

# INTEGRATION OF ICT IN THE TEACHING OF MATHEMATICS IN SITUATIONS FOR TREATMENT OF DIFFICULTIES IN PROVING

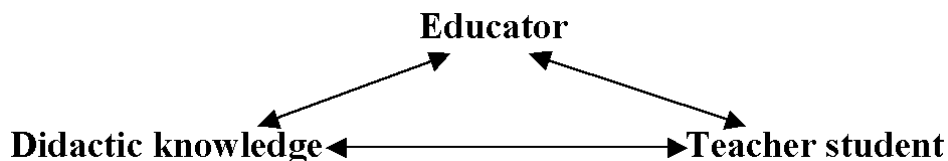
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*The study related in this paper is small-scale and deals with integration of ICT into the maths teaching. We attempted to analyse teacher education sessions addressing the integration of ICT and to investigate the impact of such sessions. We point out some elements that are critical in the design of teacher education about the integration of new technologies into maths teaching, but also, that must be object of research.*

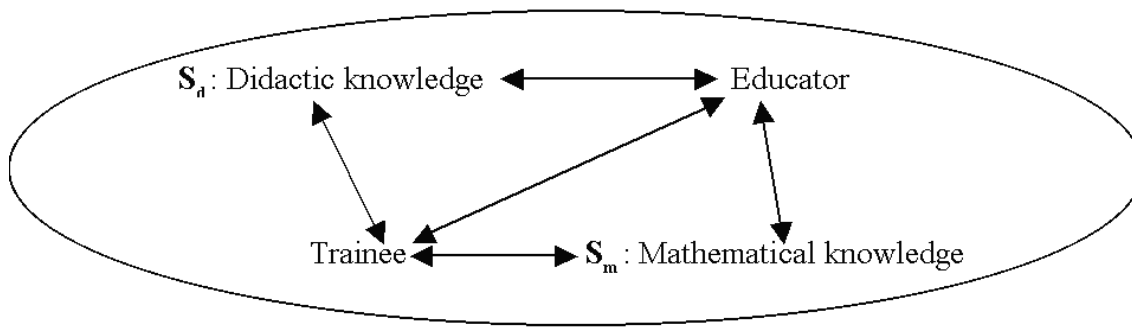
## I - EDUCATING TEACHERS FOR THE INTEGRATION OF NEW TECHNOLOGIES

The education of trainee teachers to the use of ICT considered as a teaching system is quite complex. While it is possible to analyse this system using a variety of theoretical positions, the analysis that follows is based on the didactic triangle proposed by Portugais (1999) for teacher education and adapted from Brousseau's didactic triangle (1986). This triangle is made of three poles, the educator, the trainee teacher and the didactic knowledge.

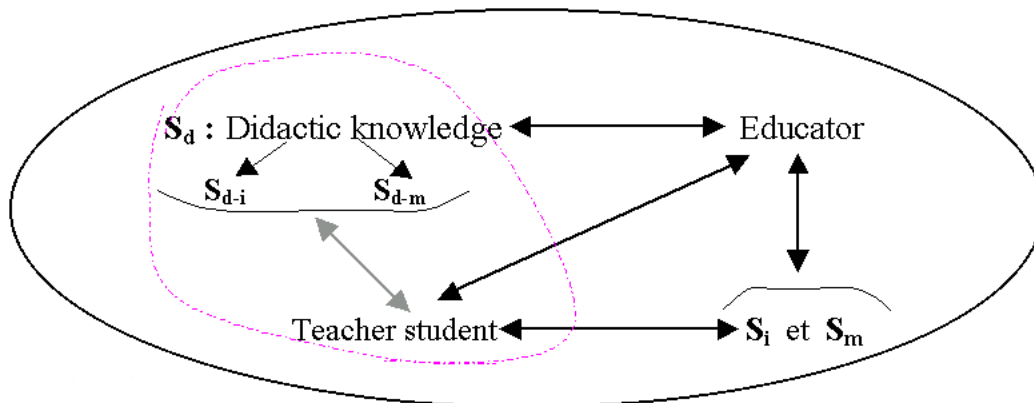


Rolet and al (1999) proposed to insert in this diagram the content-knowledge that we have called mathematical knowledge  $S_m$ . This is « the knowledge found in traditional school disciplines (French, mathematics, etc.) for which there exists a scholarly knowledge, of which the process of transposition has been studied, and which gives rise, in general, to an explicit program of studies in the framework of teacher education » (see figure, below).

By didactic knowledge,  $S_d$ , we mean « the knowledge coming from the didactics of traditional school disciplines, devoted to the teaching and learning of content knowledge. Such didactic knowledge has been lately established and there exist some reference texts, or some issues of research in progress ... Their transposition remains a topic of research. » (Rolet and al. 1999).



