

# INSTRUMENTS OF SEMIOTIC MEDIATION IN CABRI FOR THE NOTION OF FUNCTION<sup>(\*)</sup>

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*This paper is aimed at analysing the role of some Cabri tools as potential "instruments of semiotic mediation" in the construction of the notion of function. Firstly it presents the terms by which this analysis is carried out. Secondly it shows how this analysis could be invested for conceiving a suitable learning activity and, in the case of a student's report, for modelling the occurred internalisation of specific mathematical meanings.*

## Introduction

This paper is aimed at analysing the role of some Cabri tools in terms of potential "instruments of semiotic mediation" in the construction of the notion of function. Its aim is purely theoretical since it wants to clarify the modality in which it is possible to conceive the introduction of an artefact in the construction of a mathematical knowledge.

This kind of analysis represents an interesting methodology since it describes a method that, in a certain way, may be generalized. Indeed, it is conducted on a software, in which the correspondence between artefact and meaning to be mediated is not, in this case, so immediate<sup>1</sup>: a DGS is in fact used to introduce a typically analytic concept. This means that this methodology isn't exclusively a feature of certain artefacts, conceived from the beginning to mediate specific meanings.

Fundamental ingredients of this analysis are, first of all, the theoretical meaning of "instrument", and secondly, that of "semiotic mediation". Firstly, this paper will try and clarify the meaning given to these terms.

This work is rooted in a larger didactic project still to come, that constitutes the object of my PhD thesis and which is carried out under the responsibility of C. Laborde and M.A. Mariotti. A detailed and definitive description of the whole research is still premature and beyond the aim of this paper. Nevertheless, before dealing with the analysis of some Cabri tools, it is necessary to specify the way in which mathematical meanings to be mediated are considered. Then, the second paragraph will briefly explain the fundamental assumptions of this project for the construction of the concept of function.

Finally, the last part of this presentation will try and exemplify the utility of this analysis during the phases of conception, implementation and *a posteriori* analysis of a didactic activity. Indeed, it will briefly show how this analysis led to the

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<sup>1</sup> Unlike the case of "L'algebrista" (Cerulli, 2001) or "Cabri-géomètre" (Laborde and Capponi, 1994) employed to learn algebra or Euclidean geometry respectively.

elaboration of a Cabri activity on the one hand, and, through the filter of a student's report, how it could help to model the occurred internalisation of certain mathematical meanings on the other hand.

### **Artefacts, instruments and semiotic mediators**

As Vygotski (1978) writes, *the sign (or "psychological tool") acts as an instrument of psychological activity in a manner analogous to the role of technical tool in the laboratory. The divergence is that the function of the technical tool is externally oriented: it must lead to changes in objects. The sign, on the other hand, is internally oriented: it's intended to master the person himself.* The process of genesis of sign, called by Vygotski "process of internalisation", is characterized by the "internal reconstruction of an external operation" and by the "transformation of an interpersonal process into an intrapersonal one".

The educational aim, developed within the didactical theory centred on the notion of "semiotic mediation" (Mariotti (2002), Bartolini Bussi et al. (1999)) is to achieve the internalisation of a technical tool (which is used by the student to fulfil a task) into a sign, which is able to stand for a certain mathematical meaning.

The recent theories on the instrumental genesis (Rabardel (1995)), even if focusing on the fundamental distinction between artefact<sup>2</sup> and instrument<sup>3</sup>, partially clarified this internalisation process. The instrumentation and instrumentalisation processes highlighted by Rabardel, indeed, consider only certain aspects<sup>4</sup> of the real change in meaning occurring when one instrument becomes a semiotic mediator. The emergence and the evolution of mathematical meanings incorporated in the artefact, as well as the clarification of their mathematical status, involve many other elements, particularly the social dimension.

The analysis, aim of this paper, is placed, in a way, at the intersection of these two theoretical paradigms. Indeed, it takes into account some Cabri tools in terms of

1. Rabardel "instruments", by identifying artefact and relevant (social) schemes of use (SSU);
2. "semiotic potential" associated with these instruments, in a Vygotskian perspective of social construction of Knowledge.

### **Trajectory and dynamic interpretation of the notion of function**

The conquests of the abstract definition of function, as correspondence between two sets, with its achieved independence from every object of modelisation, dates back to the beginning of the nineteenth century. This definition of function refers to a static notion (Marchini (1999)). It has lost every relation with the primitive dynamic intuition tightly tied with time and movement, as it appeared, for example, in

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2 The artefact is the mere material or symbolic object.

