PROBLEMS ARISING IN TEACHERS' EDUCATION IN THE USE OF DIDACTICAL TOOLS

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The author illustrates, on the basis of her experience, some problems related to the education of future Mathematics teachers in the use of didactical tools, focusing on Geometry teaching and DGS. She analyses the difficulties she encounters in order to achieve her goals and links these difficulties to the different trainee teachers' conceptions of Mathematics. Finally, she poses some issues for further investigation.

§1. WHY USE DIDACTICAL TOOLS?

The Italian Mathematics curricula, from primary school (even from pre-school) up to high school level, *stipulate that the main goals in Mathematics education are the development of pupils' reasoning capacities and the learning of the mathematical method*. The aim is the achievement of a method and of a mentality, rather than of a certain number of notions. The Mathematics curriculum of *primary school* (pupils aged 6-11), dating from 1985, starts with the following declaration: *"Mathematics education contributes to the forming of thought in its various aspects: intuition, imagination, planning, hypothesis making and deduction, testing and therefore verifying or falsifying"*. Similar concepts are expressed in the preliminary part of the *middle school* curriculum (pupils aged 11-14), dating from 1979: "*Mathematical, Chemical, Physical and Life Sciences, with their methods and content, tend to develop logical, abstractive and deductive capacity, and a scientific mentality…"*. Further, the goal of Mathematics education is: *"To lead gradually to verifying the validity of intuitions and conjectures by means of ever more organised argumentation"*. These concepts are confirmed up to *high school level* (pupils aged 14-19), in the last year of which we find the goal: *"to develop proofs in axiomatic systems, proposed or free-constructed"*(Curriculum Brocca, elaborated from 1988 to 1991).

Convinced that didactical tools, *if appropriately used*, can help the teachers to follow these aims[1], I have carried out a number of teaching experiments at various school-level using didactical tools of different type, technological (see Gallopin and Zuccheri, 2002; Borelli, Fioretto, Sgarro and Zuccheri, 2002), or not (see Zuccheri, 1999). In these experiences I have acted directly in interaction with the pupils, or indirectly, co-operating with expert teachers. Through these experiments I have derived the conclusion that a *special* type of competence of the teacher is essential in order to obtain good results in this type of teaching\learning process. This paper attempts to describe the fundamental factors that form this competence and,

[1] As confirmed by many research studies (see e.g. the Proceedings of the meetings CERME1, CERME2, ICTM1, ICTM2).

considering the context of future teachers' education, explores whether it is possible to build such competence.

§2. FORMING TEACHERS' COMPETENCE IN THE USE OF TOOLS

For more than twenty year I have carried out many courses for Mathematics teachers of any level, about the use of computers, pocket calculators and not-technological tools, as semitransparent mirrors. I have introduced a laboratory concerning didactical tools in the university course of Mathematics Education (for Mathematics degree) on which I teach. Further, as the person responsible for the Mathematics-Physics-Computer Science Section of the SSTSS[2] at Trieste University, I have incorporated a laboratory-course about technological tools for Mathematics teaching in the curriculum of future Mathematics teachers of secondary school. I deal, in a part of this course, with the use of dynamical geometry software (DGS).

Carrying out these courses for graduates in various disciplines and for Mathematics university students, I have observed that the subject *"didactical tools"* always excites the interest of all people[3] (whether or not they are teachers in-service or pre-service, or future teachers). Many of them, at the end of the course, will continue learning about such tools themselves, sometimes asking for information about obtaining certain instruments and the relevant documentation.

Despite this positive experience, I am not completely satisfied with the results because the '*message'* I intend to transmit via this work (*that of using the tools to develop the reasoning capacities of the pupils and to facilitating the acquiring of the mathematical method*) is sometimes not completely well-understood and put in practice by those participating in these courses.

In fact, at the end of the course, the participants are generally required to produce as homework a short didactical project in which they should put in practice what they have learned before. Often, some of them do not produce teaching materials that correspond to my expectation, even if they during the lectures seem to understand both levels of explanation that I provide (*one about the use of the tool itself and the other about the various strategies of use of the same tool for Mathematics teaching*). They seem to give attention only to partial aspects of the didactical use of the software, or to technical aspects, or to enjoying the use of the tool itself.