As far as the integration of new technologies in maths teaching is concerned, we propose a diagram in which we have included the notion of artefact and instrument.



In this diagram;

$S_m$  denotes mathematical knowledge. The trainee's relation to mathematical knowledge is not in the explicit aims of teacher education in their second year of University Teacher Education Institute since the trainee teachers already passed a national examination purely based on mathematical knowledge at the end of the first year.  $S_i$  denotes the knowledge on the use of the artefact. The trainee teacher relation to  $S_i$  constitutes the first axis of education concerning in the case of our study the manipulation and the use of Cabri-geometry (a dynamic geometry software program).  $S_{d-i}$  denotes the didactic knowledge attached to the implementation of the artefact in a learning situation. The trainee teacher relation to  $S_{d-i}$  constitutes the second axis of education which aims to enable the trainee teacher to put in interrelation  $S_m$  and  $S_i$ ; that means how to integrate ICT into maths teaching. Trainees, in their future task as teachers, will have to use  $S_m$  and  $S_i$  in interrelation.  $S_{d-m}$  denotes the knowledge linked to the implementation of the content knowledge (i.e. the mathematical notions) in a teaching situation. This knowledge is to be found in the didactics of school disciplines (here mathematics) and it deals with the teaching and the learning processes without taking into account the artefact. As for the teaching competence of concerned trainee teachers, they have a classroom of mathematics in their responsibility.

These four types of knowledge, as presented above, are nevertheless formalised but it should be stressed that these types of knowledge differ in nature and they exist in very different ways. Thus they are not stated in the same way: They may be

completely explicit (for example  $S_m$ ) or they may remain implicit (for example in some cases for  $S_{d-m}$  or for  $S_{d-i}$ ); some may be communicated by writing but some are only expressed orally.

Starting from the diagram above, representing the interrelations in teacher education for the integration of new technologies, we are questioning the impact of such education.

## **II- WHAT IS THE IMPACT TEACHER EDUCATION IN THE INTEGRATION OF THE NEW TECHNOLOGIES?**

It has often been claimed that the individual learns from his/her own mistakes or errors; these latter provide opportunities for the learner to become aware of conflicts existing between old and new knowledge. This led us to choose mistakes as indicators for measuring the impact of education to the integration of new technologies in maths teaching.

We selected a mathematical topic, namely geometry and a type of problems in geometry namely proof problems. Actually proof is considered by teachers as a difficult topic, one of the key topics of middle school teaching in France. These choices allowed us to formulate our research questions.

**Q 1:** How does a trainee teacher integrate ICT, before receiving specific education about their integration: What tasks is he/she able to propose to pupils in order to help them overcome difficulties in writing proofs?

**Q 2:** What is the impact of such education to the ICT integration upon the ways trainee teachers design computer based situations intended to improve the learning of proof?

### **METHODOLOGY**

With the aim of measuring the impact of education to the use of Dynamic Geometry we carried out two experiments. The subjects were trainee teachers at the University Teacher Education Institute of Grenoble.

In the two experiments, the trainee teachers were given proof activities proposed in the paper - pencil environment with fictitious students solutions comprising mistakes; the trainee teachers were asked to point out the students' mistakes and to propose Cabri II based situations for the treatment of these mistakes. Making trainee teachers work on pupils' solutions allowed us to place them in an activity that they have to carry out in their profession as teachers. We preferred to take fictitious students solutions because it allowed us to control the type of errors on which trainee teachers had to work.

The first experiment was carried out with trainee teachers (two pairs) who had not yet followed the sessions on Dynamic Geometry in order to observe how the trainee teachers integrate ICT (and more precisely some specific Cabri features) in tasks without having received a deep education on the use of ICT in maths teaching. This first experiment was aimed to answer question Q1.

Next, we observed and recorded Dynamic Geometry sessions and we analysed the tasks to which the trainee teachers were faced from the point of view of some Cabri features.

Once the Dynamic Geometry sessions finished, we carried out a second experiment with the same trainee teachers as in our first experiment. We wanted to investigate to what extent they were able to reinvest the work done during Dynamic Geometry sessions in to the specific topic of proof. The analysis of experiment 2 was aimed to answer question Q2.

### **GRID OF ANALYSIS**

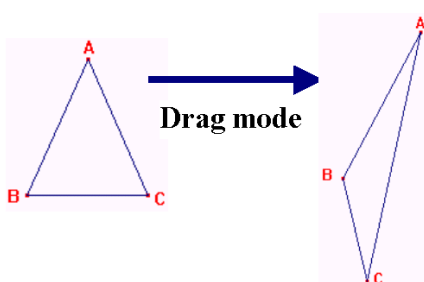
With the aim to analyse the results of both experiments as well as those of the observation of Dynamic Geometry sessions, we constructed a grid of analysis. Tools of Cabri object of analysis are: Construction, Drag mode, Black Box, History of the construction processes (session replay), Hide/Show and Ambiguity.

