SOFTWARE ENVIRONMENTS SUPPORTING AND ENHANCING SYSTEMIC THINKING

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In this paper Systemic Thinking is considered as a general philosophy that, by suggesting a "thinking globally, but acting locally" approach, can represent a major paradigm shift in how we view the world. After dwelling on the characteristics of Systemic Thinking and its importance in Mathematics, we analyse whether, how and why a widely diffused mathematical application software (Excel), implemented as a general problem solving tool, involving low-medium level mathematical knowledge, could play the role of mediator in supporting and creating the conditions for Systemic Thinking development. We maintain also that a particular microworld (Cabri), implemented in research environments for problem solving activities in a very specific mathematical topic, can play the same role.

1. Introduction

Lena Licon Khisty describes Systemic Thinking as a general philosophy that "suggests thinking globally, but acting locally". She writes that "Systemic Thinking represents a major paradigm shift in how we view the world; it is a shift away from the view of reductionism or thinking about isolated parts that fit a mechanistic model." (Licon Khisty, 1997). The term Systemic Thinking refers to the concept of "system". In this sense, the word "system" stands for "a (possibly complex) set of elements that may be of different nature and which interact towards the accomplishment of certain objectives". As this definition implies, a system's components are not only the individual elements composing it but also the network of interactions among them and their purpose.

Systemic Thinking is applied in various areas as a tool for analysing and designing systems: economic systems, health systems, educational systems, electoral systems, measurements systems, reference systems, etc. Systemic Thinking presupposes that one builds within his head a global scenario that represents the reference system to be worked on locally. Systemic Thinking is therefore a complex thinking process that develops through a sequence of individual thinking acts, even of different nature. Systemic Thinking requires to be supported in each phase by one or more representations and operating modalities concerning the elements involved, both actual and virtual, mental or external (Bruner, 1966; Chesa&Tarrago, 1998; Tall, 1994)

In this paper we affirm that Systemic Thinking is crucial in mathematics contexts as elsewhere, and we discuss whether, how and why some software environments may support and create the conditions for its development.

2. The importance of Systemic Thinking in Mathematics

In (Lemut & Greco, 1998), the authors maintain that systemic thinking is central to algebraic modelling. They show that systemic thinking intervenes when: a) identifying the entities involved in a given situation and the characteristics of all mutual influences, and when acting on each single entity in order to describe how it is influenced by others; b) controlling (through mental or external representations) each single relationship and the complete set of relationships involved in the problem situation; c) verifying the perception, in a sequence of representations, that one may or may not be dealing with a representation that can be just converted into algebraic language, according to the aims of the problem situation.

In the same paper, the authors note that, although central to algebraic modelling, systemic thinking is not peculiar to it. Although the matter is not discussed in the paper, they hypothesise that systemic thinking may also be at the basis of checking results from a sequence of algebraic manipulations in order to decide, from time to time, whether the result reached is suitable for verifying the plausibility of a conjecture, demonstrating a theorem or highlighting a property (Gallo, 1994). Lemut & Greco use an example to illustrate that systemic thinking also intervenes in the arithmetic solution of a given problem.

In geometry field for instance, the role of Systemic Thinking is fundamental in open situations where a conjecture must be formulated and thereafter demonstrated. In (Arzarello, 1998), the following case is discussed, though with a different purpose*: "Given a quadrilateral ABCD and a point Po, construct the point P1, symmetric of Po with respect to A, P2 symmetric of P1 with respect to B, P3 symmetric of P2 with respect to C, P4 symmetric of P3 with respect to D. Determine which conditions the quadrilateral ABCD must satisfy so that P0 and P4 coincide."* The key step in solving the problem is when, after establishing the relationship between the positions of points P1, P2, P3 and P4 with respect to Po, A, B, C and D, the student investigates what positions A, B, C and D take if P4 is made to coincide with Po. It is only at this stage that the relationships between the two groups of points (which we could define as inverse each other) are also made explicit, thus completing the picture of the "system" in question.

3. Software environments for supporting and mediating Systemic Thinking

This section discusses whether, how and why Systemic Thinking may be supported and developed through activities based on commonly used software. We consider a leasing situation that may be seen as an open problem; it is a very common everyday situation that contains a strong mathematical component.

A brief account of the situation is provided, followed by an analysis of the reasoning performed by an adult, Enrico, who is fairly confident with mathematical tools and is used to handling computing devices (calculator, spreadsheet and software programs for symbolic computation).

