BETWEEN ARITHMETIC AND ALGEBRA: A SPACE FOR THE SPREADSHEET? CONTRIBUTION TO AN INSTRUMENTAL APPROACH.

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In this research work, we have a didactical look on the integration of a computer tool into mathematics teaching: the spreadsheet. We have related research about spreadsheet to an instrumental approach perceived as essential to analyze the questions of technological integration. This approach allowed us to extend to this technology the theoretical framework of instrumentation. These theoretical elements are being used in the analysis of the professional resources and the teaching practices, in order to understand the difficulties in the integration of the spreadsheet. In this paper, we mainly describe this theoretical work, then some of its didactical consequences.

Nowadays, there is a firm institutional will to integrate the technologies of information and communication to the practices and curricula in France. In mathematics, teachers are encouraged to use Internet and various software: dynamic geometry and symbolic calculation software, spreadsheets, etc. Our research deals with the spreadsheet, which officially first appeared in middle school, then in high school; it was motivated by the following observations:

- Although the spreadsheet has been explicitly inscribed in the syllabus for several years now, its use still remains very rare, even marginal,
- The number of resources setting spreadsheet activities (handbooks, publications, Web sites…) has considerably increased but this does not substantially improve the integration of this technology.

These initial statements give rise to many questions both from a theoretical and practical point of view. In particular:

- What do we know about the spreadsheet potentialities for mathematics education and about the necessary conditions for their achievement?
- Does the spreadsheet integration set specific problems, and if it does, what are they?
- What are the characteristics of these resources? In what way could they (or could they not) help the integration intended by the institution?
- How do teachers, who really have integrated the spreadsheet, work? How did their practices develop and how do they keep on evolving?

These questions organized our research with the view to understand the problems of spreadsheet integration better. We began by an inventory of didactic research about this tool. This inventory focused our study on spreadsheet use for the teaching and learning of algebra. It allowed us to delimit the spreadsheet potentialities in this domain, due to its hybrid status of arithmetico-algebraic tool. However, it seemed to us that the previous studies were insufficiently considering the spreadsheet's instrumentation. That is why we resumed, at a second stage, their analysis in the light of the instrumental approach developed by the works of Artigue, Guin, Lagrange, Trouche in the context of the CAS (see for instance [Guin&Trouche eds, 2002]). By doing so, we characterized the computer transposition of this technology; we examined the possible instrumental geneses and, in these geneses, the interplay between technical knowledge and conceptualization in algebra.

This study seems to us an essential part of our research: it allowed us to build tools for analyzing both resources and real practices. That is why we chose, in this paper, to focus on it. We will then briefly present our first results.

I The spreadsheet and the problematic arithmetic/ algebra

The use of spreadsheets in mathematics teaching has been studied for instance by: Rojano & Sutherland (1997,2001); Capponi (1999); Arzarello, Bazzini, Chiappini (1994, 2001). All these studies concern the arithmetic-algebra transition at middle school and give the spreadsheet a potentially positive role in this transition. However their approaches differ by certain aspects: while Rojano & Sutherland and Capponi are situated in a constructivist frame, Arzarello and his colleagues underline the socio-cultural dimension of the learning, considering that the construction of knowledge is not only a question of cognitive conflict of an isolated individual, but a collective and social activity joining a culture. The spreadsheet is then seen as a system of social interaction where teachers and pupils build a socially shared language: the algebraic language.

In paper-pencil environment, several research have already identified certain difficulties of the learning of algebra, especially those connected to:

- The procedural/ structural duality (Sfard, 1991)
- The ruptures with arithmetic: discontinuities/ false continuities (same signs but new methods, new statuses of objects and operations: letters, symbols...) (see Grugeon, 1995 for a synthesis)
- The complexity of the semantic/ syntactic relationships in algebra (see Nicaud and Duval in Grugeon, 1995; Arzarello and al., 1994, 2001)

Taking into account these difficulties, the contributions of the spreadsheet put forward by the researchers rely both on its constraints (constraints of communication, of symbolism, of organization) and on its new modalities of action: possibilities of calculation, of representation; interactivity; interplay of various languages (natural language, numeric, algebraic, graphic).

