ANALYZING CHANGES IN STUDENT PERFORMANCE: WHAT IS THE ROLE OF TECHNOLOGY?

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Changes in technology have implications for the teaching and learning of school subjects. In this paper, we are interested in how to analyze changes in student performance when curriculum is influenced by shifts in technological media. Taken from a series of studies carried out in algebra classrooms we will offer examples suggesting that technologically-supported curriculum change can transform student learning and will reflect on the role of technology in such change.

Introduction

Technologists and educators interested in the ramifications of technology for the teaching and learning of school subjects speculate about the degree to which shift in technological media transform the content meant to be taught with these media and the cognitive difficulties one might anticipate students might meet as they try to learn that content. And, there is speculation about the degree to which new technologies will lead to the wholesale replacement of current curricula with new content (Examples of such are Papert 1996; Schwartz 1999; Noss 2001). Taken from a series of studies carried out in algebra classrooms over the last decade, in the body of this paper, we will present examples where students' performance in a technology-supported curriculum is different from the performance one might have predicted for students learning this content in a non-technology supported environment. These examples suggest an affirmative response to the question posed in the title: technology can transform student learning. However, in analyzing these particular examples, we will suggest that in understanding this particular change, it is more appropriate to focus on the curriculum than it is to focus on the technology used in the classroom. The curriculum which technology supported in this case changes the order in which students meet key concepts. This change in order allowed students to solve some kinds of problems that students typically might find difficult; it also created new points of transition and difficulty that might not have been difficulties for students learning in other ways. Thus, we argue that in examining examples of technological change it is important to consider curricular change. Change that involves the use of technology in the classroom may sometimes be understood more deeply as an example of curricular change, rather than a change in the nature of the media used in classrooms.

Critical Transitions: Identifying Changes in Cognitive Hierarchies

In the next sections we will analyze 2 examples. For each example we will look briefly at the curricular and technological resources students had in their disposal. We will proceed to describe data suggesting that students who used these resources performed in atypically powerful ways. A powerful performance might be the demonstration of a skill typically acquired at a later stage in learning or a demonstration of the non-routine use of a higher order skill to engage in a problem usually solved with teacher-taught routine procedures. Related to this powerful performance, we proceed to look at related data that tell a different story. This part of our examination of the examples identifies a critical moment of transition. In this moment of transition, students with the technological and curricular resources that supported powerful performance perform in quite a different way. We examine situations where they confront a new problem that in standard instruction would be assumed to be simpler or similar to problems on which students' performance was strong, but where these students have difficulties. With this three-tier analysis of these examples, we seek to illustrate that explanations for both strength and obstacles should be looked for when studying curriculum that makes use of new technological tools. In doing such analysis, we suggest that identifying critical transitions for students is an important research tool. And, finally we argue (in response to Noss, 2001) that technologically-supported curricular change can lead to change in students' cognitive hierarchies, though such change may have as much to do with curriculum as it has to do with technology.

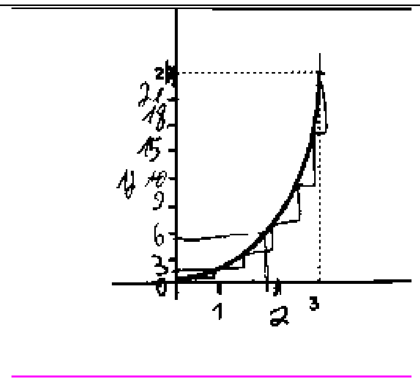
Example 1: Tools for recursive modeling of algebraic patterns

Resources: Using technology such as simulations' software, MBL or other modeling tools that includes dynamic forms of representations of computational processes, it is now possible to construct graphical models without first writing symbolic expressions with x's and y's. Several studies suggest that such emphasize on modeling offers students means and tools to reason about differences and variations (rate of change). The *VisualMath* curriculum is an example of an algebra curriculum where the learning of algebra is preceded by such semi-qualitative modeling (Yerushalmy and Shternberg 2001). In this curriculum, an MBL environment that allows users to construct and model motion generated by the movement of the computer's mouse supports the introduction of modeling of motion.