3 The instrument is a mixed entity, elaborated by the subject, which comprises, on the one hand, the artefact, on the other, the associated (eventually social) schemes of use (SSU).

4 They concern, in particular, the relationship between the single subject and the artefact.

Newton. For this reason, the crucial problem at school is that, even if students get the idea of correspondence, they don't perceive the co-variation of two variables of which one depends on the other. Very often, according to them, function is the datum of a formula that allows calculating  $y(x)$  by a given  $x$ , for a discrete and finite set of values.

The key to recover a "dynamic interpretation" of the notion of function was provided in Laborde and Mariotti (2001) and in Falcade (2001), by the idea of trajectory. According to the general assumption, this idea, developed within the study of geometrical functions in a DGS, can be re-invested to lead to a concept of function (not only geometrical) as an object incorporating an asymmetric relation of co-variation among variables.

### **Analysis of some instruments of semiotic mediation in Cabri**

#### ***Drag mode***

The dragging of an object on the screen may be considered as the instrument that immediately characterizes and determines the dynamic character of Cabri. Its material and symbolic component, that is to say the artefact, is characterized by a set of pixel properly actuated on the screen.

The associated scheme of use concerns the dragging action.

This instrument has three semiotic potentials:

- By assigning any position to an object in the screen, it materializes one of the most important aspects of the notion of variable: its "general characteristic";
- It evokes in a natural, but implicit, way the variation and the change in time.
- Considering the very instinctive nature of this instrument, its hindrance makes the subject to perceive whether the absence of freedom or the presence of constraints.

#### ***Trace***

The artefact consists in displaying on the screen the trajectory followed by one object during its movement. From a graphic viewpoint, it also keeps record of its motion speed, by drawing a sequence of points more or less close to one another.

This can be associated with three schemes of use:

Scheme 1: move a point that leaves its trace.

Scheme 2: move a point and obtain the trace of another point, depending on the first one.

Scheme 3: move a point that leaves its trace and obtain at the same time the trace (even of another colour) of another point depending on the first one.

As we can notice, the Trace instrument always involves the Drag mode instrument.

The *Trace1* instrument (Trace+scheme 1) can explicate the notion of variation in terms of a sequence of changes of state in time, and thus objectify the primitive functional dependence: that of space with respect to time. Furthermore, *Trace1* graphically externalises the total freedom of the independent variable.

The *Trace2* instrument (Trace+scheme 2) focuses on the crucial concept of dependent variable. Unlike *Trace1*, this highlights the lack of freedom degrees of this latter.

By means of *Trace3* instrument (Trace+scheme 3) both the domain and the image set of function can be contemporaneously experimented as trajectories of inter-related moving points. By implying twice the notion of trajectory, this instrument may create a valid semiotic mediator to "reify" and give awareness to the notion of co-variation.

### ***Macro***

The artefact is built as a "tool box" that allows both the creation and the running of a macro-construction, that is to say a proper sequence of operations (in general a geometrical construction) made by the user and enabled by Cabri.

Three schemes of use involving the macro artefact may be identified:

Scheme 1: the user applies an unknown macro.

Scheme 2: the user applies a known macro.

Scheme 3: the user creates a macro.

The *Macro1* instrument (Macro+scheme 1), as well as *Macro2* (Macro+scheme 2) need the explicit statement of the initial objects. On the other hand, the final objects are created by the software, which executes a given construction. Both *Macro1* and *Macro2* can represent the "assignment of a particular value" to the function; on the other hand, only the combined use of these instruments together with dragging can demonstrate the "general nature" of the function.

*Macro1* can be distinguished from *Macro2* thanks to the task that can be associated with them. In the case of an unknown macro, *Macro1* may be associated with *Trace2* to study the hidden construction. The trajectory of the different points shall then be conceived as the set of all points (or the support of that point) on which the macro acts and for which the identification of action is required. So, one starts from a global perception of the function, in terms of geometrical curves, to attain to a local meaning.

On the other hand, in the case of a macro note, (scheme 2), of which the student knows the action point by point, the request may be that of studying the effect on a given set of points. The path is then the opposite: from local to global.

*Macro1* and *Macro2*, together with *Trace2*, help creating the meaning of function in terms of a co-variation relation among variables bound by a certain rule. Thanks to the fact that the subject is really aware of the "generating process" of the dependent

variable, the state of this latter is better defined, compared to the simple use of Trace2. Macro1 and Macro2, together with Trace 2 thus allow the identification of variation domains of the independent and dependent variables.

