ANALYSING THE IMPACT OF ICT ON
MATHEMATICS TEACHING PRACTICES

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While the potential of ICT (Information and Communication Technologies) to enhance the teaching of mathematics is strongly emphasised, the integration of ICT into classrooms seems to be more difficult than expected. This paper proposes new dimensions of the integration of ICT as part of an enhanced framework for analysis.

Innovative or research papers as well as curricula emphasise the strong potentialities of ICT (Information and Communication Technologies) for teaching mathematics at all levels. Their integration into classrooms seems however to be more difficult than expected. In my understanding, the framework that authors generally use to analyse ICT in mathematics is limited to the interaction between the student, the computer and the knowledge. This limited framework explains the discrepancy between the potentialities and the reality of the integration (Lagrange *et al.*, 2003).

The goal of my present research is to enlarge this framework by putting this interaction inside the long-term process of conceptualisation and in the context of schools institutions. To this aim, new dimensions of the integration of ICT are to be considered. Then, the focus is to be put on the teacher trying to integrate ICT, because of his obvious central role in this enlarged context.

A first part of this text reports on how new dimensions were elaborated. A second part draws on research studies about teachers and technology and tries to interpret certain phenomena belonging to the teacher's practice in the light of the new dimensions.

New dimensions to analyse ICT integration

Lagrange *et al.* (*ibid.*) did a study of literature about ICT in teaching and learning of mathematics in the years 1994 to 1998 and elaborated a set of indicators in several "dimensions" (Table 1). This study showed that literature tends to analyse most often the relationship between ICT and mathematical knowledge (epistemological and semiotic dimension) and the influence of ICT in conceptualisation (cognitive dimension).

Two dimensions were embryonic in the years 1994 to 1998. I present them in details.

The first one is the "instrumental" dimension taking into account that a student using a tool to do mathematics develops knowledge on the tool together with mathematical knowledge. Basically, a calculator, for instance, is a combination of plastic and silicon. To practise mathematics with this tool, a student has to build uses and representations linking the tool and the mathematics. Along the time, his representations will evolve, sometimes in a more mathematical sense and sometimes

not. For instance, at the beginning a student is not wrong when (s)he makes an association between the graph of a function on the screen of a calculator and the function itself. (S)he gives a meaning to notions like graphic representation, variations, zeros, etc. After that, the relationship between screen and function should become more elaborate, considering the material limitations of the screen and the student should use appropriate mathematical knowledge to interpret the graph on the screen. It sometimes happens, but sometimes also the student makes no difference between "calculator truths" and mathematics. In the varied possible "instrumental genesis", situations explored in classroom and other options of the teacher play an important role. For instance, when the teacher does not consider the calculator, students use of calculator produces a personal understanding of mathematics, very different of the "official mathematics" (Trouche, 1994).

The second dimension, which was embryonic in the years 1994 to 1998, is the institutional dimension. In a given institutional context, for instance the national school system of a country, or a level of this system, or even the classroom, a role is more or less explicitly assigned to the teaching of mathematical notions. The institutional basis that implicitly rules mathematical notions lasts beyond the curriculum changes (Chevallard, 1992). The above described epistemological and cognitive dimensions cannot alone help to analyse the impact of a technology on this basis and the way it can be integrated (its "viability"). For instance, with cheap simple calculators, paper/pencil basic techniques for addition, subtraction, multiplication and division keep little practical usefulness. Nevertheless their learning remains because of the epistemic interest of their practice to learn about decimal numeration and polynomial expressions. In contrast, at high school level, a similar technique for computing the square root was abandoned and a button of the calculator is now used to get a decimal approximation of a square root. A reason is that this technique was of only marginal importance in the learning of mathematics and that no strong link was made with the underlying algebraic properties. This example of differently resisting paper/pencil techniques shows that an institutional balance related to a notion cannot be explained just from a conceptual or cognitive point of view.

This level of "techniques", intermediate between tasks and conceptualisations is important in the "institutional dimension". A technique is generally a mixture of routine and reflection. It plays a pragmatic role when the important thing is to complete the task or when the task is a routine part of another task. A technique plays an epistemic role by contributing to an understanding of the objects that it handles, particularly during its elaboration. It also serves as an object for a conceptual reflection when compared with other techniques and when discussed with regard to consistency.

Without technology, paper-and-pencil techniques cannot be avoided because of their pragmatic role. Their epistemic role is less apparent, but very important in the institutional balance, as we saw above. Technology introduces new techniques,

whose pragmatic role is obvious. It is generally more difficult to consider the possible epistemic role of "instrumented techniques" (see Lagrange, 2000) but it is essential because a technique cannot exist in a school institution without playing an epistemic role.

