THEORY – PRACTICE RELATIONSHIPS IN MATHEMATICS EDUCATION

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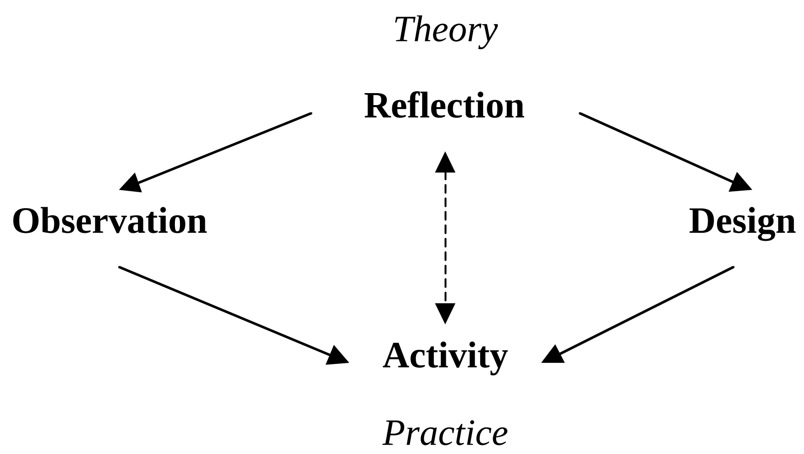
Introduction

The theory-practice relation in mathematics education is multifaceted, and it seems more proper to talk about the relationships between theory and practice than about *the* relation. This is evident when looking at the example I want to focus on here, an example where practice is more visible than its theoretical components – which seems to be commonplace within projects focusing on development of teaching practice. Thus, it is important to identify these components, or rather the kinds of relationships between theory and practice that may be found in this kind of project. Before starting the journey through the example I wish to open up with some introductory reflections, allowing us to make some concluding remarks after the journey.

Why do we need theories? Because we always ask ourselves *why*, as I just did in putting this question and to answer the why we need something to refer to – a theory.[1] To judge the relevance/importance of a theory, we can ask to which *whys* the theory might provide answers. Indeed,

> The dialectic between theory and practice reflects a tension between life as lived and life as experienced (Mason & Waywood, 1996)

This tension may be paralleled by the dialectic between activity and reflection. In educational settings reflections may grow out from observations of teaching activities and generate the (revised) design of teaching activities. Thus, in the research enterprise, as in the field of mathematics education, the following simple model of a theory-practice dialectic (which may be called a *TP-cycle*) may be identified:



[1] As in mathematics – why is the sum of the angles in a triangle180°? To answer we must refer to a theory, i.e. Euclidean geometry.

Each arrow in this diagram denotes a relationship, which in each case is as crucial for the output category as is the input category. This kind of cycle is commonly used for example in action research (see Mason & Waywood, 1996), where the theoretical component of the reflection box may vary in degree.

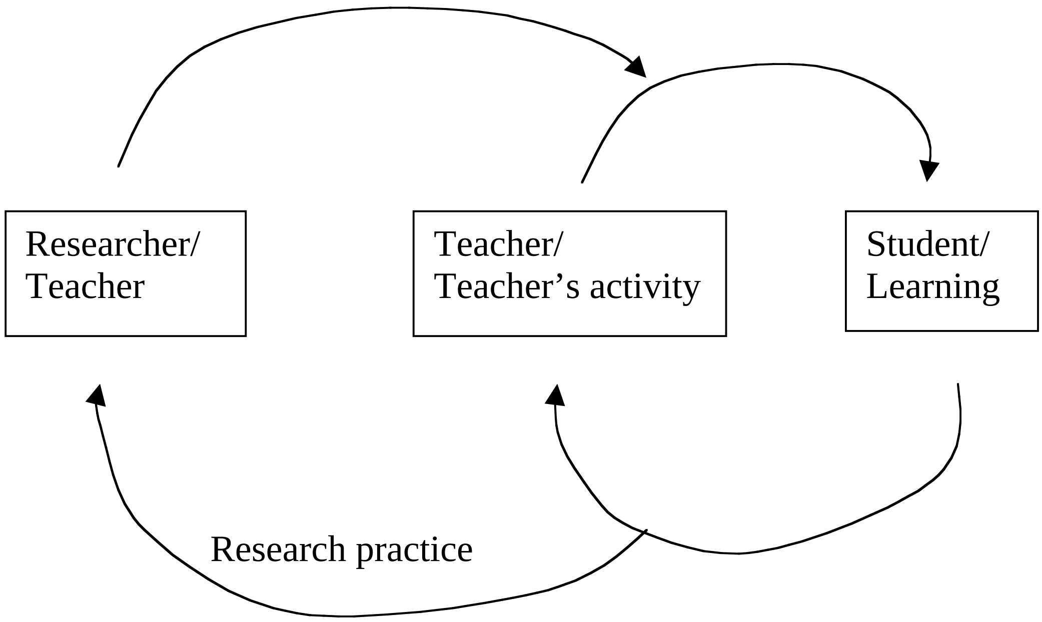
In his overview of international perspectives, Bishop (1992) puts his focus on five critical relationships in research in mathematics education, and also identifies three different "traditions" of enquiry in mathematics education. A similar classification of three traditions is made in Mason and Waywood (1996). Bishop's framework can be visualised by the following 3x5 matrix:

Relationship **Tradition**	What is/ What might be	Mathematics/ Education	Problem/ Research method	Teacher/ Researcher	Researcher/ Educational system
Pedagogue					
Empirical-scientist					
Scholastic-philosopher					

These traditions differ in the way they treat the scientific components of the goal of enquiry, the role of evidence, and the role of theory. For example, in the first of these components, the focus of the pedagogue is to directly improve teaching, the empirical scientist aims at explaining the educational reality as it is observed, while the scholastic-philosopher is trying to establish a "rigorously argued theoretical position" of good teaching. These differences give a different emphasis on the relationship between *what is* and *what might be*. (Bishop 1992) Similar reflections can be made about how the different traditions treat the relationship between (the discipline) mathematics and education (of mathematics), the problem and the research method, and so on.

How these relationships are being conceptualised is linked to the way teaching and learning are being conceptualised. When viewing teaching/learning through the transmission metaphor a linear model of the relation between researcher-teacher-student is obtained, where the researcher "hands over" research results to the teacher just as the teacher "hands over" mathematical knowledge to the students (Steinbring 1994, p. 90).

In a more systemic "modern" view, where the socio-institutional conditions of the partners in the teaching-learning practice and in the research practice are taken into account, a non-hierarchical model of relations between these relatively autonomous practices appear, as in the following diagram in Steinbring (1994, p. 91):



It is partly a question of control:

> Didactical science has no direct possibility of controlling the everyday practice of the mathematics teacher, and the teacher has no straightforward possibility of controlling the students' process of either learning or comprehension. (Steinbring 1994, p. 91)

In the example to follow, three more general key aspects of the relationship theory-practice come into focus, each referring to answers to quite different questions:

- *genesis* (of theory from practice)
- *validation* (of theory by practice)
- *design* (of educational practice from theory)

A project focused on developing practice, with a low level of theoretical input, may well in the reflective phase have important theoretical pay out. This, of course, is one source of formulating relevant research questions. Design of educational practice may be viewed as the reverse process – what practice grows out from this theory? This relationship may well be at the same time the most crucial and the least obvious, while the validation category is more open to critique using scientific criteria.

The example I now want to offer as one input to discuss the theory-practice relationships may be considered as an activity on which to reflect.

Activity: The A-course project

In Sweden compulsory school covers nine years but almost all students go to upper secondary school (grades 10-12), which (presently) is organised into 16 study programmes, by a national curriculum, with different emphasis on theoretical studies or practical training. The subjects are organised in courses that cut across the programmes. In mathematics there is a progression of courses A, B, C, D and E, where for example all students of the science programme do A-D (D is on trigonometric functions and calculus), but many of the study programmes require only course A[2]. All students must do course A, with the same national assessment test for all. Now, in many of the less theoretical study programmes a high percentage of students don't pass the national test, having low interest, motivation, and results. These kinds of problems provided the background and impetus for the *A-course project* presented here. At six upper secondary schools in west Sweden mathematics teachers, communities, the university of the region, and the national school board[3] all engaged in a project to develop and try out new working formats for the A-course, aiming at increasing students' self confidence and interest in mathematics. There were 20 teachers from 6 schools involved in the project, with two project leaders from the university.[4] It was planned as a two year project with a specific aim to keep continuous contact between the teachers and between the teachers and the project leaders.

During the first semester of the project objectives were identified based on a description of the current situation. The next task was to find educational activities for the course project, based on ideas of the integration of mathematics and other subjects, and that were highlighting the use of mathematics. Group work, rich problems, laboratory activities, and learning logs were the main new formats chosen. This work was facilitated by reading literature, as suggested by the project leaders, and by joint seminars with visiting expert speakers. The students in the project classes also made an algebra test for diagnostic purposes[5]. Finally each school chose its "local" project, fitting to the needs, aims and interests identified by the teachers at the particular school.

At the beginning of the second semester the local projects started up. Continuous contacts and discussions were accomplished by using the internet service called First Class (FC). Project leaders visited all schools for longer discussions on specific local issues and reported back on FC. There were also joint meetings with expert speakers.

During the third semester the local projects continued and finished. The project leaders visited schools, now for discussions with the teachers as well as students. The FC communication was upheld as before, with reports of discussions from the visits. On a joint meeting with a visiting expert speaker the focus was now on assessment.