Signing a lease is a fairly common operation performed, for instance, when running a shop, business or public organisation. Leasing involves two subjects, the leaser and the leaseholder. A prospective leaseholder may be given a choice between various types of contract which, depending on the value of the object to be leased, offer different contract conditions, such as the number of instalments, the frequency of repayments (monthly, six-monthly, yearly), the amount of each instalment, the deposit to be paid on signing the contract, the final price for redemption. Both parties will seek to enter the type of contract which, from their individual point of view, is the most convenient.

Problem*: Suppose you are a prospective leaseholder who wants to figure out how leasing plans work and evaluate which of various options are the most convenient.*

Phase 1.

Enrico's reasoning: Enrico examines the first leasing plan: eight six-monthly instalments of 2.4 million lire each, for a commodity costing 15 million lire (for simplicity's sake, both the deposit and final redemption are disregarded). The lessor informs him that under this scheme the applicable annual interest rate is approximately 12 percent. Using the calculator, Enrico reckons the overall expense straight away as the product of *2,400,000*8=19,200,000*.

Then, by calculating , he infers that the interest payable in four years is 28%, so the annual interest is 7% (28:4). But, in this way, the result does not match the lessor's claim.

Analysis of Enrico's thinking Process: Enrico instinctively performs a few simple (albeit incorrect) calculations suggested by the data at hand. At the end of this phase he applies a first stage of systemic thinking *(1° stage ST)* when he checks whether the amount of annual interest he worked out is consistent with the information received from the vendor. He then realises that his way of calculating the interest rate is too simplistic, but he lacks the further knowledge required.

Phase 2.

Enrico's reasoning: He therefore requests further information and obtains the following explanation: a) each instalment contains a capital share and an interest share; b) the capital and interest shares of each instalment depend on the interest rate related to the number of instalments per year; c) the capital shares that are to be paid back in succession must be calculated using the compound interest formula after *n* periods:

(instalment=capital*(1+interest)^{n}, hence capital=instalment*(1+interest)$^{-n}$).

Analysis of Enrico's thinking Process: Enrico seeks out additional information in order to find further relationships within his data that might tell him what he wants to know. In other words, he realises that he must discover other elements of the system underlying lease contracts *(2° stage ST)*. The intricate situation emerging

from the new data acquired tells Enrico that he needs a suitable aid for processing, representing and quantifying the relationships he is trying to express.

Phase 3.

Enrico's reasoning: Enrico decides to give up the calculator and resort to a spreadsheet (Fig. 1). At the top of the sheet, he describes and represent the elements relevant to the situation: value of the commodity (A1-B1), deposit (A2-B2), number of instalments (A3-B3), frequency of repayments (A4-B4), instalment amount (A5-B5), redemption price (A6-B6) and total cost (A7-B7). Further down he applies the suggested formulas and represents: a) the progressive number of instalments from 1 to 8 (from A 10 to A17); b) the capital share paid back within each instalment (from B10 to B17); c) the related interest share (from C10 to C17). Finally, he describes the present value in A19-B19.

	A	B	C
1	commercial value	15000000	
2	deposit	0	
3	instalments number	8	
4	periodicity	six-montly	
5	instalment amount	2400000	
6	redemption	0	
7	total cost	=B2+B3*B5+B6	
8			
9	periodic rate		
10	1	=$B$5*(1+$B$9)^(-A10)	=$B$5-B10
11	=A10+1	=$B$5*(1+$B$9)^(-A11)	=$B$5-B11
12	=A11+1	=$B$5*(1+$B$9)^(-A12)	=$B$5-B12
13	=A12+1	=$B$5*(1+$B$9)^(-A13)	=$B$5-B13
14	=A13+1	=$B$5*(1+$B$9)^(-A14)	=$B$5-B14
15	=A14+1	=$B$5*(1+$B$9)^(-A15)	=$B$5-B15
16	=A15+1	=$B$5*(1+$B$9)^(-A16)	=$B$5-B16
17	=A16+1	=$B$5*(1+$B$9)^(-A17)	=$B$5-B17
18			
19	present value	=SOMMA(B10:B17)	
20			

Fig. 1 Formulae used

leas.M.2

	A	B	C
1	commercial value	15000000	
2	deposit	0	
3	instalments number	8	
4	periodicity	six-montly	
5	instalment amount	2400000	
6	redemption	0	
7	total cost	19200000	
8			
9	periodic rate	0,035	
10	1	2318841	81159
11	2	2240426	159574
12	3	2164662	235338
13	4	2091461	308539
14	5	2020736	379264
15	6	1952402	447598
16	7	1886378	513622
17	8	1822588	577412
18			
19	present value	16497493	
20			

Fig.2 Present value in case the six-monthly was 3,5%

Analysis of Enrico's thinking Process: Enrico sets out a systemic table *(3° stage ST)* representing the data he has, the results he wants to achieve and the relationships between them. This organisational effort is to a large extent aided by the software tool he employs, the spreadsheet.