The spreadsheet thus appears as:

- a good tool of semiotic mediation (Arzarello, Bazzini, Chiappini)

- a tool occupying an intermediate situation: post-arithmetical or pre-algebraic both at the level of its features, of the knowledge involved and of pupils behavior and errors (Capponi)
- and a tool allowing pupils to progress, from their arithmetical intuitive methods towards more algebraic ones (Rojano/Sutherland).

However, for Capponi, this double position of the spreadsheet can also maintain the pupil completely on the arithmetical side. He insists on the fact that they are the proposed situations that might assure the transition towards algebra.

Therefore, let's examine in more details the activities presented in these studies: on what levers do they play on to favor the transition to algebra? We notice that:

- Various "control levers" are considered according to the authors. Thus, Arzarello, Bazzini, Chiappini play on the semantic/ syntactic complexity, Rojano&Sutherland on the methods of resolution, Capponi on the duality of the objects.
- It is never an isolated activity which "realizes" the transition but a "series" of activities following a precise progression instead.
- This progression is, for Capponi, established via the instrumental/ technical side: namely the tool features required to solve each exercise.
- Finally, our analyses reveal the existence of implicit elements. Every activity has its own mathematics “pre-requisites” and objectives. Beside these mathematical elements, the activity also carries some technical “pre-required” elements. These technical elements are not explicit in the terms of the exercises; yet, they're absolutely necessary so that the task will be properly devolved and that the actual activity will be the activity aimed at. In order to delimit and understand the problems of spreadsheet integration in the practices, we wonder whether the available resources take into account these implicit components. If they do, then how do they suggest the teacher to manage them? Is it by way of the progression or by playing on some effects of the didactical contract with pupils? Papers do not tell. Thus, the full use of the interesting features of the spreadsheet is obviously not self-evident.

We make out implicit elements, new tasks, revealing that the work will be different from the one in the paper-pencil environment. What are exactly these differences and what impact could they have on the learning of algebra, on the expected conceptualizations in this domain? All of this shows that the “technical side” of the tool cannot be overlooked, given that we use this tool in a didactical purpose. In order to favor the arithmetic/ algebra transition, researchers underline the situations' importance. But which didactical variables are they playing on? Whereas we can point out the mathematical variables used in their situations, the “instrumental” ones (that is regarding the tool's functionalities) mostly remain implicit as we've shown it. Yet, if these implicits are not taken into account, they might generate several misunderstandings, pupils using the spreadsheet "otherwise" than what is expected by the teacher.

These analyses about the technical aspects of the tool (various functionalities, features) and their impact on conceptualization, their link with mathematics lead us directly to the question of instrumentation.

II AN INSTRUMENTAL APPROACH

II.1 Conceptualization and instrumentation: the general framework

Learning with software introduces questions linked to the tool itself insofar as it includes performing some technical tasks. The introduction of these technical tasks in teaching modifies the classical didactic triangle by questioning each of the three poles: learner, teacher and knowledge. What is the status of such a task for the pupil, the teacher, the institution? How is it situated with regard to the mathematical knowledge? Further, what are the relationships between mathematics and technicality? More precisely, when exactly does the pupil have a mathematical activity within a computer work? This is neither in a report of sheet, neither in the ability to use a toolbar of the software, nor in visual results… The instrumental approach provides us with some answers: for Rabardel (1999), instrumentation *comes along with conceptualization*. Likewise, the research on the use of technologies mentioned below report the relationships between the technical part and the conceptual (said “noble”) part of mathematics. They show that these relationships are rather to think in terms of dialectics than opposition. Their observations and conclusions can be applied to the case of the spreadsheet. Thus, as regards to the traditional teaching, the use of the spreadsheet for the learning of algebra does not settle any more only in terms of: "*the learning of the one is there to remedy the incapacities of the other one*" but also in terms of "*transmission of the elements of a technical/conceptual dialectics*" fitted in a certain mathematical culture which **is precisely** instrumented (Lagrange, 2000; Rabardel, 1999). So, we have to analyze the way this instrumentation takes place in the case of the spreadsheet.