Relative strengths of students using these resources: Apparently, throughout the curriculum's focus on qualitative modeling, the students we will describe had developed ways of using tools to solve complex problems that concern non-constant rate of change. Graphs, and what we will call staircases (a graphical depiction of differences in y value for a set change in x) emerged as models of situations, and also as models for reasoning about mathematical concepts. For example, when the students looked at a distance-time graph to evaluate the speed in the middle of the interval they discussed the possible results according to the change of the stairs.

David: there is no speed -- we have time and distance. Towards the end he started to drive faster. Let's do stairs -- that would be the best. The distance increases in time and he starts to drive faster.

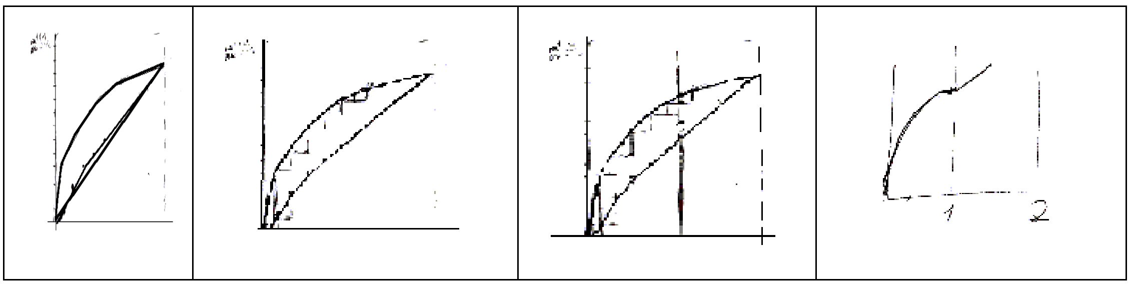
But it is hard to say. Each stair starts slow and continues faster.



In other work, Shternberg (2001) interviewed thirty-four advanced calculus students, as they solved the following problem from a qualitative calculus unit (Taylor 1992).

A cook has a large portion of meat at room temperature that should be cooked as quickly as possible. He has at his disposal a conventional oven and a microwave. In the microwave oven the meat temperature increases at a constant rate and in the conventional oven the meat temperature increases at a changing rate. In a cooking trial the cook found out that although the meat temperature in the conventional oven is always higher than in the microwave, cooking time is 2 hours in both ovens. Could cooking time be less than 2 hours using these 2 ovens?

Typically, these advanced students tried to solve this problem by fitting an expression to the process of the change in the temperature of the meat. Since the problem does not include any specific data most of the interviewees failed to represent the situation symbolically and did not devise any other solution. Shternberg also interviewed 7th graders who had studied the *VisualMath* curriculum's materials on qualitative problem solving. The 7th graders, unlike most of the calculus students, started to represent the problem by exploring iconic sketches that would match the problem conditions. The following sequence of graphs from one interview represents the type of work done by the 7th graders to model and solve the problem:



From left to right: A model that represents the two processes, analysis of the curved process using stairs, identification of a location with an equal rate of change for both processes, and the creation of a single more efficient process.

The different components of the lexical system introduced by the curriculum (As described in Schwartz and Yerushalmy 1995, this system includes: graphical icons, descriptive verbs, and the staircases.) seem to have been adopted by these 7th grade students as manipulable objects that provided them with support when solving problems that were far too complicated for them to describe symbolically (they had only begun to learn algebra!). Using the grammar of objects and the operations on them, students constructed complex mathematical models, based on qualitative analysis of variations using the staircases.