I have reflected many times about the reasons for this. My conclusion is that the fundamental factors, rather than a lack of knowledge of Mathematics (and, possibly, of Computer Science), are both their low interiorization of the mathematical method and their *conceptions*[4] of Mathematics. This last aspect, as stressed in many research

[2] Secondary School Teaching Specialisation School: it is a two-years post-graduated university course for teachers qualification.

[3] This is confirmed in other experiences, e.g. [Ponte and Oliveira, 2002].

[4] For the notion of *belief* and *conception* we refer here to [Thompson 1992]. In particular, we adopt the following definition referring to teachers' conception (of

works[5], also influences their *beliefs and conceptions* of the way in which Mathematics should be presented to pupils.

This conclusion comes from the analysis of the results I obtained up to now. This is mainly based on observation of students' behaviour during the laboratory lessons and on didactical projects produced by them as homework.

To better explain my point of view, I will distinguish the case of trainee teachers and future teachers with a good mathematical background from those with a lower mathematical background.

§3. THE CASE OF GOOD MATHEMATICAL BACKGROUND

Let we consider first the case of future teachers and pre-service teachers with a good mathematical knowledge: Mathematics university students and graduated in Mathematics, Physics or Engineering.

I say at first that the number of this type of students participating to the courses I mentioned before is low (10 at most, for any year of course). So I can usually form a personal opinion about these persons (dividing them in two different types I will explain in the following) observing them and working with them in many occasions not involving the use of DGS. In fact, I teach them also in the courses of Didactics and History of Mathematics, I examine them almost one time during any year (*it is relevant to observe that a part of this examination consists in a lesson performed by the student*); in some cases I have been also the proposer of their doctoral dissertation in Mathematics.

I state beforehand that there are maybe three different didactical uses of DGS those require competence levels of increasing difficulty for the teacher:

A) To realise geometrical constructions by giving instructions to the pupils (this can be done in a more or less critical way, requiring different mathematical competence).

B) To show geometrical concepts, relations or properties dynamically (this requires mathematical competence, but also a certain creative ability in planning a clear, well-ordering and interesting visual presentation).

C) To stimulate the pupils to investigate problems and produce conjectures, possibly in autonomous way (this requires creativity and ability in investigating, solving

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Mathematics): "... *a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like*" ([Thompson 1992], p. 130).

[5] For an overview see [Thompson 1992]. Further, as stressed in Boero, Dapueto, Parenti (1996), the problem of teachers' education is very complex and can be examined by numerous points of view. A special volume of CERME1 Proceedings is devoted to it, analysing the various factors, among them the teachers' beliefs and conceptions (chap.2, see [Ponte, 1999]), which can affect it.

and posing problems).

In the courses about DGS carried out for students with a good mathematical background, I have achieved what I judge the best results with persons who have not only the capacity to easily understand and reproduce complicated formal proofs of theorems, but are also able to investigate, solve and pose problems. Such competencies are the same as those a Mathematics researcher needs. This type of students, which I would categorise as "*creative* (or *constructivist*) *students*", easily (perhaps even, *spontaneously*) learns to plan didactical projects up at level C. In contrast, what I would categorise as *"formalist students"*, among them some graduates with high marks, having a static idea of Mathematics and rigour, have reached only levels A and, at most, B. Further, I observed that this "*formalist students*" are inclined to separate the lesson *"of Mathematics"* from that *"of Computer Science"* and that their projected laboratory lessons with DGS are merely an appendix of the theoretical one. In this lessons they only reproduce the theory that they suppose to have explained before, in a preliminary traditional lesson without computer.

§4. THE CASE OF LOWER MATHEMATICAL BACKGROUND

If the future teachers have a lower mathematical background, the situation is more complicated. To illustrate it, I will analyse more in detail in this section the results obtained with a group of future middle school teachers who have degrees in Life Science or Earth Science. The consideration of this case is important because the majority of middle-school Italian Mathematics teachers, actually, are graduates in Life Sciences. In fact, for the qualification in Mathematics teaching at middle school level (11-14 years old pupils), beyond graduates in Mathematics or Physics, graduates in Life Science, Earth Science, Chemistry and others are also admitted. All these degrees are obtained attending university courses of almost 4 years, ending with the presentation of a thesis. Unfortunately, the percentage of Mathematics in the traditional study curricula for the Life Sciences degree at University is low: there is generally only a one-year course of Mathematics. Further, *the way in which Mathematics is treated in this course is instrumental*, addressed to a future Life Scientist and not to a future teacher. The same is true for the degree in Earth Science. Further, there is no specific course of Computer Science in these traditional curricula[6].