Errors in proof formulating selected for the study are: Use of information just visualized on the diagram: This type of mistake often occurs at middle school when moving from the geometry of observation to the geometry of deduction. This move causes an important change of didactic contract in geometry in the secondary school;

- Confusion in the operative status of statements: Duval (1989) shows that every proof step has a ternary structure which is made up of earlier given statements, a rule of substitution and a new statement. This type of mistake consists generally in making no distinction between hypothesis and conclusion where pupils use conclusion as hypothesis;

- Confusion between the status and the content of a statement: We mean by this type of mistake, mistakes where pupils do not take into account the status of a statement and reason only on its content. Indeed, a statement, having the same content can change status according to proof. This is a difficulty for pupils who are accustomed to reasoning on the content of statements.

Thus, we established a relation between these proof mistakes and some selected Cabri tools. For example, for the mistake of visualisation on the diagram and the drag mode of Cabri we established the relation “Drag - pure visualisation”. It is about using a Cabri tool (drag) to highlight pupils’ mistakes (pure visualisation). For example; it is about providing the pupil with a Cabri-diagram which seems to have some geometrical properties. Those properties are possibly properties that the pupil reads on the static drawing, and the drag mode can be used to show to the pupil that there is something wrong and to make him think about what doesn't work and why.



Faced with the drawing of triangle ABC drawn, the pupil is likely to immediately say by direct “reading” on the drawing that triangle ABC is isosceles.

As, with the drag mode, the pupil is going to be able to understand that he just reads on the drawing and that

ABC may indeed not be isosceles. Then, we can ask the pupil: What further property is necessary so that ABC is isosceles?

### **EXPERIMENTS Experiment 1**

Experiment 1 was achieved with two pairs of trainee teachers at the University Teacher Education Institute of Grenoble. These trainees followed a first initiation to Cabri, « Initiation to Cabri »: a compulsory unit for all trainees ; and they didn't follow a deeper optional teaching « Dynamic Geometry ».

We proposed to the trainee teachers three proof tasks for middle school level in paper-pencil environment with fictitious pupils' solutions in which were inserted proof mistakes. The trainee teachers were asked to propose tasks in Cabri for the treatment of those mistakes.

Results of our first experiment showed that the drag mode feature was not used by trainees. On the other hand, the specific features of Cabri related to « construction » were used. The other features of the environment were not used.

Following experiment 1 we observed and analysed Dynamic Geometry sessions.

#### **Observation of Dynamic Geometry sessions**

In order to analyze Dynamic Geometry sessions, we defined a priori five levels of treatment of the Cabri tools as follows:

- the tool is explicit in the session, the use of the tool is both object of teaching from the perspective of the use of the artefact and from a didactic perspective,
- the tool is explicit in the session from a didactic perspective but implicit from an artefact perspective
- the tool is explicit as an artefact but implicit from a didactic perspective,
- the tool remains implicit from both perspectives
- the tool is absent from the sessions

Thanks to this analysis we could note that

Tools « **drag** » and « **construction** » were explicit in the Dynamic Geometry sessions. The specificity « **black box** » was at an intermediate level between the level « explicit from a didactic perspective but implicit from an artefact perspective » and the level « implicit from both perspectives ». Tool « **hide/show** » was explicit in the sessions as an artefact but implicit from a didactic point of view. The functionality « **ambiguity** » was implicit from a didactic perspective and from an artefact perspective. The tool « **session replay** » of Cabri was absent from the Dynamic Geometry sessions.

Once the Dynamic Geometry sessions ended, we carried out our second experimentation.

## **Experiment 2**

Our second experiment was achieved with the two same pairs of trainee teachers at the University Teacher Education Institute of Grenoble, but this time after the « Dynamic Geometry » sessions. We designed experiment 2 in two parts: In the first part, the trainee teachers were asked to use Cabri for the treatment of mistakes in fictitious productions. In the second part, the experimenter prompted the trainee teachers to use the tools « black box », « ambiguity », « session replay » and « hide/show » for addressing the mistakes of fictitious productions.

While constructing experiment 2 in two parts, we based it on the results of experiment 1 as well as on the observation of Dynamic Geometry sessions. On the one hand, results of experiment 1 showed that the trainee teachers use the tool « construction » even before the training sessions; on the other hand, the analysis of Dynamic Geometry sessions shows that the tools « drag » and « construction » were treated explicitly during the sessions. So we thought that the experimenter did not need to prompt the use of those two tools. However, the tools « black box », « ambiguity », « hide/show » and « session replay » being little or not at all addressed during the sessions, we decided to prompt their use in the second part of experiment2.