New transitions: When Richard Noss (2001) asks "what is natural anyway?" he suggests that a new “digital culture” may shift our notions of pedagogical order. One of the examples he gives concerns the "closed form" (that we call explicit here) and the 'recursive form' for representing symbolically a function's table of values. Recursive definitions are usually viewed as more complex and are usually studied late in high school. Technology, like the one demonstrated above or like a spreadsheet or graphing software may suggest that the explicit algebraic rule is no longer a more natural way to describe a function (for related thoughts based on student performance see Hershkovitz et al., 2002;Stacey and MacGregor 2000). But, if students become familiar first with recursive forms, explicit forms may ironically become more difficult. Here is an episode from an interview with Yael and Ronit (9th grade VisualMath students) who tried to solve a problem given as a table of numerical data that describes a quadratic process.

Yael and Ronit are trying to answer the question: If the following table describes the post office rates for shipments, how much a 54 kg parcel would cost?

They solve the problem by developing the rule of squaring the weight at the end of a

Weight in kg	Price in marks
From 0 to 1	1
From 1 to 2	4
From 2 to 3	9
From 3 to 4	16
From 4 to 5	25

given range. And, they developed this rule quickly in line 3. But, they express disappointment at not being able to connect idea of constantly increasing differences that they identified with the explicit rule they have found (6-9). The explicit rule gave the answer but it did not connect with the recursive thinking that provided an explanation for how the cost grew. Earlier experience with recursive forms complicated their use of explicit forms. This complication might not have arisen if they had not had earlier support for recursive reasoning.

1. Yael: … we *(already)* know that it increases by 2 each time. 5, then 7, then 9. That's not the purpose. I know there is a way, because it's not some... We can also according to the graph. Increasing constantly. As if it would not be constant.

2. Ronit: If I'm looking at the table of values, I see...From weight 6 to 7 I have 49.
3. So I want to propose a connection between the two. I see that 49 divided by the larger number yields the larger number. I'm squaring.
4. The large number between two numbers and I have the price. It should be 54.
5. It would be 2916 the price of this package. Or it would be more. Or less.
6. Yael: We are raising weird ideas.
7. Interviewer: Why do you feel this way?
8. Yael: That's my feeling. You asked what is our feeling. That's my feeling. I need something to feel sure there.
9. Ronit: That's it. I don't feel sure. I want to find the rule. I know that the slope … I'm sure that the slope of my function will be price relative to weight. Y is actually the price and X is the weight. If I do such a stair, I'll have the weight relative to the price or the price relative to the weight.

Example 2: Conceptualizing equations as comparisons of functions

Resources: The lion's share of the early parts of the Visual Math Algebra curriculum focuses on functions of one variable and equations of one variable conceptualized as the comparison of two functions of one variable. With this way of thinking about equations, students acquire alongside the algebraic procedures alternative methods to solve equations. Students are encouraged to use systematic guessing, intuitive numerical and graphical analysis strategies to narrow down the search interval, all of this without the use of symbolic manipulations. Students in many algebra curricula that use linked graphic, tabular, and symbolic representations similarly learn to solve equations in a variety of ways (see for example:(Huntley, Rasmussen et al. 2000; Hershkowitz, Dreyfus et al. 2002).

Relative strengths of students using these resources: Algebra beginners who studied the VisualMath curriculum and whose class had not learned procedures beyond the linear equation were able to analyze and solve correctly the following problem, with *and* without use of graphic software (Yerushalmy and Gilead 1997):

Jo was asked to determine the number of solutions for the following equation:

$$x^2 + 4 = x + 24$$

1. Using the table of values he decided that the equation has one solution.. Explain how he arrived at his conclusion.

2. Was Jo right? Can you suggest how he could check his answer? Explain.

X	*f(x)*	*g(x)*	*f(x)-g(x)*
4	*20*	*28*	*-8*
5	*29*	*29*	*0*
6	*40*	*30*	*10*
7	*53*	*31*	*22*
8	*66*	*30*	*36*

The problem yielded a crop of answers from a group of students who had not learned to solve quadratic equations. Most of their suggested strategies blended numerical and graphical analysis with algebraic symbols and sometimes even included reasoning about procedures. For example, Eli sketched the two functions, marked the intersection points and expressed objection to Jo's domain interval.

