundly different (Vygotskij, 1978). Vygotskij distinguishes between two types of mediator tools for each human activity: technical tools and psychological tools. The first, directed to the control of nature, mediate the action of the individual toward the outside. The use of such tools brings a transformation in the objects, that is, it produces external effects on the object that can be controlled at the perceptive-motor level. The psychological tools, instead, are directed to the mastery or the control of one's own processes of behaviour. They are directed internally and do not produce any transformation of the object. Vygotskij, moreover, notes that the development of the higher psychological functions, involved in all the processes of comprehension and of the construction of meanings, is the result of an integration of technical tools and psychological tools in the activity. It is precisely by means of this integration that it is possible to realise that which Vygotskij defines as the process of *internalisation*. This process consists of an internal reconstruction of an external practice, in which is involved the use of technical tools.

The acquisition and application of technical tools extends the sphere of influence of the individual, allows the developing of new experience, while the acquisition and application of new psychological tools raises the levels of influence and awareness of the processes and phenomena which characterise human activity. Both the technical tools, controllable by means of a perceptive-motor approach and the psychological tools, controllable by means of a symbolic-reconstructive approach, are important for the learning process in mathematics.

In this regard, we note that "traditional" teaching in mathematics, which we have characterised as transmissive, is based, almost exclusively, on a symbolic-reconstructive approach. The teacher tries to put the student in contact with the mathematical objects by means of the use of psychological tools which require a high level of abstraction to reconstruct, in the student's mind, the properties which characterise such an object.

We observe, however, that this type of approach, disconnected from the construction of a rich experiential base, can create obstacles for learning. For example, various studies have shown that supplying an interpretation or an explanation of a mathematical concept, without having created among the participants in the activity the experiential conditions which are at the base of its sharing, generally produces resistance, is useless or causes confusion.

The MLA is characterised by the integrated use of technical and psychological tools in didactic practice and is finalised to the construction of the experiential base which is necessary for the appropriation of the mathematical meanings involved in the teaching/learning activity. Such an experiential base can be developed only through activities in which appropriate tools, which can carry out their mediation function both at the external and internal levels, are used.

Saying this, various questions are left open.

- By means of which processes can a piece of abstract mathematical knowledge be incorporated into a tool? In which way can it be made available for a learning experience?

- Which characteristics must a tool possess to allow the developing of an appropriate experiential base in relation to a determined domain of mathematical knowledge? What role do the tools play in the construction of an experiential base in mathematics?

- If a tool incorporates some mathematical knowledge, is the construction of the meaning sufficiently guaranteed by the use of the tool or does it depend on the type of activity that is done with its mediation? What is the rôle of the exploration of the tool in the construction of the meaning? What are the conditions that allow the exploration of the tool to be productive at the learning level?

In the following paragraphs, we will point out several hints for answering these questions.

The reconfiguration of the teaching objects mediated by the ICT

The notion of the MLA is based on the idea that the mathematical objects of teaching are not objects which are accessible only by means of a symbolic-reconstructive approach, but also starting from a perceptive-motor approach. As a matter of fact, they are objects that can be re-configured by means of the appropriate use of specific technologies. The transformation of the mathematical object of teaching begins to come alive through its incorporation in a technological tool of *specific mathematical "resources"* for the activity. It is important to observe that the use of the term "resources" in this context calls into question the notion of *"culture".* As a matter of fact we intend "culture" as the field of resources that can be taken as reference in the development of a specific activity. Culture is seen here as something concrete which gives shape to each activity and human practice aimed to transform the material world itself, assigning it meaning and value.

We observe that the re-configuration of the mathematical objects carried out with an ICT occurs by utilising the interactivity and the potentiality of visualisation and computation made available by modern computer technology.

Through the process of incorporation of a mathematical resource in a computer system an important cultural transformation is accomplished. A resource that, up to that moment, could exist only on a purely mental plain because it was accessible only through a symbolic approach, is reified and is made available in a tool with which the subject can interact being able to observe the effects produced by the system. For example, Cabri-Géomètre incorporates, in its interface, the axioms of Euclidean geometry (and also some basic theorems) which constitute the basic resources for the realisation of geometric constructions. The *Algebrista* (didactic software which allows the carrying out of some algebraic transformations in a symbolic form) incorporates in its interface the associative, commutative and distributive properties

of the operations which are the basic resources for algebraic transformations (Cerulli & Mariotti, 2002). The functions of the CAS incorporate, for example, algebraic transformations, resolutive formulae of some equations, the notion of Cartesian plan and graphics. Some microworlds of Ari-Lab incorporate, in their interface, the potentialities offered by a numbering system that constitutes the basic resources for the solution of arithmetic problems (Bottino & Chiappini, 2002).