The analysis of this second experiment showed that:

Functionalities « drag » and « construction » of Cabri being explicit in the sessions from both didactic and artefact perspectives, were used by the two pairs in the first part of the experiment (i.e. without any solicitation).

The functionality « black box » being at an intermediate level was used by one pair of the two pairs in the first part of the experiment whilst it was used by the two pairs in the second part of the experiment (after being prompted to use it).

The functionality « hide/show » was explicit as an artefact but implicit from a didactic perspective, « ambiguity » was implicit from a didactic and artefact perspective and the functionality « Session replay » was absent from the sessions. These three functionalities of Cabri were not used by trainees during the first part of the experiment. We can assume that this absence of use comes from the fact that these three features were not object of teaching from a didactic perspective.

This clearly shows the impact of Dynamic Geometry sessions.

On the other hand, in the second part of the experiment, we could note a difference among the use of tools « hide/show », « ambiguity » and « session replay ».

The tool « session replay » was used in the second part of the experiment for all activities. This occurred because schemes of instrumentation of this tool and of the « construction » tools of Cabri are quite similar.

Functionalities « hide/show » and « ambiguity » are both implicit from a didactic perspective in the Dynamic Geometry sessions. However in the second part of the experiment trainees used the functionality « ambiguity » but they did not use the functionality « hide/show ». We think that this difference essentially arose because trainees did not see any other utility of the tool « hide/show » than that of the aesthetic aspect. We could observe that «session replay», absent from the sessions was used by the two pairs during the second part of experiment 2.

We infer from this observation that not only the explicitness in the sessions affect the integration of Cabri tools by the future teachers but that there are also other more complex factors that come into play.

### **III- DISCUSSION**

Our points of discussion are the following:

To give to the mathematical problems a context in the computer environment, the « Dynamic Geometry» session intended to make use of the interrelation of  $S_m$  (mathematical knowledge) and  $S_i$  (the knowledge on the use of the artefact). The « Dynamic Geometry» session also had the other aim to modify the trainee's relation to  $S_{d-i}$  (didactic knowledge attached to the implementation of the artefact in a learning situation) while asking them to analyse or to design didactic situations.

It turns out from the experiment that didactic knowledge  $S_{d-m}$  (knowledge linked to the implementation of the content knowledge in a teaching situation) is indispensable for trainees to construct didactic situations making use of Cabri even though they had knowledge at levels  $S_{d-i}$  (didactic knowledge attached to the implementation of the artefact in a learning situation),  $S_i$  (the knowledge on the use of the artefact) and  $S_m$  (mathematical knowledge).

Another point of discussion is related to the construction of schemes of instrumentation (Rabardel, 1999). For the pupils the construction of schemes of instrumentation takes place in interaction with the construction of mathematical knowledge. But, the future teachers construct schemes of instrumentation in interaction with their available mathematical knowledge but not in interaction with the construction process of mathematical knowledge.

### **IV- CONCLUSION**

Proof mistakes as well as Cabri tools of Cabri are not treated in the same way by the trainee teachers. For example, addressing the immediate reading on the drawing by using the drag mode doesn't pose any difficulty for trainees. But, addressing recognition problem of the operative status of statements by using the tool ambiguity seems to be more difficult for them.

Firstly this is probably because addressing the reading on the drawing requires less knowledge at the level  $S_{d-m}$  (knowledge linked to the implementation of the content knowledge in a teaching situation) than addressing the recognition problem of the operative status of statements; secondly, the use of the tool ambiguity requires more

knowledge at the level  $S_{d-i}$  (didactic knowledge attached to the implementation of the artefact in a learning situation) and  $S_i$  (the knowledge on the use of the artefact) than that of the drag mode ; and lastly, the drag mode played a more important role in the Dynamic Geometry sessions than ambiguity.

This example reflects the complexity of a fine analysis which must take into account not only the type of difficulty, but also the Cabri tool, and the place of the tool in the Dynamic Geometry sessions in relation to the different types of knowledge. Therefore on the one hand  $S_m$  (mathematical knowledge),  $S_{d-i}$  (didactic knowledge attached to the implementation of the artefact in a learning situation) and  $S_i$  (the knowledge on the use of the artefact) come into interrelation but on the other hand something absent from the Dynamic Geometry sessions but which is indispensable:  $S_{d-m}$  (knowledge linked to the implementation of the content knowledge in a teaching situation) should also interact with other kinds of knowledge in order to allow the trainee teachers to be able to cope with students difficulties by using Cabri features.

This brings us to the final conclusion: The preparation of future teachers cannot be seen as a juxtaposition of specific modules; but on the contrary, their education only makes sense if a coherence between modules is constructed.

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