II.2 Spreadsheet and conceptualization in algebra

What solving processes, methods or techniques does the spreadsheet favor? How do the usual algebraic objects live in this environment, especially those, which have already been reported as problematic in the paper-pencil environment? What are the new objects?

We present here two main results of our analysis: on the one hand the solving process, on the other hand the study of the usual objects and the new objects introduced by the tool: their apprehension, their status, their symbolism.

The solving process: an analysis of the “trial/refinement” method

A fair number of problems (optimization, equation) can be solved in the spreadsheet with a method that is close to a paper-pencil one, named the “trial and refinement (T/R) method” (Rojano&Sutherland). So, we started to study the T/R for such a problem by comparing it to the "arithmetic" and the "algebraic" methods on the same problem.

In paper-pencil: We've defined some criteria characterizing the 3 methods: type of calculation (numeric/literal), type of resolution (direct/indirect), nature of the involved objects (expressions/equations), step (perform calculus/ solve equation/ test equality), type of used data (known/unknown). The comparison shows that the T/R, in paper-pencil environment, is intermediate: arithmetic for some of its characteristics, algebraic for some others or still "mixed".

With the spreadsheet: the tool brings some specificity making the method closer to the algebraic one: the comparison shows that some criteria, that were arithmetic for the T/R in paper-pencil, become algebraic with the spreadsheet. For instance, by reporting the data of the problem to different cells and the relations between these data to relations between cells (formulas), some intermediate formulas emerge, which are very close to the equations algebra would have led to. That was not the case for the paper- pencil method. *The spreadsheet adds an algebraic organization to an arithmetic resolution*.

Finally, the method (accessible even to pupils who usually meet difficulties) favors, by the organization of the sheet itself, the transition to algebra.

<u>The objects and new objects (apprehension, and status, symbolism)</u>

Beside the elements problematic for algebra that have already been listed in paper-pencil research, we show new objects introduced by the spreadsheet itself. They deal with its new possibilities, its external/ internal constraints and the gestures it requires. We shall detail here some examples of the role played by instrumentation on conceptualization through the analyze of the spreadsheet functionalities.

Let's start by the study of 2 cells connected by a formula. We shall detail so a new object put in evidence there: the **" variable cell "**:

In paper-pencil, variables in formulas are written by means of symbols (a letter generally for the level concerned here). This variable "letter" is connected to a set of possible values (numeric here) and exists <u>in reference</u> to this set.

In the spreadsheet, let us take the example of the formula to calculate squares: we have a cell argument: A2 and a cell where the formula was edited: B2, referring to this cell argument:

	A	B
1		
2	5	=A2^2

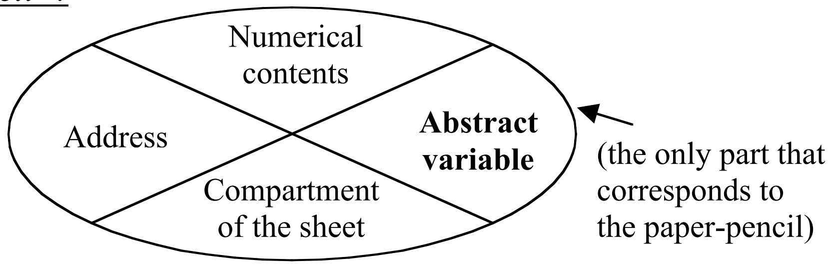
A2 is the cell argument; B2 calculates the square of the value in A2

Here again, the variable is written with symbols (those of the spreadsheet's language) and still exists in reference to a set of possible values. But this referent set (abstract or concretized by a particular value as 5 on the figure) appears, here, through an important intermediary: the cell argument A2 that is both:

- an abstract, general reference: it represents the variable (indeed, the formula does refer to it, making it play the role of variable)
- a particular concrete reference: it is here a number (even if nothing is edited there, some spreadsheets attribute the value 0)
- a geographic reference (it is a spatial address on the sheet)
- a material reference (it is a compartment of the grid, some pupils can see it as a box)

So, where in paper-pencil, we stick a set of values, a cell argument overlaps here, embarking with it, besides the abstract/ general representation, three other representations, without any equivalent in paper-pencil. Henceforth, to remind this difference, we shall call it " variable cell ":

The "variable cell":



Notice that the cell B2 has a double, or two-faced, status: it is both a formula and a possible variable for a new formula in another cell!

For more examples, let us add now to the previous situation some of the most interesting features of the tool: the "**re-copy**", the possibility of **assigning names** and the **automatic re-computation** (the two last mentioned functionalities are still in process of being analyzed):

The functionality of re-copy, available in spreadsheet, still complicates the situation: for instance, the preceding formula in B2 can be recopied automatically, by "drawing down" the handle of recopy, generating a "variable column", which is another different object from the previous one.

Then, it is also possible to assign a name to a group of cells, for instance "n" for the group "A2:A5", and use this name in a formula, for instance "=n^2" in B2. By doing so, we generate another notion of variable: this time, the variable is "n" and the intermediary is a finite number of "cells arguments", each of them having the characteristics of a variable-cell. Yet, this "variable-group" is not a mere group of variable-cells placed side by side, the fact that they are linked by the same name "n" adds a new dimension to this notion of variable: the numeric multiplicity. This dimension carries along a conception of variable very close to the traditional one.

But, as we could observe it, afterwards, this functionality has never been used in the professional resources analyzed up to now!

At last, when formulas include absolute references (like A1…) the dynamic aspect of the created sheet is given by the functionality of automatic re-computation. The automatic re-computation of the formulas (when a value is changed) plays also an important role in the evolution of arithmetic strategies towards algebraic ones: here, the notion of parameter, as a variable of the problem, emerges not only through a cell but precisely through the gesture of automatic re-computation.

Likewise, the numbers, the equality sign, the "unknown factor", the formulas live in a different way in the spreadsheet. By a similar study, we can, for instance, focus on the formulas and analyze them through each of the previous functionalities. For another example, if a formula is recopied, its usual operational invariance is not translated by a syntactic invariance in the spreadsheet: in the previous example, the formula of the square, if copied downwards, becomes: A2^2, A3^2, A4^2 etc. Then how does this invariance make sense for the pupil? Is it through the gesture of copying out?

II.3 Conclusion

In spite of an apparent simplicity of use, the tool generates some complexity: new objects are created, usual objects are modified and new action modalities are available (resolution process). We can relate these elements to the implicit ones highlighted in our first part. These elements are added to the usual difficulties identified in paper-pencil: symbolism, writing/decoding of formulas, apprehension, status of the objects. At the very moment when the pupil lives these transitions to algebra (ruptures, false continuities) when he must both give new status to known objects and change his methods of resolution; several elements specific to the spreadsheet come to intermingle and interfere with the concepts of variable, unknown factor, equation… Do these interferences have a positive, negative or negligible influence on the expected conceptualizations (namely in reference to the paper-pencil environment)?

Besides, insofar as the number of machines is often limited, we shall also consider the changes introduced by the collective work: the role played by interpersonal processes in conceptualization can't be underestimated and must also be investigated.

We wanted to show in this work how the instrumental genesis, in the case of the spreadsheet, comes along with many questions that are essential in a didactic perspective. Thus, this theoretical work provides us with a framework to tackle the problems of integration through the analysis of the resources and the practices: is this instrumental genesis taken into account in the resources, in the teaching practices? Can these questions explain certain success of integration or, on the opposite, certain failures?