In 2001, a group of 14 Life and Earth Science graduates attended a laboratory-course of the SSTSS in which I treated the DGS, using *Cabri Géomètre II*. I stress the fact that four of the graduates were attending this course for the first time; *the others ten for the second time*, being in the second year of SSTSS (this laboratory-course must be iterated). I briefly describe in the following the structure, the contents and the methodology of the course, for better explaining my expectation about the results.

[6] This refers to graduates before the recent reform.

The laboratory-course consisted of 5 sessions, each of 3 hours. During the first 3 sessions I explained the fundamental commands and tools of *Cabri* and the use of macro-constructions. The method I used was to teach them, realising, at the same time, geometrical constructions together with the students (they had at their disposal a computer between any two persons, I used a computer connected by a video-projector). *Some constructions were realised by a problem-solving method, others by illustrating*[7] *them.* In the (collective) problem solving sessions I guided the discussion giving hints, explaining auxiliary constructions and proving them, writing on the white-board the conjectures, the attempts of proof, the partial results and, finally, the proof of the solution. At the end, I also stressed the characteristics, from didactical point of view, of the work we had made together. We used only elementary Geometry. The Science graduates participated actively to the lessons; *many of them showed that they are able to perform deductions and to support them with valid arguments*.

During the whole of the fourth session, the graduates had to begin a project to design a Geometry lesson (or more) for a middle school class, about a suitable subject, using *Cabri*. They had to work individually or in pair (as choice), with a computer. During the session I helped them with technical and didactical suggestions. *I explained to one of them, by chance, the use of measure and tables with Cabri II*. Following the session they had to complete the project individually at home, writing it up for submission. In the last session some graduates sketched to the whole class their work in progress. *Accidentally, the graduate to whom I had explained measure and tables, illustrated their use.*

At the end of the course, the Sciences graduates were very enthusiastic, as they said, about the didactical possibilities offered by DGS. They had been very involved during the laboratory activity. Further, by means of my explanations, in which I used the dynamicity of Cabri, some of them had learned new Geometry and wanted to reproduce it in their homework.

They submitted their projects to me after approximately one month. All the projects were written using a word-processor. Some projects were very accurate and reported *Cabri*-figures, others were of very poor quality. I evaluated them taking in account their use of DGS, but I also considered the characteristics of the proposed didactical use of the software, the correctness and the completeness from mathematical point of view.

Reading the projects, I was very perplexed about the Science graduates' interpretation of what I wanted to teach them about DGS. To better understand their interpretations, I accurately analysed the texts, constructing a grid. The main data are reported in the table. In the following I summarise my observations, based on them.

a) About explicit knowledge of Mathematics and mathematical method. Except

[7] It means: I) drawing them at the white-board, II) geometrical proving them, III) realising them using Cabri.

for one project, *the main feature was that they failed to contain any geometrical proofs or argumentation even though during the lessons we spent a lot of time justifying any result*. Overall, the projects were quite poor in terms of mathematical content: generally there was reported only the title of any subject, without explanation. Three of the reports also contained some things that were incorrect. Further, only six graduates had correctly stated the mathematical prerequisites and four had confused some prerequisites with some objectives of learning.

b) **About previous teaching experience at middle school.** *Nine Life-Sciences graduates had previous autonomous experience in Mathematics teaching at middle school*. Further, the trainee teachers in the second year of SSISS (10) had already made a *pre-service period* nearby teachers in service.

c) **About competence in using Cabri.** *All the graduates proved that they had acquired some competence in using Cabri,* by reporting *Cabri*-figure in the text, or writing correctly (or sketching) a sequence of instructions, or appropriately mentioning the use of some tools.

d) **About the aims of a lesson by using Cabri.** *Only five recognised as a goal the improvement of the logical ability of the pupils (but only two of these declared that they wanted to require argumentation from the pupils)*. *All recognised as a goal the approach to computers* (7 in general, 7 in using *Cabri* itself). Four stressed the goal of improving manual dexterity (3 of them did not recognize the importance of improving, at the same time, the logical ability). Seven stressed the aim of facilitating the mathematical learning (two of them observed that enjoying using computers can motivate pupils to learn Mathematics).