Phase 4.

Enrico's reasoning: To use the sheet in an effective way, Enrico needs to approximate roughly a six-monthly rate (SR) he considers reasonable and enter it in cell B9. Having done so, he automatically obtains a first lease plan, but the sum of the capital shares therein is out by 15 million lire (commercial value of the commodity (fig.2). He subsequently realises that, to be consistent with the plan at the top of the sheet, he needs to come up with a six-monthly rate that produces a sum equal to 15 million lire. He is told that a "search objective" command (fig.3a) fulfils this purpose and so applies it (Fig.3b). In this way, he can read on the sheet (cell B9, Fig.3b) the interest rate that has been actually applied.

Analysis of Enrico's thinking Process: This phase is clearly guided by the characteristics of the spreadsheet itself. Until Enrico comes up with a preliminary hypothesis regarding the six-monthly rate (B9,Fig.3a), the sheet calculates evidently an incoherent (B19,Fig.3a) result as the sum of the cells from B10 to B17. When Enrico applies the "search objective" command correctly, he is prompted to clarify the relationship between the entities in question, i.e. the amount of the individual instalments and the interest rate applied, the sum of all capital shares and the present value of the commodity. Enrico uses this command to calculate at what interest rate the sum of the capital shares is actualised as 15 million lire, given the instalment rate set in the leasing plan (B5) (*4° stage ST)*. The same command also reckons the instalment amount the vendor should fix if there was a rise or fall in the applicable interest rate. This facility offered by the spreadsheet allows Enrico to contemplate the situation from two points of view, his own as a buyer and the vendor's, thus expanding his systemic view of the situation *(5° stage ST)*.

leas.M.2

	A	B	C
1	commercial value	15000000	
2	deposit	0	
3	instalments number	8	
4	periodicity	six-montly	
5	instalment amount	2400000	
6	redemption	0	
7	total cost	19200000	
8			
9	periodic rate	0,035	
10	1	2318841	81159
11	2	2240426	159574
12	3	2164662	235338
13	4	2091461	308539
14	5	2020736	379264
15	6	1952402	447598
16	7	1886378	513622
17	8	1822588	577412
18			
19	present value	16497493	
20			

	A	B	C
1	commercial value	15000000	
2	deposit	0	
3	instalments number	8	
4	periodicity	six-montly	
5	instalment amount	2400000	
6	redemption	0	
7	total cost	=B2+B3*B5+B6	
8			
9	periodic rate		
10	1	=$B$5*(1+$B$9)^(-A10)	=$B$5-B10
11	=A10+1	=$B$5*(1+$B$9)^(-A11)	=$B$5-B11
12	=A11+1	=$B$5*(1+$B$9)^(-A12)	=$B$5-B12
13	=A12+1	=$B$5*(1+$B$9)^(-A13)	=$B$5-B13
14	=A13+1	=$B$5*(1+$B$9)^(-A14)	=$B$5-B14
15	=A14+1	=$B$5*(1+$B$9)^(-A15)	=$B$5-B15
16	=A15+1	=$B$5*(1+$B$9)^(-A16)	=$B$5-B16
17	=A16+1	=$B$5*(1+$B$9)^(-A17)	=$B$5-B17
18			
19	present value	=SOMMA(B10:B17)	
20			

Fig.3a -3b - Seeking for the six-monthly rate resulting in a present value of 15 million Italian lira

Analysis of Enrico's thinking Process: This phase is clearly guided by the characteristics of the spreadsheet itself. Until Enrico comes up with a preliminary hypothesis regarding the six-monthly rate (B9, Fig.3a), the sheet calculates evidently an incoherent (B19,Fig.3a) result as the sum of the cells from B10 to B17. When Enrico applies the "search objective" command correctly, he is prompted to clarify the relationship between the entities in question, i.e. the amount of the individual instalments and the interest rate applied, the sum of all capital shares and the present value of the commodity. Enrico uses this command to calculate at what interest rate the sum of the capital shares is actualised as 15 million lire, given the instalment rate set in the leasing plan (B5) (*4° stage ST)*. The same command also reckons the instalment amount the vendor should fix if there was a rise or fall in the applicable interest rate. This facility offered by the spreadsheet allows Enrico to contemplate the situation from two points of view, his

own as a buyer and the vendor's, thus expanding his systemic view of the situation *(5° stage ST)*.