The resources embedded in these systems are presented to the users in the interface by means of specific objects with which the pupil can interact. We call such objects *computational objects*. Through the interaction the users receive feedback from the system in terms of an external effect, mainly an effect of visualisation.

We observe that the computational objects reify mental processes involved in the interaction with the aspects of mathematical culture of reference for the system. They reflect the experience of other people who in the past have tried to resolve problems similar to those with which the student is involved, or who have invented the system in use to make the solution more efficient (professional software) or the learning of a resolutive strategy easier (didactic software).

Moreover, we observe that comparable systems are different for the granularity in which the mathematical resources have been incorporated in them, other than for the modes of interactions and the way in which the results are presented by means of the interaction.

At this regard, it is important to note that, in general, software produced with a didactic aim is built in a such a way that a command produces an easily identifiable external effect, while in other software, made for professional purposes, the mathematical knowledge incorporated can be, one might say, "hidden". Let's compare for example the transformations of a simple algebraic expression done with didactic software (for example the above mentioned *Algebrista*) with a much-used CAS in didactics, like *Derive*, but not finalised for the learning of the first elements of algebraic language. In the first case, it will be possible to identify the application, for example of the distributive property and to have feedback from the system, while in the second case this will be difficult to separate from the other transformations incorporated in the command "Factorise". Both of the types of software can be used in an efficient way in an MLA. The didactical usefulness of a system depends on how the students, on the basis of their own experience of the domain of knowledge of the activity, can interpret the use of the resources incorporated in the system in relation to the task to be resolved and to the way in which the teacher manages the teaching/learning activity.

By means of the process of incorporation of mathematical resources in a tool, new technical tools are made available which permit to make experience within the mathematical domain of reference for the system. Incorporation allows the student to work with mathematical resources through perceptive-motor operations as in concrete practice. This is particularly evident when the systems involves graphic aspects, as

happens when one constructs or transforms a geometric figure with Cabri or one represents the graphic of a function with Derive, while it seems to escape more easily the perceptive control when one does a transformation on an algebraic expression using *Derive* or *Algebrista*. In reality, also in this second case, the perceptive-motor aspect is always involved (the application of the object "commutative property" on a part of the expression produces an effect on the form of the expression which can be grasped at the visual level).

By means of the process of incorporation described, the possibility to utilise mathematical resources for developing experience in the solution of specific mathematical tasks is offered to the student.

Role of the tools in the construction of the experiential base

As we have seen before the notion of MLA is based on the idea that, in order to develop genuine mathematical knowledge with the pupils, it is necessary to integrate the perceptive-motor approach with the symbolic-reconstructive one in the didactical practice.

This integration will be possible if, in the activity, a productive dialectic between the use of technical tools and psychological tools is developed.

On the basis of our experience, we can confirm that this can occur only if the interaction with the tool available for the activity:

- Produces effects, controllable at the perceptive-motor level, able to suggest to the students possible actions in the solution of the task at hand and/or encourage the emergence of objectives for it

- Makes available possibility of validation of the strategy employed in the solution of the task and/or supports for the planning of a strategy which are controllable at the perceptive-motor level

- Produces effects that, in the context of the activity, can then be interpreted as mathematical phenomena, by way of discussions that metaphorically use that which the interface *dynamically and concretely* exhibits in the interaction.

To clarify what has been said, we can consider some activities that can be carried out with Cabri. In particular, let's consider the simple construction of a geometric figure, for example a rectangle, in such a way that it "stands up to" dragging, that is, that, transformed with the dragging function, it remains a rectangle. In the first phase it is possible that the pupils construct the figure basing it solely on the perceptive aspects even though choosing procedures which are different one from the other. The application of the dragging function to various points of the figure allows the validating of the constructions and triggers the process of elaboration of strategies for the carrying out of the task. The different constructions and the mathematical objects used in them (for example perpendicularity and parallelism) constitute the point of departure for the symbolic-reconstructive phase that can bring one to meaningful formulations from the point of view of the construction of concepts (Pesci, 2001).