At one of the participating schools a lot of work had been done to prepare laboratory mathematics teaching, and there was a wish by all the participating teachers to have a

[2] For the Swedish school system, see web page: www.skolverket.se/english/system/index.shtml
[3] Skolverket, see www.skolverket.se
[4] See the report (in Swedish) by the project leaders, Löfwall, S. & Löthman, A. (2000).
[5] This test was developed within the Norwegian KIM project (see e.g. Brekke 1996).

joint meeting at this “exemplary” school. This was then organised during the fourth semester and led to a discussion on why the specific ways of working were used (list of benefits). Teachers now also completed an overall project evaluation, with the following main comments on the project: *Teachers have changed their views on the teaching of mathematics; Teachers will continue to use group work and labs; The “exemplary” school visit gave good input; It was positive with an “open” project; More time was needed during project work; The informal contacts was the most valuable outcome*.

The teachers also completed and submitted their local project reports. From these a number of “discoveries” could be identified, indicating some reflecting parts of the activities: *With an increased student responsibility an increased importance of assessment/evaluation became necessary; The new ways of working must be used continuously for students to learn how to learn this way; Reflective phase of lab activities important; The method of variation of presenting parts of mathematics has a positive effect on student learning (on computers); The rich problems created a relaxed and nice working climate (as important as good results); Group work produced high level of activity; Our project was most beneficial for weak students.*

In the report from the project leaders one interesting “discovery” was that most of the teachers stressed that it was the fact that mathematics could be used in practical situations and other subjects that motivate students – but student interviews showed that the crucial thing was that you understand, not what it is about. In their final report (Löfwall & Löthman, 2000), the project leaders made the following major conclusions: *Qualitative change in teaching was observed: a move away from traditional tasks towards open rich problems and labs; Assessment became an important issue; The project contributed to the pedagogical discussion at schools; Continuous communication necessary (FC); Theoretical and practical aspects balanced well, as did time.*

In the teachers’ reports only very few theoretical considerations or conceptions were visible (teachers’ own written words): *Group work was organised according to the principles suggested by the project leaders; This way of working develops not only the math knowledge but also the students’ ability to communicate and collaborate (this was related to the text of the national curriculum); The result was not what we had expected (on learning logs); The project leaders and the visiting speakers inspired me to do more practical group work, for the benefit of more learning styles; Last year we worked with applications but not as well structured as this time…was the main contribution to the improved results; The ideas from the project will go on living.*

Reflections

The A-course project can be classified as a problem-driven developmental project: by starting from *what is* it explores into *what might be*. In the classification scheme by Bishop (1992) this project belongs to the pedagogue tradition. Two comments seem appropriate here. One concerns the before mentioned relative autonomy of the different practices of teaching and research. In the teachers' reports the theoretical component is almost invisible, the comments being focused on within practice issues. The theoretical inputs into the project, provided mainly by the project leaders and visiting expert speakers, seem to have been "translated" to the language of practice:

> Theories need to be personalised and made concrete by the teacher to be of use in practice (Blomhøj et al 2002, p. 52)

The apparent invisibility of theory in the teachers' reports may reflect that this kind of personalisation was in play. A remark in the same vein was made by Piet Verstappen at ICME6:

> Theory is characterised by reflection, and practice by action. But theory cannot guide practice like a map. Theory can be useful only as far as it becomes a part of the reality of the teacher (in Hirst & Hirst 1988, p. 385)

The other comment concerns the design of the developmental (or research) enterprise (in this example) as a whole. When the key issue is to investigate what might be, the intriguing question by Skovsmose (2003, p. 210, my translation) is highly relevant: *What does it mean to research something that doesn't exist?* For this kind of project an imagined (better) situation is outlined, by using what Skovsmose calls *pedagogical fantasy*, from the current situation. Since that (imagined) situation is not available in its full scope a situation must be arranged that holds at least some of the imagined features. This situation can be researched by pedagogical experimentation. By the results the arranged situation can be used as a window to explore, by using critical reasoning (in Danish *udforskende analyse*), the imagined situation. This triangular design Skovsmose calls *pedagogisk udforskning* (in English he uses the term *critical research*; Skovsmose 2003, p. 216), and it encompasses different degrees of scientific level, making it possible to erase the apparent sharp borders between developmental and scientific work in education. The A-course project seems to fit well into this conceptualisation, which, to make it possible, asks for a full cooperation between the different actors in the developmental/research process. In the A-course project this is underlined by the comments of the project leaders, and by the teachers, that the continuous communication was a necessary component for the success of the project. The description above of the outcome shows the potentials and benefits inherent in such a project of developing and strengthening the relationships between theory and practice, as outlined above.

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