Phase 5.

Enrico's reasoning: To compare the six-monthly rate (SR) he obtained with the annual lessor's rate, Enrico tries to find out the annual rate (AR) on the basis of the six-monthly rate produced. He notices that, if an initial capital H is expected to yield the same amount of interest no matter whether this is calculated on an annual or six-monthly basis, the equivalence $H(1+SR)^2=H(1+AR)$ is obtained. From this, the relationship between the six-monthly rate and the annual rate can be worked out. Enrico describes the annual rate in A60-B60 (Fig.4).

Analysis of Enrico's thinking Process: Enrico formalises a new relationship he has found in the system *(6° stage ST)*. In this phase, Excel cannot provide any significant help and could be replaced by the pocket calculator Enrico used earlier on.

Phase 6.

leas.1

	A	B	C	D
1	commercial value	15000000		15000000
2	on account	0		0
3	instalments number	8		48
4	periodicity	six-monthly		monthly
5	instalment amount	2400000		400000
6	redemption	0		0
7	total cost	19200000		19200000
8				
9	periodic rate	0,058		0,010562829
10	1	2267633	132367	395819
11	2	2142567	257433	391682
12	3	2024399	375601	387588
13	4	1912748	487252	383537
14	5	1807254	592746	379528
15	6	1707579	692421	375561
16	7	1613402	786598	371635
17	8	1524418	875582	367751
18	9			363907
19	10			360103
20	11			356339

58				
59	present value	15000000		15000000
60	annual rate	0,120151569		0,134383335
61				
62				

Fig. 4 – Comparing B60-D60 suggests the leasing plan more convenient for the buyer

Enrico's reasoning: At this point, Enrico wonders whether he may be better off paying back in monthly instalments, which are one sixth the amount of the six-monthly ones. He then inserts the data concerning this hypothesis on the sheet (column D, Fig.4), so as to compare the two situations. After the calculation he finds the monthly interest rate that would be applicable (D9, Fig.4) and calculates

the related annual rate. He then realises that this second hypothesis would be to his disadvantage since the annual rate is higher in this case (D60) than before (B60).

Analysis of Enrico's thinking Process: Enrico is guided by his curiosity and supported by the easy of representing a situation and getting information in a spreadsheet environment. Making small changes to the sheet layout, he can compare the two hypotheses *(7° stage ST)* he himself formulated (Fig.4) and weigh up their financial impact both from his point of view and from the lessor's. In this phase the computer is no longer just a mere support, but guarantees the conditions that make systemic thinking possible and allow it to develop *(8° stage ST)*.

Phase 7.

Enrico's reasoning: Enrico, who had never before wanted anything to do with matters of this kind (even when he might have needed to), starts wondering what is actually behind leasing plans. He especially wonders what relationship there is between the capital and interest shares of each instalment. Studying the sheet, he notices that *C10=B9*B10* and so wonders whether *C11=B9*B11*, *C12=B9*B12* and so on. However, he can see that this relationship does not hold true because the capital share (B10-B17) diminishes while the interest share (C10-C17) grows (Fig.5, columns C and D).

leas.M.6

	A	B	C	D	E
3	instalments number	8			
4	periodicity	six-montly			
5	instalment amount	2400000		free exploration	hypothesis :
6	redemption	0		Cn = B9*Bn ?	N?
7	total cost	19200000			
8					
9	periodic rate	0,058372132			
10	1	2267633	132367	132367	B9*B10
11	2	2142567	257433	125066	B9*B11
12	3	2024399	375601	118168	B9*B12
13	4	1912748	487252	111651	B9*B13
14	5	1807254	592746	105493	B9*B14
15	6	1707579	692421	99675	B9*B15
16	7	1613402	786598	94178	B9*B16
17	8	1524418	875582	88984	B9*B17
18					
19	present value	15000000			
20	annual rate	0,120151569			
21					

Fig. 5 – The first free exploration

Adding temporary columns or using empty cells in sheet areas that do not interfere with the calculations, Enrico enters formulas to explore the sheet and make conjectures. For instance, he notices that *C11-C10* is equal to *B9*B10*, so he calculates *C12-C11* and notices that the result is equal to *B9*B10+B9*B11*, and so on. He thus assumes that the interest payable for the nth instalment is the result of *B9*B10+B9*B11+B9*B12+ …+B9*Bn*

i.e.

interestrate*(first.instalment.capital+second.instal.capital+.....+nth.instal.capital).