e) **About the type of use of Cabri.** One graduate proposed a lesson well enough organised with geometrical argumentation, but proposed to use *Cabri* only for drawing the relative figures, without linking DGS with the geometrical reasoning (*auxiliary use*). Five proposed a lesson using *Cabri* for realising geometrical constructions (§3, A), six for showing geometrical facts (§3, B) and two for leading the students to conjecturing some geometrical properties (§3, C).

f) **About the role of the computer lesson.** Eight graduates (among them 5 in the second year of course) proposed at first a lesson without using a computer, and only in second time the laboratory-lesson (one of them supposed a previous lesson of Technical Drawing, made by the teacher of Technical Education). The others proposed a lesson with computer.

g) **About the way to present Mathematics to the pupils.** *Six graduates proposed giving their lessons to the pupils without any justification*. Only one graduate proposed explaining, with geometrical argumentation, the subject to the pupils. *Verification, by measuring angles or segments, was the only argument used by the other seven* for convincing pupils of the correctness of geometrical constructions, for stressing geometrical facts, or for leading to conjecturing. *The use of measures* *or of tables was the second, after dynamicity, among the Cabri tools* used.

Further, as the three emblematic examples that follow illustrate, from a mathematical or didactical point of view the students made incorrect use of measures and tables (the names are referred to Table 1).

Example 1. E has a good competence in using *Cabri*. She describes precisely, step by step, a complicated construction of the regular octagon, given its side, without giving any mathematical justification. At the end, she measures all the sides and, by dragging, modifies the figure in order *to convince* the pupils that really they obtained a regular octagon.

Short comment. *Instead of this, the main mathematical facts to observe here were that the eighth point obtained coincides with the first and that the interior angles are equal. Further, the didactical approach without any reasoning does not really reflect in this project any mathematical learning objective. It should be more fruitful to propose, as problem, the construction of a regular octagon inscribed in a circumference, starting from the (previous known) constructions of the inscribed square and of the axis of a segment.*

Example 2. C has a good competence in using *Cabri*. She describes without any reasoning the classical construction of the angle bisector and *verifies by measuring* that in this way the angle has been divided into two equal parts, dragging further the figure. After constructing the three bisectors in the triangle, she shows their intersection point, constructing it by intersection of only two lines, without any comment about the fact that the third line also passes through the same point. The pupils must drag the figure to observe the positions of the incentre. She finally writes that, in the following, the pupils *"will have to prove with Cabri... that, in an equilateral triangle, the barycentre, the orthocentre, the incentre ... coincide"*.

Short comment. *In this project, beyond the lacking presentation and investigation of the problem, it becomes explicit the confusion of its author between "proof" and "verify".*

Example 3. K has a certain competence in using *Cabri*, but the explanation is confused. She wants to lead the pupils to the conjecture that any side of a triangle is less than the sum of the two others. She constructs a segment AB (horizontal) and constructs in A and B two segments AC' and BC" equal to given others. Reporting them with circles of centres A and B and intersecting the circles, she gets a triangle ABC. She measures the sides and constructs a table of the measures, from which the pupils should get the intuition of the triangular property (she does not suggest calculating any sum). She modifies only the measure of BC.

Short comment. *The choice of dragging BC is not favourable to visual intuition of the property, whereas it could become evident by dragging AB, without measuring. She follows this method because probably she attempts to favour the understanding, focusing only on numerical aspects and not really on geometrical facts.*

In the projects submitted by the Life Science graduated future teachers emerges, in my opinion, *their instrumental view of Mathematics* and what they sincerely think about *the way to teach Mathematics at middle school*. Except for one, following a diffuse trend, it is to teach computing rules and geometrical rules, numerical verifies of rules and so on, even though they, appropriately stimulated during the course, had shown a greater level of competence and knowledge. This is put in evidence also by the fact that one half of them used measures and tables, even though, in practice, I had not dealt with measures and tables during the taught session.

§5. CONCLUSION

I have observed in other occasions the same phenomenon that I have just described about DGS also for other didactical tools. Even if the intention is to educate future teachers *"to use didactical tools for Mathematics teaching"*, some people learn it (as the future teachers with good mathematical background I called before "*creative or constructivist students*"), but some others learn *"to use didactical tools and to teach Mathematics"* (as the future teacher with good mathematical background I called before "*formalist students*"), others only learn *"to use didactical tools"* (as the majority of trainee teachers with lower mathematical background described in §4, *having an instrumental view of Mathematics*). I think that it occurs because the type of use of didactical tools reflects the conceptions of Mathematics of the person that is using them. In fact, the different behaviour I observed in the three types of trainee teachers and future teachers, I described before, agrees, except for some detail, with the classical distinction in three conception of Mathematics (*the problem solving view*, *the Platonist view*, *the instrumentalist view*) coming from history and philosophy of Mathematics and summarised in [Ernest 1988][8].