Tools, activities, construction of meaning

In the preceding section we have highlighted that a tool is always the result of a cultural evolution, is produced for specific aims and consequently incorporates an idea.

That does not necessarily imply that the tool is the source of the meaning or that, at the didactic level, the meaning can emerge from the interaction between the student and the tool. Which meanings, for example, could be included in tools such as *Cabri* or *Ari-Lab* or *Algebrista* in relation to the domain of mathematical knowledge of each one of them?

We consider those positions, which attribute a meaning to the tool because of the fact that in it specific mathematical resources are incorporated, to be profoundly mistaken. We think, instead, that such positions can be very dangerous in didactic practice.

The meaning resides in the aims for which the tool is used; in the plans which are developed for using it. These plans, expressed in socially shared language, gradually realised in the course of the activity with the tool, constitute the meaning, in that they reflect the course of the activity mediated by the tool and orientated toward an aim.

This reflection brings us to affirm that an MLA is characterised by two general types of aims:

- There is one aim relative to the task or problem that is confronted during the activity

- There is one aim directed to set the resolutive activity of the task or of the problem within a theoretical frame

We note, as a matter of fact, that when, in an MLA, one uses resources incorporated in a tool, one always has to deal with a double valence of their meaning: there is a meaning connected to the practical and concrete use of the resources in relation to the task which is being confronted, and there is also a meaning relative to a rationalization of such use and to its organisation within a theoretical construction.

In relation to the first type of aim, we observe that an MLA mediated by the use of a specific tool can be productive at the didactic level if in it there emerge breakdowns or contradictions which can be resolved exploring and exploiting, in a direct way by the pupil or in the social practice of the class, the resources that the tool makes available. It is by means of the resolution of such contradictions, through the exploration and use of the resources of the tool, that the resolutive strategies of the students can become objectives for the task and can evolve.

From what has been said, it comes out that the exploration of a tool is useful not in itself, but if it is the expression of a contradiction that emerges within the activity. In an MLA a contradiction manifests itself when the operative work with a resource of the tool used in the solution of the task is interrupted because a gap occurs between that which the student had anticipated obtaining and that which truly he has obtained by the use of the resource or because contradictions emerge between the participants

in the activity. In this case the exploration of the tool must be seen as a change of focus in the activity. Attention is moved from the task to the study of how the resources of the tool function and to the effects that they produce. If the tool is appropriate for supplying support for the resolution of the contradiction (directly by means of interaction with the system or in the class' social practice mediated by the system) through exploration new objectives for the task can emerge. In the opposite case the solution of the contradiction will have to be managed by the teacher using different tools.

In relation to the second type of aim, we observe that an MLA can be productive at the didactic level if the actions and the effects produced during the resolutive activity of the problem, with the available tool, can be used as reference in the class discussion to interpret and justify, on the theoretic level, what was tried on the practical level. In this regard, let's consider three examples.

Each geometric construction done with *Cabri* incorporates one or more theorems in the Euclidean geometric environment. Mariotti (Mariotti, 2002) has shown that the appropriate use of the various function and commands in Cabri (computational objects, drag, history) can be used in the class' social practice to bring the pupils to exercise a control in the passage from perceptive data (invariant in dragging) to the relationships between hypothesis and thesis relative to the theorem incorporated into the construction carried out. In this framework, Cabri can be seen as a tool of support, utilisable in the social context of the class, for the start of the demonstration process in the geometric field.

The actions and the effects involved in the algebraic transformations carried out with Algebrista can be used in the class discussion for organising the activities of algebraic manipulation as activities of formal demonstration that utilise the basic axioms present in the interface of the system and the theorems demonstrated by the pupils in previous activities (Cerulli & Mariotti, 2002).

In other situations the activity with a tool can be productive at the level of theoretical reflection beginning from the contradictions that can emerge during the solution of the task.

For example, if with Derive one proposes a situation in which the use of the commands can produce a result, non easily readable, (it's sufficient to imagine some apparently simple equations whose solution is supplied using the trigonometrical form of the complex numbers or some functions whose graphic, calculated in the complex field, is different from that which was expected) a useful contradiction can emerge to start a phase in which the resolutive activity of the task is framed in a theory.

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