Analysis of Enrico's thinking Process: The same applies in this phase *7,* where Enrico interacts continuously with the sheet in order to make conjectures about further relationships between the system's elements *(9° stage ST)* and to formalise them. He becomes increasingly bold in manipulating the formulas, looking at the spreadsheet globally but operating on it locally *(10° stage ST)*.

Phase 8.

Enrico's reasoning: He now verifies his last conjecture in formal terms. In doing so, he needs to draw upon his knowledge of algebra with the aid of a software program for symbolic computing. This check is based on the assumption that the lessor, at the end of each instalment term, has the same capital s/he would have had if s/he had invested the leased amount at the same interest rate s/he charged the lease-holder.

Analysis of Enrico's thinking Process: The verification of his conjecture in this phase leads Enrico to concentrate on the economic significance of the objects involved *(11° stage ST)* in order to find a key for formalising his conjecture. Once again he is encouraged to put himself in the vendor's shoes and get a deeper insight into the system *(12° stage ST)*. At this level, the new software employed (a tool for symbolic computation) is once again able to support the process.

4. Conclusions

The analysis of Enrico's thinking processes has made it possible to clarify how Systemic Thinking progresses during the study of an open situation. At the same time it has shown how common software programs such as Excel may play the role of mediator in supporting and creating the conditions for the development of Systemic Thinking, that has to be considered as a very important and cross-disciplinary way of reasoning. Specifically, our analysis points out that:

a) open problem solving activities require students to perceive and gradually untangle the "system" that lies at the core of the problematic situation examined;

b) achieving a global view of a situation (e.g. Phase 4) and being able to understand and use mathematical models (algebraic models and the "search objective " model embedded in Excel) may help discover and formalise local actions that describe particular relationships between certain elements of the context;

c) reasoning on local relationships (e.g. Phases 6 and 7) that are to some extent formalised may improve the skill of re-thinking a system globally and reveal aspects that might otherwise have remained hidden.

Applying the same methodology as in leasing case, we analysed also the Enrico's thinking process when approaching the situation described in the footnote[1]; situation

[1] Let *d* and *d'* be two intersecting straight lines and P a point, which does not belong to either of those lines. Construct two points A and B such that A belongs to *d*, B belongs to *d'* and P is the midpoint of the segment AB. (Laborde, 1998).

that, unlike the leasing case, belongs exclusively to mathematics and has been solved in Cabri-Geometre environment. We found a strong analogy between the two situations as concerns what highlighted, making reference to the leasing case, in a), b) and c).

We feel that among the reasons why certain software programs are able to support and create the conditions for Systemic Thinking development, the following can be included: a) they offer a great variety of representation modalities and inherent functions; b) users can be forced to individuate a "system" on which to operate; c) users' attention is concentrated on interpreting and finding out relationships within the system; d) the focus is on how to operate and not on executing operations; e) it is possible to satisfy users curiosity as well as to formulate, validate and verify conjectures, getting a better understanding of a given "system"; e) users can be encouraged to view matters from various points of view and to analyse the meaning of specific software outputs.

From our analysis, it appears also that certain software environments can play a crucial role in Systemic Thinking development when the following conditions occur: users are asked to solve open situations; tutors are very attentive in grasping what suggestion the user needs, and when; users have a strong motivation or curiosity to acquire new knowledge; users are inclined to gamble on the potential of the software in use.

In conclusion, we feel that follow-up studies should focus on whether conscious activation of Systemic Thinking in significant problem situations under tutor guidance (that is crucial in several reasoning phases as in our example) could enable students to apply this intellectual tool in other cases. Preliminary experience gained with adults and 14-15 year-old students seems to provide a positive answer to this, and appears to be in line with the statement that systemic thinking is a "...way of thinking that, once adopted, permeates all thinking regardless of situations or context." (Licon Khisty, 1997).

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