III First results concerning the analysis of the resources

In the preceding study, we showed the existence of implicit elements at the level of the tool functionalities-elements that were, yet, absolutely necessary for realizing the activity intended. We also showed the existence of new objects in instrumental genesis, interfering with the conceptualizations expected in reference to paper-pencil. Then, we wondered whether the professional resources were taking into account all these various elements. So we needed to include the "instrumental" dimension in our analyses. Thus, we are using the preceding study to build a grid, which takes into account not only the mathematical elements but also the instrumental ones with the purpose of analyzing the activities. The grid is composed of different categories : mathematics, didactic, instrumentation, spreadsheet, each of them having different criteria. For instance, here is a part of the pole "Spreadsheet", in the last column, we define the different values of each criteria for a given activity:

Criteria	Values
Kind of work with the spreadsheet and related tasks:	« data edition », « calculator », « protected sheet, ready for use », « work on formulas », « work on different systems of referring to a cell », « work on resolution methods », « graphical work »,…
Required functionalities	Articulation of various registers, New abilities, Instrumental variables, Other possible strategies

Since the categories have some criteria in common, we can place them in parallel. Here is a part of the grid (simplified for reason of space) for the criteria "Analyse of a given task" showing the inter-play between categories:

Mathematics	**Instrument : including**	**a part of the Artefact**
Task →	Translation into spreadsheet task ↓	
corresponding mathematical technique to solve the task ↔	instrumental technique to solve the task ↔	corresponding gestures
	Eventually technology/ theory ↘	
Back to an answer in the mathematical domain	←	Answer/ solution in the spreadsheet environment

We currently continue to work on this grid, however, it already allowed us to produce some results. We briefly present them in what follows:

1) Raised problems

As seen in theory, the activities carry some implicit elements: presumed abilities and knowledge about the functionalities of the spreadsheet, place in a progression with respect both to the spreadsheet and to the mathematical contents, didactical contract "pupil-teacher" established within the instrumentation of the spreadsheet. The origin of these questions probably lies, for a part, in the nature itself of the suggested resources: scattered activities locally discussing an item of the program, difficulty to propose a work, which is situated in the long term as well as in an explicit progression.

2) Disparities/richness of the activities

Activities of the same category can, in spite of a significant set of common points (common variables), present a great variation regarding the pupil's mathematical work. Some of them, are successions of closed instructions (executive work, "push-button" work, purely formal considerations), others are extremely rich. This richness is often due to:

- the presence of interplay between instrumental environments (spreadsheet, paper-pencil or sensitive world)
- the presence of institutional management elements: explicit mention of the place in the mathematical progression, in the spreadsheet progression, etc.

3) A shifting towards the middle school

Some activities are typically exercises usually coming within the high school, they "move" here towards the middle school where a resolution is made possible by the spreadsheet and the "trial and refinement" method identified previously. Here again, the functionalities offered by the spreadsheet are essential.

IV Perspectives

We must complete now the analysis of the resources so as to obtain a landscape rather representative of what is offered to the teachers. Once this landscape described, we will turn to the teaching practices in order to understand how the consideration of the instrumental genesis influences the integration of the spreadsheet: How do teachers, who really have integrated the spreadsheet, work? How did evolve their practices and why? Do the "disturbances", bound to the questions of instrumentation and to the lack of their consideration in the activities, play a role in the integration of the spreadsheet? Does the teacher feel these elements unfamiliar to his way "of making mathematics"; to the representation he wants to give to his pupils? Or is there no more than the reasons usually evoked: fear of changing practices, refusal to provide the efforts necessary for this integration, material problems, lack of training, fear of being inefficient? We make the assumption that the teacher, non expert of the tool:

- is rather poorly sensitive to the expressed potentialities of the instrument,
- first sees the differences, has a presentiment of the added complexity,
- is badly armed to combine instrumentation and mathematical learning

and that for these reasons, teachers can hardly get benefit from the resources.

Our present research is aimed at testing these assumptions and studying how is made or why is not made the integration of the spreadsheet by the teachers: which resistances are there, what are the reasons?

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