Therefore, If the problem is to act in order to change beliefs and conceptions, I think that, to overcome it, it is not sufficient to accurately organise longer laboratory courses about DGS: it is necessary to experiment other strategies, more linked for instance to teaching practice, as a training period nearby to a teacher in service which uses correctly this tools. Could be useful also stimulate the interaction among trainee teachers of different competence level and different conception about Mathematics. I think that it could be a good starting point for further investigation, about the questions I posed in this paper, and, more in general, about the problem of teachers' education.

In addition, I observe that the use of didactical tools put clearly in evidence the mathematical and didactical competence of the person that is using them. In fact (as shown in §4, Ex. 1, 2), *the attempts to illustrate, in detail, elementary mathematical concepts and properties laid bare possible misconceptions or incorrectness*. This aspect can be advantageously exploited in the teachers' education process. In fact, requiring the trainee teachers to produce detailed exercises in using didactical tools and examining their production, we can get useful information about their cultural

[8] Quoted in [Thompson 1992, p.132].

background and their conceptions. Successively, analysing and discussing with them errors and misconceptions, both from didactical and mathematical point of view, we can act in order to improve their professionalism.

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Name	Year of course	Previous autonomous teaching experience[1]	Type of use of Cabri[2]	General contents of the didactical project					Aims and goals of the didactical project				Teaching method		Cabri functionality and tools mentioned in the project			
				Cabri figures[3]	Mathematical contents[4]	Proof or argumentation[3]	Explanation of Cabri constr.[3]	Error in prerequisites[5]	To develop Logical abilities[3]	To teach Computer Science[6]	To facility Math. learning[3]	To develop manual dexterity[3]	I -Lesson II- Laboratory[3]	To require argumentation[3]	Reconstruction step by step[3]	Tables or measures[3]	Macro constructions[3]	Dinamicity[3]
A	1	Y	B	N	VF	N	N	O	N	Ca	Y	N	N	N	Y	N	N	N
B	1	N	B	N	VF	N	N	L	Y	G	N	Y	Y	N	N	N	Y	Y
C	1	Y	B	Y	F	N	Y	N	N	Ca	Y	N	Y	N	Y	Y	N	Y
D	1	Y	B	N	VF	N	N	N	N	G	N	Y	Y	N	N	N	N	Y
E	2	Y	A	Y	VF	N	Y	L	Y	G	Y	N	N	N	Y	Y	Y	Y
F	2	Y	A	N	I	N	Y	O	N	G	Y	Y	Y	N	N	N	Y	Y
G	2	YG	B	N	F	N	N	N	N	Ca	N	N	N	N	N	Y	N	Y
H	2	Y	A	N	F	N	Y	N	N	Ca	Y	Y	N	N	Y	Y	N	N
I	2	Y	Aux	N	I	Y	Y	N	Y	Ca	N	N	Y	Y	N	N	N	N
J	2	Y	B	Y	I	N	N	O	Y	G	Y	N	Y	Y	N	Y	N	N
K	2	YG	C	N	F	N	Y	L – S	N	Ca	Y	N	N	N	N	Y	N	Y
L	2	Y	A	N	VF	N	Y	N	N	G	N	N	Y	N	N	N	Y	N
M	2	N	A	Y	VF	N	Y	L	Y	G	N	N	Y	N	N	N	Y	N
N	2	N	C	Y	F	N	Y	L– O	N	Ca	N	N	N	N	N	Y	N	Y

[1] Y = Yes, N = No, YG= Yes, as naturalist guide
[2] A = To constructing, B = To showing, C = To showing for conjecturing, Aux = Auxiliary
[3] Y = Yes, N = No
[4] F = Few, I = Incorrectness, VF = Very few
[5] N = No, L = Lacking prerequisites, S = Superfluous prerequisites, O = Some prerequisites equal to an objective
[6] G = General, Ca = (only to learning) Cabri