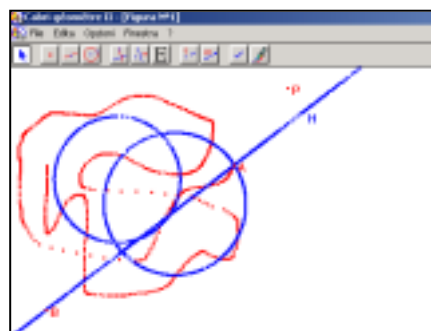
The *Macro3* instrument (Macro+scheme 3) implies that, starting from a construction made by the user, this latter defines the objects depending on the others. By wanting the strict identification of the initial and final objects, Macro3 makes explicit the different status of independent and dependent variables. This leads to a particular aspect of the notion of function: that of being a functional relation between variables, that is defined on the basis of a specific but general object and that, for this reason, is potentially applicable to all the objects of the same type.

Therefore, Macro3 can help the construction of the meaning of function, by characterizing its constitutive elements and by condensing<sup>5</sup> the process into object. It is important to underline that, as well as in the case of the function definition, to validate a macro it is necessary to name it.

### Implications of the above illustrated analysis in conceiving a didactic activity

To exemplify the utility of the said analysis, in terms of methodology, I will try to briefly demonstrate how this contributed to the conception of a didactic activity. In particular, I will describe the first part of a sequence of experiences<sup>6</sup> with Cabri, during which students worked in pairs<sup>7</sup>. After each activity, it was required to take part in a general discussion and to write at home an individual report specifying, on the one hand, what one has experienced and understood, and, on the other hand, doubts and questions arisen. According to the terms of the didactic contract, every written part had to be duly justified since it would become the subject matter of the subsequent discussions.

In the first question, students were asked to create three points A, B, P, to apply the unknown macro-construction, "Effect1" (which was pre-recorded on their screen), to the points given in that order, and to verify that a fourth point, called H, appeared. Students were also asked to try and move all the points on the screen and to write down which ones allowed dragging or not. This first question aims at introducing the Drag mode tool. "Effect1" acts as a black box. Starting from three given points, A, B, and P, it constructs a new point H, obtained as the orthogonal projection of P on the straight line passing through A and B. This first question aims the activation of the Drag mode instrument.



<sup>5</sup> We use here the term « condensation » with the meaning given by A. Sfard (1991)

<sup>6</sup> The in-depth analysis of all the sequence is still *in fieri* at this moment and represents a part of my PhD thesis

<sup>7</sup> The students involved in this experimentation were 17. They were at the second year (15/16 years old) of an Italian secondary school specialising in scientific studies.

In the second question, students were asked to describe in geometrical terms the trajectory followed by the different points, by using the Trace tool to materialize it. This second question aims at making topical the Trace1 and Trace2 instruments. The request to describe the trajectory of different points in geometrical terms meets the need to pass from the simple perceptive observation of one trace left on the screen to a re-elaboration in geometrical terms.

Finally, in the third question students were required to find and create again the macro-construction hidden in "Effect1". The aim of this last question is to mobilize the Macro1 and Trace2 instruments.

All the students answered to these three first questions by adhering completely to the artefact world; the instruments provided were exactly used to carry out their original purpose: move the points, observe the traces and discover the hidden macro. For example, Mattia and Nicola answered to the second question: *“Moving A externally to the segment B-P from P to B and from B to B, H traces a circle that includes in itself in addition to the point H, the B and P points, the latter being extremes of the diameter”*. Whereas Igor and Filippo wrote: *“Moving the points A and B, H describes a circumference; moving the point A, the diameter of the circumference is PB and moving B the diameter of the circumference, which is described by H, is PA. Therefore the width of the circumference depends on the distance between P and the point that we aren't moving. If we move P, the straight line, which is traced by H, passes through A and B”*.

In Vygotskian terms, one can say that, at this stage, these instruments function as mere technical tools. Obviously, the links between these activities and the notion of function (psychological tools) are completely obscure and inaccessible.

### **The beginning of a change in meaning, through the filter of a report**

During the hour following the above-presented activity, the teacher comprehensively analysed the experiences made by each pair. On the one hand, she depersonalised and “de-contextualised”, thus making collective the situation lived by each pair. On the other hand, she reviewed the activity by introducing a “translation code”, necessarily inter-subjective, based on the ambivalence<sup>8</sup> of those experiences within both the “artefact world” and the “mathematical one”<sup>9</sup>. New mathematical terms such as variable, independent and dependent variable, domain and image were then associated with the situation experienced in Cabri.