"Jo determined the number of solutions by looking at the 0 in the difference. Since 0 difference indicates an intersection point where there is no difference between the two graphs. And since the y values are the same at that x value. Jo was wrong since he did not construct the negative values part of the table. I would suggest that he check the number of solutions using graphs."

As this example illustrates, viewing an equation as a comparison of two functions, students can solve problems for which they have not yet been taught an algorithmic solution method.

New transitions: Elsewhere (Yerushalmy & Chazan 2002), we described at some length why a class teacher wondered whether his students could solve a task involving systems of equations in two variables without being taught a method for solving such systems. We also described why he decided instead to explore how his students would approach a single equation in two unknowns rather than a system of equations. He asked his students: "How would you represent the equation x+y=2x-y?" And, we described three different groups in the class and their correct and faulty suggestions. One group of students decided to view both x and y as independent variables and to view this equation as a comparison of two functions of two variables. They then began to explore three-dimensional representations. Another group viewed y as a parameter that could take on a range of values. For every value of y, a new linear function in x would be generated. Another group began approaching the question by thinking of the equation as a comparison of two functions: f(x,y)= x+y and g(x,y) = 2x-y. By using the difference function and symbolic manipulations, they reduced the question of finding the solution of an equation in two variables, to a question about the zeros of a single function in two variables. In the end, they changed $-x+2y=0$ to $y=1/2x$.

In typical algebra instruction, the transition from equations in one variable to equations in two variables is not a large one. One uses similar solution techniques and learns how to apply them to a wider class of equations. The main change is in the nature of the solution set one might uncover. When viewing equations as comparisons of two functions, this transition seems more substantial. The graphical and tabular representations of functions of two variables must be developed in order to help students see their connections to the ways of representing functions of one variable with which they are familiar.

Discussion

One important way in which mathematics education research findings are used is to help teachers anticipate student difficulties. For example, one might use research by Filloy and Rojano (1989) in this way. They found that students in the classrooms they studied found problems of the form ax+b=cx+d substantially harder than problems of the form ax+b=c, for example. In explaining this phenomenon, they suggest that algebra beginners bring to the classroom experiences of arithmetic. The harder problems frustrate arithmetic solutions and thus form a key transition point, or what they call a "didactical cut." One might imagine that this work would provide a solid basis for teachers' knowledge of student performance in algebra. Yet, assuming that new understanding is build upon prior knowledge and experiences, a key question in mathematics education research is the degree of stability that one might expect to find for such didactical phenomena. On the one had, some researchers seem to feel that some didactical phenomena reflect deep cultural understandings and are thus quite stable over time. From this perspective, understanding difficulties that unfolded historically sheds light on difficulties students may experience in the classroom (the notion of an epistemological obstacle, see Sierpinska 1994 , seems to function in this way). On the other hand, technologists seem to believe that didactical phenomena are more mutable; they depend on the media in which instruction takes place.

In this paper, we have provided examples that suggest that didactical phenomena are mutable. Are phenomena, like Filloy and Rojano's didactical cut approach-independent? Or are they, as suggested by several studies (e.g. Noss 2001, Papert 1996), related to how students learn and to the cognitive tools they have to tackle such problems? Our evidence suggests that didactical cuts are related to the tools students have and are not fixed in stone. The use of new resources can lead students to exhibit strengths not exhibited by students not utilizing such resources. At the same time, use of these resources can lead to new didactical transitions or difficulties. While we might thus seem to be supporting the position of technologists, our analysis of the factors leading to this mutability are not media-based. Although we are interested in the use of technology and in software design, in the examples we have just given, we would emphasize the importance of considering the curricular resources provided to students in these examples. What seems as a contradiction might be explained by our view that in analyzing such examples technology is important, but not as something separate, rather as but one component of the curriculum with which students interact.

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