Epistemological and semiotic dimension	Influence of ICT • on the mathematical knowledge and practices, • on the representatives used in this activity
Cognitive dimension	Cognitive frame (constructivist, socio-cultural,..) Concepts used (schemes, webbing, etc.) Cognitive role of ICT (visualisation, expression, connection, etc.)
Institutional dimension	Interaction of ICT with tasks and techniques in the culture of a school institution, Role of instrumented techniques in conceptualisation of mathematics.
Instrumental dimension	The tool's possibilities and constraints, Instrumentation processes.
Teacher dimension	Teacher's beliefs and representations of mathematics and of ICT New teaching situations Influence of research and pre/in service programs
Table 1: Indicators of dimensions	

The conclusion of Lagrange *et al.* (*ibid.*) was that the focus of literature on the epistemological and cognitive dimensions clearly produces a corpus of didactical knowledge useful to understand aspects of educational use of ICT. On the other hand this understanding cannot alone support the integration of ICT. Using ICT for teaching and learning mathematics in schools institutions involves more than just epistemological considerations and the student is not just a "cognitive subject". To analyse the use of an ICT tool in the complex classroom reality and the evolving relationship of the students to the mathematical knowledge and to the tool, analysis of the integration needs to include the institutional and instrumental dimensions.

In the meta-study, we planed to look also into a teacher dimension because we saw him (her) as a central "actor" of the integration. We tried to characterise this dimension by indicators (Table 1). We found very few mention of these indicators in papers of the years 1994-1998. My interpretation is that innovators and researchers made an implicit assumption: new technologies and the associated didactical knowledge could easily be transferred to teachers by way of professional development and training. I think that this assumption is questionable because in a country like France, uses of technologies are deceptive although efforts have been

made to train teachers. In my hypothesis the existing corpus of didactical knowledge about ICT use is not sufficient to really help teachers integrate technology. Thus research has to study the teacher and try to look at his(her) action in the light of new dimensions. The second part of this paper aims to contribute to this study by analysing a set of publications.

A selection of research studies about the teacher

I will take into consideration a selection of research studies about the teacher and ICT from the year 1994 until now. As a difference with the above-mentioned study, this selection of papers is not systematic. I selected papers to show an evolution. All but the first paper are about Computer Algebra Systems (CAS). This is the technology I know the best and experimenting its use, discrepancy between potentialities and integration was particularly noticeable.

The first publication (Abboud Blanchard, 1994) is a philosophical dissertation discussing the use of software applications in the learning of mathematics and associated teacher training.

The second (Lagrange, 1996) is a study of the use of Computer Algebra by "expert" teachers. It stresses discrepancies between teacher's expectation and the reality of classroom observation and students' attitudes.

The other publications (Monaghan, 2001, Kendall, Stacey, 2002) are about more "ordinary" teachers. They tend also to show discrepancies and a great variability of professional practices.

> *Software applications to learn mathematics and teacher training* (Abboud Blanchard, 1994)

This philosophical dissertation was written during the years following the IPT ("Computer for Everybody") program. This program was initiated in 1984 by French government and aimed to a generalised use of 8-bits computers in schools to learn various subjects. Much was expected from this use and the reality was deceptive. At a guess, the publication says that 15% only of mathematics teachers use at least sometimes computers in classroom. Teachers think that they are not enough trained: 23% say that they had no training, 46% think they are poorly trained, 17% medium and 11% highly trained.

The publication notices that the "external" analysis of technical, ergonomic, of pedagogic characteristics of software applications that literature offers is not enough to help teachers build a classroom use. To take advantage of potentialities of a computer application, the teacher would need an understanding of "internal" characteristics like the implementation of mathematical notions and the impact on learning situations. To achieve this understanding, teachers as well as software designers should be able to use didactical knowledge about learning situations and not just "naive" approaches.

A section of the dissertation develops a typology of in-service teacher training strategies in three types. The first two types are oriented towards the presentation by the teacher of educational software applications and of classroom situations. The trainer offers his own practice, without distanciation.

The first type aims to justify ICT use by showing interesting applications and varied uses. It is characterised by a strong "personnalization". The trainer offers a lot of different applications and "rich" situations with stimulating prospects. The participants have to do the tasks that would be proposed to students.

The second type is concerned by a realistic integration. The trainer shows a more limited number of applications and offers more usual situations. The participants have to do the tasks like in the first type.

The third type of training is based on the "generation of situations". Doing this task, participants are supposed to be teachers preparing the details of a classroom session using the computer. This strategy aims to make the participants aware of a "gap" between paper/pencil situations and computer use.