I will not enclose here the analysis of the first discussion that followed the first activity<sup>10</sup>. On the other hand, I chose to show what happened with Cabri and what

8 This position is consonant with that expressed by Schwarz e Hershkowitz (2001) and based on the meaning of inter-subjectivity and ambiguity.

9 The term “artefact world” refers to the world of objects and events experienced in Cabri and incorporating the semantic domain of space and time. The term “mathematical world” refers to the theoretical domain relative to the notion of function.

10 Indeed, it is not possible to summarize the discussion in a few lines.

was said during the discussion by means of one report<sup>11</sup>. In this way I think it is already possible to have an idea of the change in meaning of the instrument and of the internalisation process that has just started.

Igor writes:

*In the last lesson, we broached the topic of functions [1]. We started from three points on Cabri, and we gave them a certain effect, called "Effect1" [2], which represented our function [3] and which drew another point [4]. We saw that by moving the three points previously drawn [5], the fourth point moved according to particular circumferences and to one straight line involving the first three points [6].*

*By discussing this aspect [7], we understood that the function is a relation involving several elements and allowing the linking of the first elements to the second ones by having them do something [8]. This is just an initial definition since we have not yet reached the mathematical definition of function [9]. In our case [10], the function [11] allowed us to move the fourth point by moving the other ones [12]. But the fourth point could not be directly moved, by dragging it with the mouse [13]. We understood that the first three points, called  $A$ ,  $B$ ,  $P$  are in our case [14] called independent variables [15], while the other point,  $H$  is called dependent variable [16] since it depends on  $A$ ,  $B$ ,  $P$  [17].  $A$ ,  $B$ ,  $P$  move on a plane [18] and the "locus in which the independent variables move" is called "Domain" [19], while that in which  $H$  moves [20] (dependent variable) is called "Image" [21]. We saw that  $H$  formed two circumferences of diameter  $AP$  and  $BP$  ...[22]"*

The report structure is particularly interesting: Igor is aware that they are doing mathematics, that is to say that what they do in Cabri finds a correspondence in the mathematical world ([1], [3]). First he describes the artefact world, he evokes the use of Macro1 ([2], [4]), Drag mode [5], and Trace2 [6], then he comes off, he begins a new paragraph and starts describing what he considers the implied mathematical meaning. His formulation doesn't belong to the "artefact world": he speaks about elements, not about points; he refers to a relation that has them do something, not to the shifting produced by a dragging [8]. Neither his formulation belongs to the "mathematics world" [9]. It springs from the internalisation and the appropriation of the sense of the activity that has come out during the previous discussion [7]. Then Igor goes back to the description of the activity in the "artefact world" and evokes the use of Macro1 and Drag mode [12]. But, according to him, these instruments begin to signify something else; all what is experimented in Cabri corresponds to a particular "instance" of the concept of function (indeed he begins with "In our case..." [10] and uses a term from the world of mathematics, "...the function..." [11]). Here, we can already detect an embryonic change of meaning of these instruments and a turning out of their semiotic potential. The hindered dragging begins to feature the dependent variable [13], whereas the relation of

<sup>11</sup> Considering the experience of the class, already involved for two years in similar experiments, and the well-established didactic contract, I assume that all texts provided by students are genuine.

functional dependence is perceived in terms of co-variation of objects [12]. The mathematical terms of independent and dependent variables, domain and image begin to be associated to elements in the Cabri activity ([15], [16], [19], [21]). Nevertheless, there's always the awareness that these actions and objects have an equivalent in the mathematical concept of function. Igor, in fact, states once again "...in our case..." [14]. We can notice how this activity bases the construction of the meaning of logical dependence on the one of causative dependence [17]; whereas, the meanings of domain and image are founded on the ambivalence of the idea of trajectory, which is at the same time the trace of a moving point ([18], [20]) and a geometric set globally perceived [22].

Each other report testifies of a particular learning process; therefore the observations carried out for Igor cannot be generalized *tout court* to the other students. Nevertheless the change in meaning, which has been highlighted by this kind of analysis, is findable in 14/17 provided reports: the results, even if partial, are, so, encouraging.

### Conclusions

In this paper we have presented the analysis of some tools of Cabri in terms of semiotic mediators for the notion of function. We have also shown how this analysis could be useful for conceiving a suitable learning activity.

The work of construction of mathematical meanings, illustrated through the filter of a student's report, is still far from being accomplished. The terms introduced by the student have indeed a poor and limited semantic field. Nevertheless, it is already possible to identify the co-existence of two worlds, the artefact one and the mathematical one, and the change in meaning of the instruments involved. How the discussion about the activity in Cabri has contributed to this change still remains an open and interesting research question.

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