The third type would the more effective for a real integration. On the other hand, the adaptation to the project of the participants requires that the trainer had a lot of didactical knowledge. In the years following this dissertation, in France especially, efforts have been done to train teachers in the use of technology by trainers more aware of didactical aspects of this use. The problems remains of a poor return of this training. The big number of research studies in the field helps trainers and teachers to do a more "internal' analysis but, in my hypothesis, this analysis, when restricted to the above described epistemological and cognitive dimension does not provide the teacher with means to take a real advantage of technology. In the next section, I consider the case of "experts" teachers who were also qualified teacher trainers and informed of research about the use of technology.

Observing "experts" teachers (Lagrange, 1996)

During the years 1994-1996, a French research team did a survey of the use of the CAS DERIVE at French upper secondary level. We observed the classrooms work of 17 teachers. These teachers were very experienced in the use of new technologies and most of them were doing innovative design of learning sessions with ICT. We compared the expectations expressed by these teachers before the course to their perception after the sessions and the "attitudes" observed among the students.

With regard to "external" characteristics, the teachers expected that DERIVE would help them to teach with more conviviality and with attention to special needs of students, and to tackle richer and more interesting problems. In their "internal" analysis, they thought that DERIVE was able to make up for students' difficulties particularly in elementary algebra and numeric calculations, and help them to do mathematics in a more reflective, strategic and conceptual way. They expected that DERIVE would free the students of technical tasks and valued much the possibility of multi-register representations.

We questioned students and teachers after the courses. Students understood DERIVE most often as a tool to make tedious calculations and double check paper/pencil results. Students who used a lot DERIVE did appreciate the new situations offered by the teacher, but stressed rather on the novelty than on a contribution to their learning. Thus teachers could not make the students share their views. They also had a somehow different opinion after the courses. They noticed difficulties encountered when putting their ideas into operation. For instance DERIVE did not compensate as much as they expected students' weaknesses and, even with DERIVE, they found free problem solving sessions not easy to manage.

Classroom observation helped us to understand this discrepancy between teachers' expectations of the support of technology and the reality of the integration. "Expert" teachers looked at CAS use at a very conceptual level and did not see the real impact of technology on teaching and learning situations. They particularly ignored the role of "instrumented techniques" (see above &1), their articulation with paper/pencil techniques and their possible epistemic contribution to mathematics education.

Differences between teachers in the same project

More recently, Kendall & Stacey (1999) considered two Australian teachers, Andre and Benoit, involved in a project of integration of the TI-92 calculator. The project planned the same activities for the two teachers, but striking differences were observed in their practices.

Before this project, Andre did not use technology a lot. With symbolic calculators, he enhanced much his technological ability and confidence. He was attracted by the exact answers, the large screen and the menus. He found a great utility to the overhead projector screen and "hooked" it systematically. His procedural approach of mathematics expressed in the production of detailed instruction guides about the calculator and of "memos" about its use for several mathematical tasks.

In contrast, at the beginning of the project Benoit was an experienced user of graphic calculators. He did not consider students effective use of the symbolic calculator to be really important. Authors observed that his students under-utilised the machines and often made errors that calculator use would have avoided. Benoit's method was based on classroom discussion without the help of an overhead projector. According to the authors, it worked thanks to exceptional classroom management skills.

Andre let his students use freely the calculator even for tasks that could be done by hand. The time saved by not doing paper/pencil calculation was used to study the use of the calculator thoroughly. This study was based on precise references to button sequences on the machine: press F4, then F6, and so forth. He made no difference between using the derivative button on the calculator and a by hand technique for the same task.

As for Benoit, he used mathematical terms, even to talk about techniques using the calculator. He limited and controlled students use of the calculator's symbolic facilities. The main utility of the symbolic capabilities was to get results in order that

students observe stable symbolic phenomena. Benoit believed that doing algebra by hand was extremely important for understanding.

Following the authors, Andre adopted the calculators as a new tool without modifying the tasks he offered to the students. He brought effective procedures to do these tasks, together with a somehow weaker mathematical reflection. Benoit's use of the symbolic capabilities of the calculator was more limited. He did not emphasise the technical characteristics, visibly mistrusting the effect of techniques of use of the calculator on conceptualisations.

This difference in approaches to teaching using a technology has no simple interpretation. Teachers' relationship to mathematics is important in the way they adapt their teaching to integrate the calculator, but factors linked to the professional habits or "habitus"[1] and other personal features play also a role. Overcoming these limitations seems to be difficult because they are so intimately linked to personal features of the teachers.

The authors say that: " the different strategies that the teachers employed suggest that there will be a variety of successful solutions to the problem of teaching both mathematics and technology use". As for me, I think that they also give evidence of limitations in the integration of CAS: Andre tended to promote CAS techniques just for their pragmatic function, thus giving no mathematical content to the instrumented work. Benoit took full advantage of the epistemic role of graphic calculators and paper/pencil techniques. He was not comfortable with the new calculator because he did not recognise this dimension in CAS techniques. In the two cases, my interpretation of the teachers behaviour is that they were not aware of the epistemic role that CAS instrumented techniques could play.

The difficulties of "ordinary" teachers

I reported above on the approach of French "expert" teachers to the use of DERIVE. Lumb, Monaghan and Mulligan (2000) report on successes but also on problems encountered by two "ordinary" teachers trying to make "intensive" use of this CAS. Like the French experts, these teachers saw great potentialities in the use of DERIVE. They also stressed the counterpart: using DERIVE as a regular resource was "at the upper end of the 'effort' scale for mathematics software", and much time was required "to get a 'feel' for how to use DERIVE".

One of the teacher recognised that a number of ideas he had at the beginning of the project were not realistic. A close examination of his classroom activity shows, among other changes, a decrease in the time devoted to the "coaching", a term that the authors use for an activity consisting in "pointing out mathematical features without revealing the answer." Monaghan (2001) found the same decrease on a

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[1] In sociology, the "habitus" is how somebody exists in social life, by way of a set of socially coded marks.

sample of 13 teachers and a simultaneous increase of another coaching devoted to explaining or facilitating technological features. He notes that this observation does not support the idea of teachers acting as 'a catalyst for self-directed student learning'.

The two teachers broke with their usual use of a textbook and devoted much effort to prepare work sheets for students. They claim that it was impossible to use material prepared with no use of DERIVE in mind. They also did not use a number of texts that their research group prepared for classroom use. As authors say: "teachers who plan to incorporate significant use of computer algebra in their teaching are presented with a re-evaluation of the mathematics they were taught and are familiar with. These re-evaluation are quite specific to the individuals and someone else 'route' is not easy to accommodate."

Like their Australian colleagues, these teachers were not directly comfortable with the use of a new technology. The paradoxical decrease of the mathematical coaching seems to me linked to the difficulty to 'feel' the use of DERIVE. In order to be able to recognise and point out mathematical aspects in the classroom, the teacher needs to anticipate very varied aspects of the situation. Without technology, (s)he has a knowledge of current situations, and (s)he is used to refer their different aspects to mathematical ideas. Thus (s)he or an observer may think that (s)he is easily improvising. The use of technology jeopardises this improvisation by introducing factors whose mathematical meaning is not directly clear for the teacher. I stressed above, from the observation of French and Australian teachers, that it is not easy to give a mathematical meaning to instrumented techniques. Lumb, Monaghan and Mulligan's study suggests that many personal aspects of the teaching practices have to be re-evaluated when a teacher attempts to integrate technology. An in-depth study of this re-evaluation should help to understand the role of instrumented techniques and to identify other aspects.

Conclusion

The selection of research studies about the teacher and ICT strengthens the idea of a difficult integration, contrasting with researches centred on epistemological or cognitive aspects. "Expert" and "ordinary" teachers meet classroom phenomena different from their expectation. Abboud (1994) emphasises the didactical dimension of an "internal" analysis. The consequence is that offering teachers examples of innovation is not enough, teachers need didactical knowledge about the use of technology. Recent observation suggests however that no transfer of didactical knowledge can at the present time bring an easy integration. Taking the example of Computer Algebra Systems, a wealth of ideas, experiment and didactical analysis has been published along the past twenty years. Only recently, researchers tried to observe classroom practices and saw that this wealth does not provide teachers with means for a satisfactory integration. Far from pyramidal knowledge transfer, the observation shows that integration is done neither "at a stroke" nor uniformly. This is consistent with the idea of a teacher in "transition" (Zbiek, 2001): the integration of

technology requires a deep transformation of the "habitus", of the classroom management and of the representations of mathematics that cannot be done in a single year.

The new dimensions of analysis, instrumental and institutional, can help this transition by better accounting for the complexity of the integration. My hypothesis is that instrumented techniques are a key in the teacher practices linked to integration. In an institutional approach, these techniques are a potential support to conceptualisations. Institutions of all levels (classroom, school, educational systems, etc.) have to decide of their acceptance, taking into account factors like the time required for their elaboration, their links with conceptualisations and their social visibility. In the classroom the teacher is the person in charge of "official mathematics". (S)he has to install "instrumented" and "by hand" techniques in the common knowledge, selecting the most significant out of them. Instrumented techniques are also the visible part of instrumental geneses. During their elaboration, the teacher will have to guide the student in his (her) construction of technological and mathematical knowledge. The teacher is not prepared for this new role.

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